Name: SOLUTIONS

Read each question carefully before answering. Answer all parts. Show all work, calculations, and/or reasoning, otherwise no points will be awarded. K-maps may be used to double check your work, but may NOT be used as your actual work. Point values are as indicated.

1. (5 points) Convert 110101 from base 3 to base 2.

① CONVERT TO DECIMAL

$110101_3 =$

$(1\times3^5)+(1\times3^4)+(0\times3^3)+(1\times3^2)+(0\times3^1)+(1\times$

$= 243+81+0+9+0+1 = 334_{10}$

② CONVERT TO BINARY

2 | 334
2 | 167 r0 LSB
83 r1

2 | 83
2 | 41 r1
20 r1

2 | 20
2 | 10 r0
2 | 5 r0
2 r1

2 | 2
2 | 1 r0
0 r1 MSB

101001110_2

2. (5 points) Convert 16.34 from decimal to **unsigned** binary. Use four decimal places to express the non-integer component.

2 | 16
2 | 8 r0 LSB
2 | 4 r0
2 | 2 r0
2 | 1 r0
0 r1 MSB

0.34 × 2 = 0.68
0.68 × 2 = 1.36
0.36 × 2 = 0.72
0.72 × 2 = 1.44

10000.0101

SIGN 16 8 4 2 1

3. Express the following as 6-bit signed binary numbers and add them, using **2's complement** for negative numbers. Indicate if there is an overflow in any of the answers, and, if so, how you know there was an overflow situation.

(a) (5 points) $30_{10} + 13_{10}$

```
  1 1 1
  0 1 1 1 1 0
+ 0 0 1 1 0 1
-------------
  1 0 1 0 1 1
```

ADDING 2 POSITIVE NUMBERS GAVE A NEGATIVE ANSWER

OVERFLOW

(b) (5 points) $(-32)_{10} + 9_{10}$

```
  1 0 0 0 0 0
+ 0 0 1 0 0 1
-------------
  1 0 1 0 0 1
```

"SANITY CHECK"

$-32 + 9 = -23_{10}$

```
  101001
   - 1
--------
  101000  → 010111 = 23
```

1 0 1 0 0 1

$+4 = 0\,0\,0\,1\,0\,0 \rightarrow 1\,1\,1\,0\,1\,1$

$+\,1$

$1\,1\,1\,1\,0\,0$

4. Express the following as 6-bit signed binary numbers and multiply them, using **2's complement** for negative numbers. Indicate if there is an overflow in any of the answers, and, if so, how you know there was an overflow situation.

   (a) (5 points) $2_{10} \times (-4)_{10}$

$1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,0\,0$

$\times\ 0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,1\,0$

$1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,0\,0\,X$

NO OVERFLOW

$1\,1\,1\,1\,1\,1\,1\,1\,1\,0\,0\,0$

   (b) (5 points) $(-10)_{10} \times 4_{10}$

$+10 = 0\,0\,1\,0\,1\,0 \rightarrow 1\,1\,0\,1\,0\,1$

$+\quad 1$

$1\,1\,0\,1\,1\,0$

$1\,1\,1\,1\,1\,1\,1\,1\,0\,1\,1\,0$

$\times\ 0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,1\,0\,0$

$1\,1\,1\,1\,1\,1\,0\,1\,1\,0\,X\,X$

OVERFLOW BECAUSE O-FLOW BITS DON'T MATCH SIGN BIT!

$1\,1\,1\,1\,1\,1\,0\,1\,1\,0\,0\,0$

5. (10 points) Express the following minterm expression as a truth table, a minimized Boolean expression, and draw the corresponding minimized circuit diagram.

$$F(A, B, C) = \Sigma m(0, 1, 5, 6)$$

A	B	C	F
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

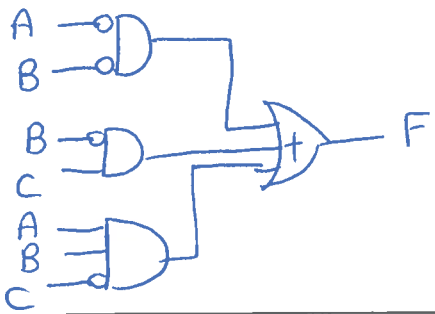
$F = A'B'C' + A'B'C + AB'C + ABC'$

UNITING

$A'B' + AB'C + ABC'$

SOP

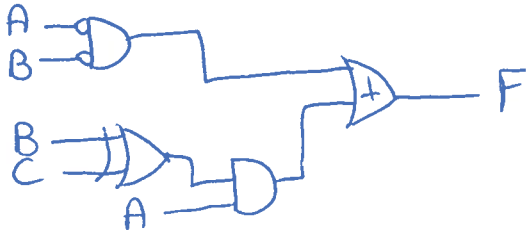
$A'B' + B'C + ABC'$



NON-SOP

$A'B' + A(B'C + BC')$

$A'B' + A(B \oplus C)$



6. (10 points) Determine if the following expressions are equivalent. Explain your reasoning.

$$F = AC + A'C' + BC + A'B$$
$$G = (A' + B + C)(A' + B' + C)(A + B + C')$$

UNITING G

METHOD 1: MINIMIZE G

$(A' + C)(A + B + C')$

$A'B + A'C' + AC + BC$

THESE ARE IDENTICAL— THE EXPRESSIO ARE THESAME.

METHOD 2: TRUTH TABLE

A B C	F	G
0 0 0	1	1
0 0 1	0	0
0 1 0	1	1
0 1 1	1	1
1 0 0	0	0
1 0 1	1	1
1 1 0	0	0
1 1 1	1	1

THE COLUMNS ARE IDENTICAL— THE EXPRESSIONS ARE EQUIVALENT

7. (10 points) Express the following minterm expression as a minimum **sum of products** expression. For each don't care term, indicate whether or not it has been included in the final expression.

$$F(A,B,C,D) = \Sigma m(0,4,7,13) + \Sigma d(2,5,10,15)$$

0. 0000 $A'B'C'D'$
4. 0100 $A'BC'D'$
7. 0111 $A'BCD$
13. 1101 $ABC'D$

2. 0010 $A'B'CD'$ -NO.
5. 0101 $A'BCD'$ -YES!
10. 1010 $AB'CD'$ -NO
15. 1111 $ABCD$ -YES!

START W/ OBLIGATORY TERMS

$A'B'C'D' + A'BC'D' + A'BCD + ABC'D$

UNITING

$A'C'D' + A'BCD + ABC'D$

UNITING

$A'C'D' + A'BC + ABC'D$

UNITING

$A'C'D' + A'BC + ABC$

$A'C'D' + BC$

8. (10 points) Express the following maxterm expression as a minimum **product of sums** expression. For each don't care term, indicate whether or not it has been included in the final expression.

$$F(A,B,C,D) = \Pi M(1,2,14)\Pi D(0,3,8,15)$$

1. 0001 $(A+B+C+D')$
2. 0010 $(A+B+C'+D)$
14. 1110 $(A'+B'+C'+D)$

0. 0000 $(A+B+C+D)$ -YES!
3. 0011 $(A+B+C'+D')$ -YES!
8. 1000 $(A'+B+C+D)$ -NO
15. 1111 $(A'+B'+C'+D')$ -YES!

$(A+B+C+D')\,(A+B+C'+D)\,(A'+B'+C'+C$

UNITING

$(A+B+C)\,(A+B+C'+D)\,(A'+B'+C'+D)$

UNITING

$(A+B+C)\,(A+B+C')\,(A'+B'+C'+D)$

$(A+B)\,(A'+B'+C'+D)$

UNITING

$(A+B)\,(A'+B'+C')$

9. (10 points) Convert the equation $F_{POS} = (B' + C' + D)(A' + B + C')(C + D)(A + B + C)$ to a reduced **sum of products** expression.

USE 2ND DISTRIB. LAW

$(D + C(B'+C'))(B + (A'+C')(A+C))$

$(D + B'C)(B + A'C + AC')$

$BD + A'CD + AC'D + A'B'C$ $A'CD$ = CONSENSUS TERM

$F_{SOP} = BD + AC'D + A'B'C$

10. (10 points) Convert the equation $F_{SOP} = BC + C'D' + A'B'C'$ to a reduced **product of sums** expression.

DOUBLE DEMORGAN

$F' = (B'+C')(C+D)(A+B+C)$

$(B'C + B'D + C'D)(A+B+C)$

$AB'C + B'C + AB'D + B'CD + AC'D + BC'D$

ABSORPTION

$B'C + AB'D + AC'D + BC'D$ $AC'D$ = CONSENSUS TERM

$F' = B'C + AB'D + BC'D$

$F_{POS} = (B+C')(A'+B+D')(B'+C+D')$

11. (10 points) You receive a 5-bit signed binary number $ABCDE$ corresponding to the temperature of a freezer in degrees Celsius. If the temperature rises above $-14°C$, the compressor should turn on. If the temperature rises above $-5°C$, a warning light should turn on. Write equations for both outputs as a function of the binary number input.

ABCDE is signed so C & W will be 1 any time A=0. Therefore an A′ term will be present in both expressions.

ABCDE	T(°C)	C	W
10000	-16	0	0
10001	-15	0	0
10010	-14	0	0
10011	-13	1	0
10100	-12	1	0
10101	-11	1	0
10110	-10	1	0
10111	-9	1	0
11000	-8	1	0
11001	-7	1	0
11010	-6	1	0
11011	-5	1	0
11100	-4	1	1
11101	-3	1	1
11110	-2	1	1
11111	-1	1	1

WARNING LIGHT W:

Every time W=1, A=1 B=1 C=1

Therefore W = A′ + ABC

elimination

$$W = A' + BC$$

COMPRESSOR C:

Every time C=1,

AB , AB′C , AB′C′DE

$C = A' + AB + AB'C + AB'C'DE$

ALL A TERMS DROP OUT FROM ELIMINATION

$C = A' + B + B'C + B'C'DE$

ALL B′ TERMS DROP OUT

$C = A' + B + C + C'DE$

C′ DROPS OUT

$$C = A' + B + C + DE$$