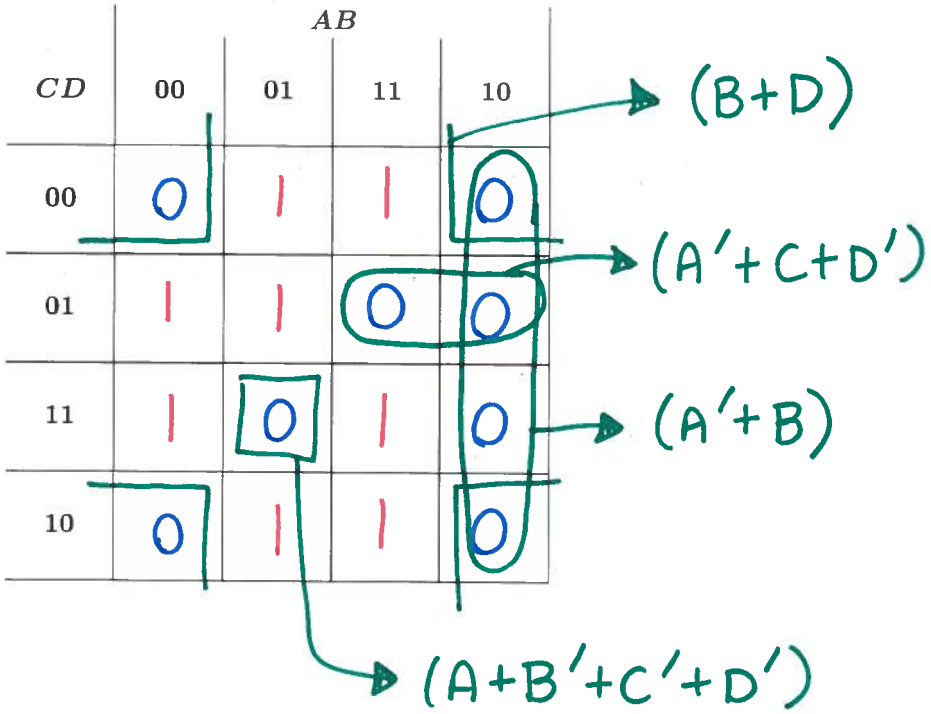
Name: SOLUTIONS

Read each question carefully before answering. Answer all parts. Show all work, calculations, and/or reasoning, otherwise no points will be awarded. Properly labeled loops **must be shown** on K-maps to receive credit. Assume that you have access to gates with as many inputs as you need. Point values are as indicated. Usage of XOR and XNOR gates is **not allowed** on this exam!

1. (10 points) Use a K-map to find the **minimum POS** implementation for the following expression. Label all loops or no credit will be awarded.

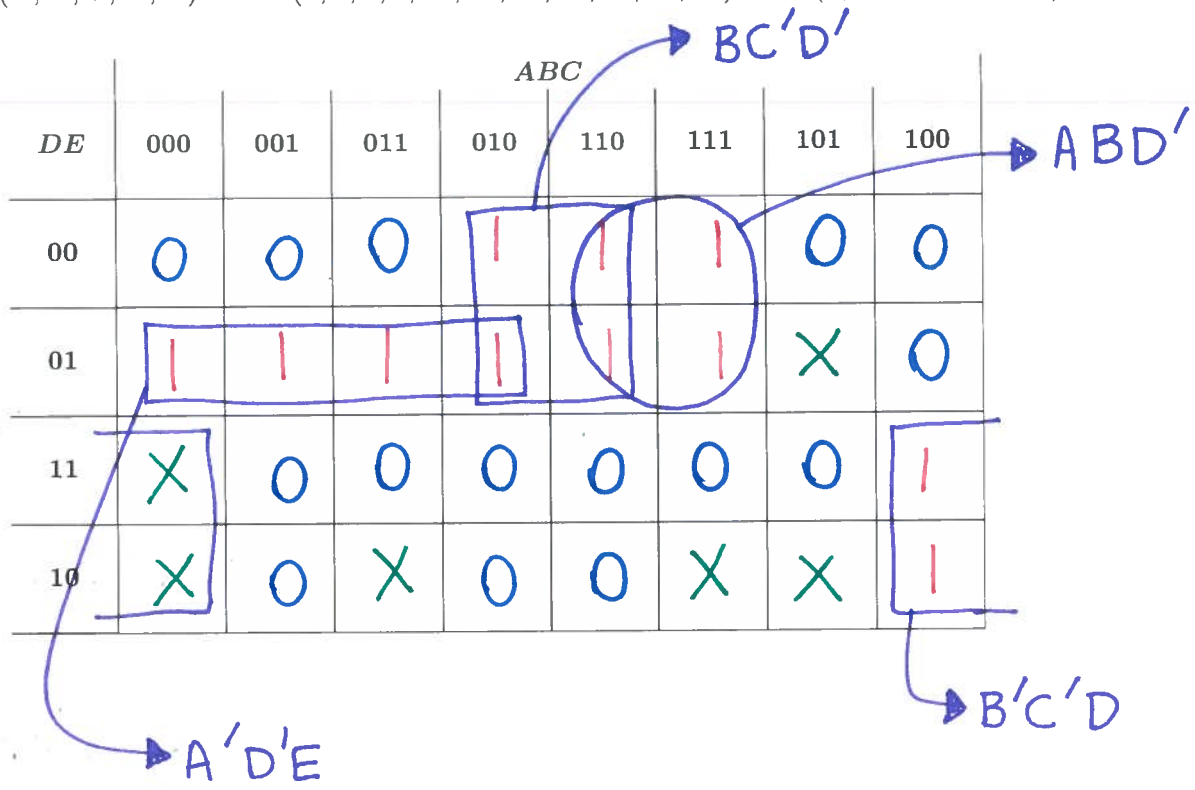
$$F(A, B, C, D) = \Sigma m(1, 3, 4, 5, 6, 12, 14, 15)$$




$F_{POS}$= (B+D)(A'+C+D')(A'+B)(A+B'+C'+D')

2. (10 points) Use a K-map to find the **minimum SOP** implementation for the following expression. Label all loops or no credit will be awarded.

$$F(A,B,C,D,E) = \Sigma m(1,5,8,9,13,18,19,24,25,28,29) + \Sigma d(2,3,14,21,22,30)$$




$F_{SOP} =$ A'D'E + BC'D' + ABD' + B'C'D

3. (15 points) Use the Quine-McCluskey method to find a **static-hazard free** implementation of the following expression.

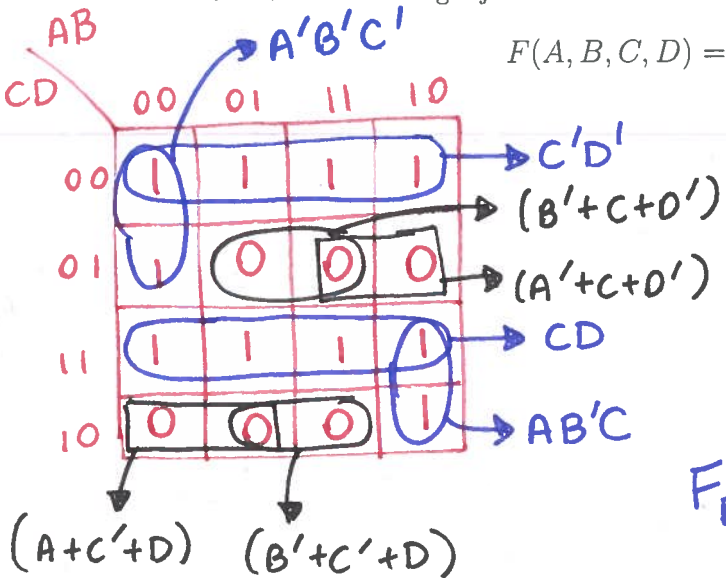
$$F(A, B, C, D, E) = \Sigma m(5, 12, 21, 23, 24, 28, 29, 31) + \Sigma d(6, 7, 14, 15)$$

| Column 1 | Column 2 | Column 3 |
|---|---|---|
| TWO ✓5. 00101 | ✓5-7 001-1 | 5-7-21-23 -01-1 |
| ✓6. 00110 | ✓5-21 -0101 | ~~5-21-7-23~~ repeat |
| ✓12. 01100 | ✓6-7 0011- | 6-7-14-15 0-11- ← ONLY DON'T CARE TERMS |
| ✓24. 11000 | ✓6-14 0-110 | ~~6-14-7-15~~ repeat |
| THREE ✓7. 00111 | 12-14 011-0 | 7-15-23-31 --111 |
| ✓14. 01110 | 12-28 -1100 | ~~7-23-15-31~~ repeat |
| ✓21. 10101 | 24-28 11-00 | 21-23-29-31 1-1-1 |
| ✓28. 11100 | ✓7-15 0-111 | ~~21-29-23-31~~ repeat |
| FOUR ✓15. 01111 | ✓7-23 -0111 | |
| ✓23. 10111 | ✓14-15 0111- | |
| ✓29. 11101 | ✓21-23 101-1 | |
| FIVE ✓31. 11111 | ✓21-29 1-101 | |
| PIs | 28-29 1110- | |
| | ✓15-31 -1111 | |
| | ✓23-31 1-111 | |
| | ✓29-31 111-1 | |

$F_{HAZARD\text{-}FREE}$ = A'BCE' + BCD'E' + ABD'E' + ABCD' + B'CE + CDE + ACE

4. (20 points) Find the lowest cost implementation of the following expression. Draw the circuit diagram of the minimum-cost circuit. Use any minimization method of your choice, showing all work and making a justification for the best implementation.

$$F(A,B,C,D) = \Pi M(2,5,6,9,13,14)$$




$F_{SOP} = C'D' + A'B'C' + CD + AB'C$

5 GATES
14 INPUTS

$F_{FSOP} = C'(D' + A'B') + C(D + AB')$

7 GATES
14 INPUTS

OR

$C'D' + CD + B'(A'C' + AC)$

6 GATES
14 INPUTS

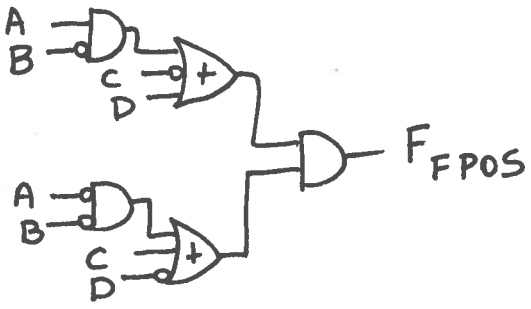
$F_{POS} = (A+C'+D)(B'+C'+D)(A'+C+D')(B'+C+D')$

5 GATES
16 INPUTS

$F_{FPOS} = (D + C' + AB')(C + D' + A'B')$

5 GATES
12 INPUTS

LOWEST COST CIRCUIT



5. Find the optimized implementation of the following two circuits. Show all work. How many gates and/or inputs do you save by implementing circuits together rather than individually?

$$X(A, B, C, D) = \Sigma m(2, 3, 4, 6, 7, 10, 12)$$

$$Y(A, B, C, D) = \Sigma m(4, 6, 7, 10, 12, 14, 15)$$

| X | AB | | | |
|---|---|---|---|---|
| CD | 00 | 01 | 11 | 10 |
| 00 | 0 | 1 | 1 | 0 |
| 01 | 0 | 0 | 0 | 0 |
| 11 | 1 | 1 | 0 | 0 |
| 10 | 1 | 1 | 0 | 1 |

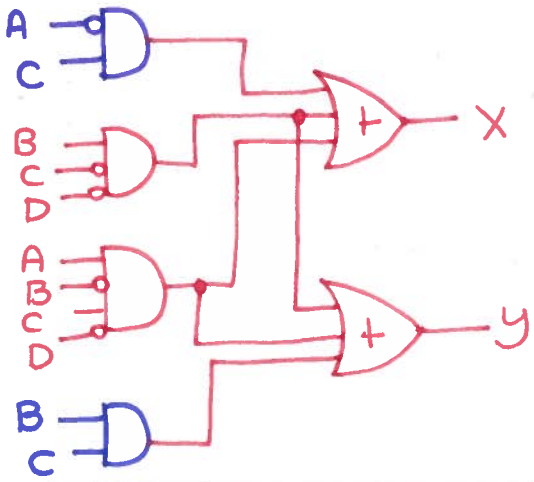
| Y | AB | | | |
|---|---|---|---|---|
| CD | 00 | 01 | 11 | 10 |
| 00 | 0 | 1 | 1 | 0 |
| 01 | 0 | 0 | 0 | 0 |
| 11 | 0 | 1 | 1 | 0 |
| 10 | 0 | 1 | 1 | 1 |

BC'D'

A'C

AB'CD'

BC




6 GATES
17 INPUTS

THIS SAVES
2 GATES &
4 INPUTS!

INDIVIDUALLY

X = A'C + BC'D' + B'CD'

y = BC + BD' + ACD'

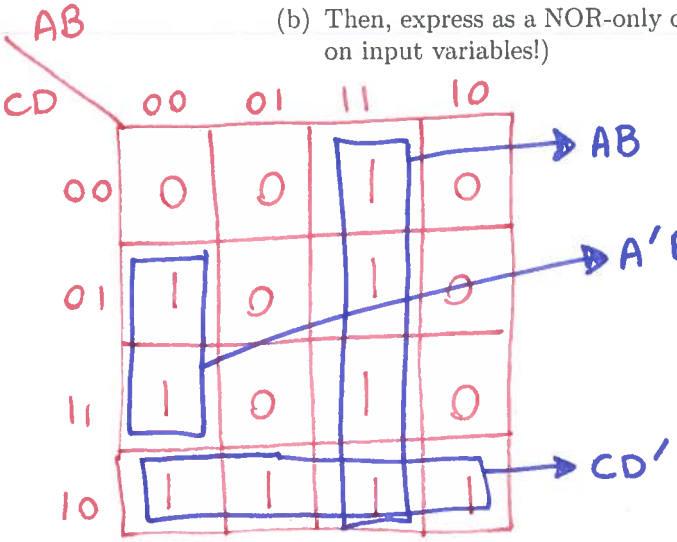
NO SHARED GATES

6. (15 points) Find the **minimum SOP** implementation of the following expression.

$$F(A, B, C, D) = \Sigma m(1, 2, 3, 6, 10, 12, 13, 14, 15)$$

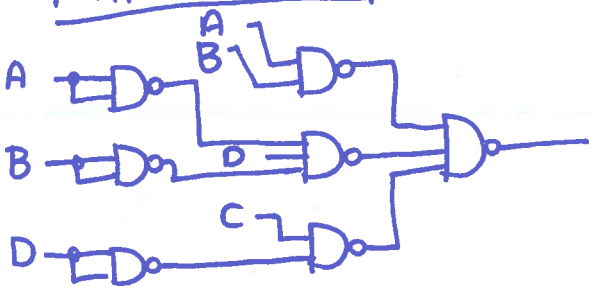
(a) Then, express as a NAND-only circuit (no inverters are allowed, not even bubbles or primes on input variables!)

(b) Then, express as a NOR-only circuit (no inverters are allowed, not even bubbles or primes on input variables!)

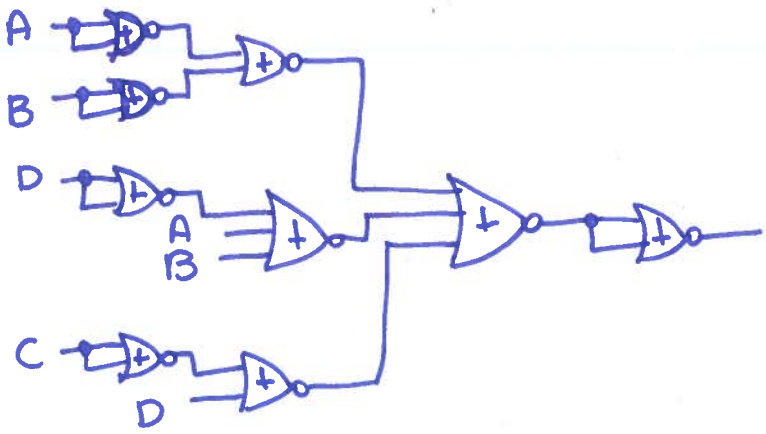



$F_{SOP} = AB + A'B'D + CD'$

NAND-ONLY




NOR-ONLY



7. (10 points) Draw a timing diagram for the following circuit, given gate delays of 2 ns for NOT gates, and 5 ns for AND and OR gates. Indicate any static hazards in the output signal