Name: SOLUTIONS

Read each question carefully before answering. Answer all parts. Show all work, calculations, and/or reasoning, otherwise no points will be awarded. K-maps may be used to double check your work, but may NOT be used as your actual work. Point values are as indicated.

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1. Express the following as 5-bit signed binary numbers and **add the binary numbers together** (i.e. do not add the two decimal numbers together and convert the result to binary). Use 2's complement to represent negative numbers. Indicate if there is an overflow in any of the answers, and, if so, how you know there was an overflow situation.

   (a) (5 points) $-14_{10} + 7_{10}$

$+14_{10} =$ 0 1 1 1 0

FLIPBITS 1 0 0 0 1

ADD 1 1 0 0 1 0

$+7_{10} =$ 0 0 1 1 1

$$\begin{array}{r} {\scriptstyle 1\ 1\ \ \ \ } \\ 1\ 0\ 0\ 1\ 0 \\ +\ 0\ 0\ 1\ 1\ 1 \\ \hline 1\ 1\ 0\ 0\ 1 \end{array}$$

1 1 0 0 1

   (b) (5 points) $11_{10} + 9_{10}$

$+11_{10} =$ 0 1 0 1 1

$+9_{10} =$ 0 1 0 0 1

$$\begin{array}{r} {\scriptstyle 1\ \ \ 1\ 1\ } \\ 0\ 1\ 0\ 1\ 1 \\ +\ 0\ 1\ 0\ 0\ 1 \\ \hline 1\ 0\ 1\ 0\ 0 \end{array}$$

ADDING 2 POSITIVE NUMBERS GAVE A NEGATIVE RESULT.

OVERFLOW

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2. Express the following as 4-bit signed binary numbers and **multiply the binary numbers together with all overflow bits present from the start** (i.e. do not multiply the two decimal numbers together and convert the result to binary). Use 2's complement to represent negative numbers. Indicate if there is an overflow in any of the answers, and, if so, how you know there was an overflow situation.

(a) (5 points) $2_{10} \times -4_{10}$

1 1 1 1 1 1 0 0
× 0 0 0 0 0 0 1 0
1 1 1 1 1 0 0 X

NO OVERFLOW

1 1 1 1 1 0 0 0

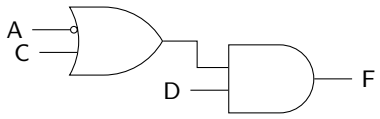
(b) (5 points) $-6_{10} \times 3_{10}$

1 1 1 1 1 0 1 0
× 0 0 0 0 0 0 1 1
1 1 1 1 1 0 1 0
+ 1 1 1 1 0 1 0 X
1 1 1 0 1 1 1 0

OVERFLOW

OVERFLOW

3. (10 points) Express F as a minterm expansion of four variables.



$(A'+C)D = A'D+CD$

$A'(BC+B'C+BC'+B'C')D + (AB+A'B+AB'+A'B')CD$

IDEMPOTENT TERMS

$A'BCD+A'B'CD+AB'C'D+A'BC'D+ABCD+A'BCD+AB'CD+A'B'CD$

$F(A,B,C,D) = \Sigma m(1,3,5,7,11,15)$

4. (10 points) Determine if the following expressions are equivalent. Explain your reasoning.

$$F = A \oplus BC$$
$$G = A'BC + AB' + AC'$$

$G = A'(BC) + A(B'+C')$

$= A'(BC) + A(BC)'$

$= A \oplus BC$

THE EXPRESSIONS ARE EQUIVALENT!

5. (20 points) Express F as a minimum sum-of-products equation. Show and label all steps in your minimization process.

$$F = (A \equiv D)(CD \oplus B) + ABCD$$

$(AD + A'D')(CDB' + (CD)'B) + ABCD$

$(AD + A'D')(B'CD + BC' + BD') + ABCD$

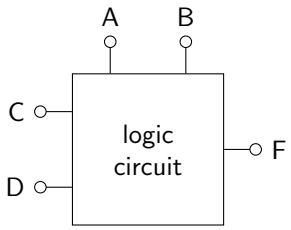
$AB'CD + ABC'D + \underbrace{A'BC'D' + A'BD'}_{\text{ABSORPTION}} + ABCD$

$AB'CD + \underbrace{ABC'D}_{\text{UNITING}} + A'BD' + \underline{ABCD}$

$\underbrace{AB'CD + ABD}_{\text{ELIMINATION}} + A'BD'$

$\boxed{ACD + ABD + A'BD'}$

6. (25 points) You receive two 2-bit numbers designated as AB and CD. If $AB > CD$, an LED should turn on. It is not possible for $AB = CD$. Derive a minimum sum-of-products expression to control the LED. Show and label all steps in your minimization process.



A	B	C	D	F	
0	0	0	0	X	$A'B'C'D'$
0	0	0	1	0	
0	0	1	0	0	
0	0	1	1	0	
0	1	0	0	1	$A'BC'D'$
0	1	0	1	X	$A'BC'D$
0	1	1	0	0	
0	1	1	1	0	
1	0	0	0	1	$AB'C'D'$
1	0	0	1	1	$AB'C'D$
1	0	1	0	X	$AB'CD'$
1	0	1	1	0	
1	1	0	0	1	$ABC'D'$
1	1	0	1	1	$ABC'D$
1	1	1	0	1	$ABCD'$
1	1	1	1	X	$ABCD$

$A'BC'D' + AB'C'D' + AB'C'D +$ UNITING
$ABC'D' + ABC'D + ABCD'$ UNITING

$A'BC'D' + AB'C' + ABC' + ABCD'$ ELIMINATION

$A'BC'D' + AB'C' + ABC' + ABD'$ UNITING
$+ A'B'C'D'$ UNITING

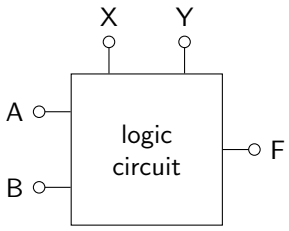
$A'C'D' + AC' + ABD'$ elimination

$C'D' + AC' + ABD'$ elimination

$C'D' + A(C' + BB' + BCD)$ absorption elimination

$C'D' + AC' + AB = F$

7. (25 points) Use a truth table to derive a minimum product-of-sums expression for the following programmable logic gate. Show and label all steps in your minimization process.



If X=0, F=AB
If X=1 and Y=1, F=A+B
If X=1 and Y=0, F=0

X	Y	A	B	F	
0	0	0	0	0	
0	0	0	1	0	
0	0	1	0	0	
0	0	1	1	1	$X'y'AB$
0	1	0	0	0	
0	1	0	1	0	
0	1	1	0	0	
0	1	1	1	1	$X'yAB$
1	0	0	0	0	
1	0	0	1	0	
1	0	1	0	0	
1	0	1	1	0	
1	1	0	0	0	
1	1	0	1	1	$XyA'B$
1	1	1	0	1	$XyAB'$
1	1	1	1	1	$XyAB$

$X'y'AB + X'yAB + XyA'B + XyAB' + XyAB$

UNITING UNITING

$X'AB + XyA'B + XyA$

ELIMINATION

$X'AB + XyB + XyA$

CONVERT TO P.O.S.

$F' = (X + A' + B')(X' + y' + B')(X' + y' + A)$

$= (X + A' + B')(X' + y' + A'B')$

$= Xy' + XA'B' + X'A' + y'A' + A'B' +$ ABSORPTION

$X'B' + y'B' + A'B'$ IDEMP.

$= Xy' + X'A' + y'A' + A'B' + X'B' + y'B'$

CONSENSUS

CONSENSUS

XX'	$X(y')\ X'(A')\ X'(B')$	$y'A', y'B'$
yy'	$y(\emptyset)$	$\emptyset$
AA'	$A(\emptyset)$	$\emptyset$
BB'	$B(\emptyset)$	$\emptyset$

$F' = Xy' + X'A' + A'B' + X'B'$

$F = (X' + y)(X + A)(A + B)(X + B)$