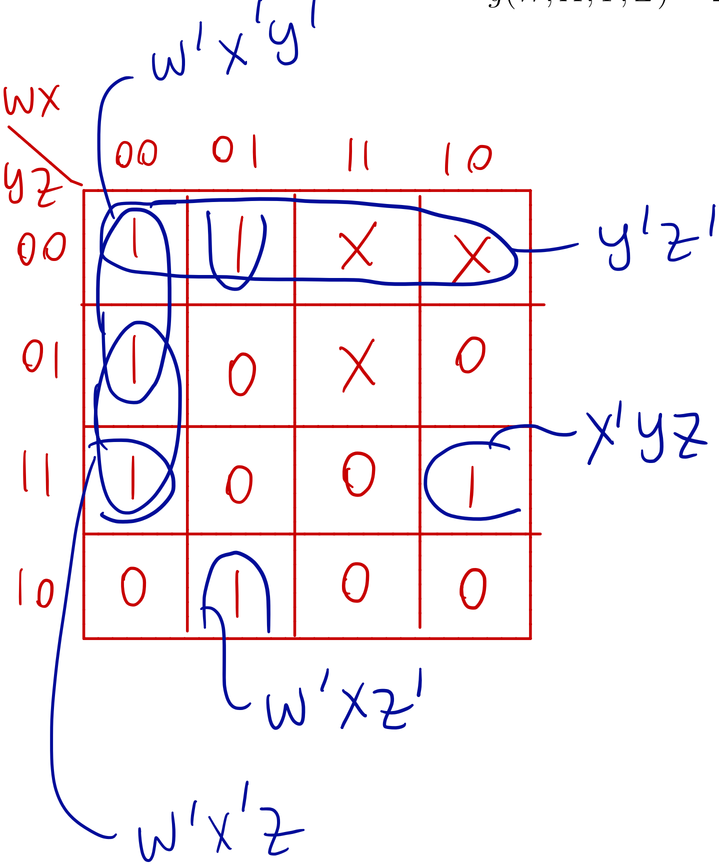
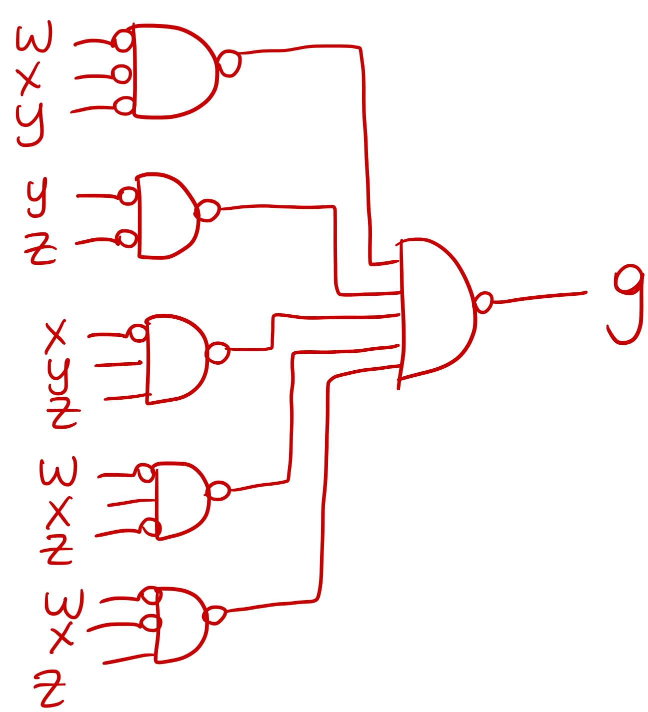
Name: SOLUTIONS

Read each question carefully before answering. Answer all parts. Show all work, calculations, and/or reasoning, otherwise no points will be awarded. Properly labeled loops **must be shown** on K-maps to receive credit. Assume that you have access to gates with as many inputs as you need. Point values are as indicated. Usage of XOR and XNOR gates is **not allowed** on this exam!

1. (10 points) Draw the following as a hazard-free NAND-only circuit. **You may use bubbles on inputs, but not as output inverters!**

$$g(W,X,Y,Z) = \Sigma m(0,1,3,4,6,11) + \Sigma d(8,12,13)$$



2. A sensor is capable of determining whether or not a car is speeding (driving faster than the speed limit) or driving dangerously (driving faster than 65 m.p.h. or driving more than 10 m.p.h. above the speed limit). The sensor receives the following codes. `AB` corresponds to the speed limit, and `CD` corresponds to the speed of the vehicle.

| `AB` | **Speed Limit** | `CD` | **Car's Speed** |
|---|---|---|---|
| 00 | 45 m.p.h. | 00 | $\leq 45$ m.p.h. |
| 01 | 55 m.p.h. | 01 | 46–55 m.p.h. |
| 10 | 65 m.p.h. | 10 | 56–65 m.p.h. |
| 11 | unused | 11 | $> 65$ m.p.h. |

(a) (10 points) Use a k-map to solve for $F$, which indicates if the car is speeding. All loops must be labeled to receive any credit.

| | | 45 | 55 | ***AB*** | 65 |
|---|---|---|---|---|---|
| | ***CD*** | **00** | **01** | **11** | **10** |
| $\leq 45$ | **00** | 0 | 0 | X | 0 |
| 46-55 | **01** | 1 | 0 | X | 0 |
| >65 | **11** | 1 | 1 | X | 1 |
| 56-65 | **10** | 1 | 1 | X | 0 |

CD

A'C

A'B'D

F= $A'B'D + A'C + CD$

(b) (10 points) Use a k-map to solve for $G$, which indicates if the car is driving dangerously. All loops must be labeled to receive any credit.

| | | 45 | 55 | | 65 |
|---|---|---|---|---|---|
| | | **AB** | | | |
| | **CD** | **00** | **01** | **11** | **10** |
| ≤45 | **00** | 0 | 0 | X | 0 |
| 46-55 | **01** | 0 | 0 | X | 0 |
| >65 | **11** | 1 | 1 | X | 1 |
| 56-65 | **10** | 1 | 0 | X | 0 |

CD

$A'B'C$

G= $A'B'C + CD$

3. The following codes are used by a vending machine to determine the item being purchased (represented by the variables DE) and the number of money inserted into the machine (represented by the variables ABC). The machine has an output, V, which is 1 if enough money has been inserted to pay for the item.

| ABC | **Money Inserted** | DE | **Item & Cost** |
|---|---|---|---|
| 000 | \$0.20 | 00 | Snickers – \$0.70 |
| 001 | \$0.30 | 01 | Coke – \$0.60 |
| 010 | \$0.40 | 10 | Water – \$0.50 |
| 011 | \$0.50 | 11 | Chips – \$0.85 |
| 100 | \$0.60 | | |
| 101 | \$0.70 | | |
| 110 | \$0.80 | | |
| 111 | \$0.90 | | |

(a) (10 points) Find the minterms of V.

| DE \ ABC | 20¢ 000 | 30¢ 001 | 50¢ 011 | 40¢ 010 | 80¢ 110 | 90¢ 111 | 70¢ 101 | 60¢ 100 | |
|---|---|---|---|---|---|---|---|---|---|
| 00 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 70¢ |
| 01 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 60¢ |
| 11 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 85¢ |
| 10 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 50¢ |

$$\Sigma m(14, 17, 18, 20, 21, 22, 24, 25, 26, 28, 29, 30, 31)$$

**THIS QUESTION CONTINUES ON THE NEXT PAGE**

(b) (20 points) Use the Quine-McCluskey method to derive an equation for `V`.

| | **Column 1** | |
|---|---|---|
| TWO | 17 | 10001 |
| | 18 | 10010 |
| | 20 | 10100 |
| | 24 | 11000 |
| THREE | 14 | 01110 |
| | 21 | 10101 |
| | 22 | 10110 |
| | 25 | 11001 |
| | 26 | 11010 |
| | 28 | 11100 |
| FOUR | 29 | 11101 |
| | 30 | 11110 |
| FIVE | 31 | 11111 |

| **Column 2** | |
|---|---|
| 17-21 | 10-01 |
| 17-25 | 1-001 |
| 18-22 | 10-10 |
| 18-26 | 1-010 |
| 20-21 | 1010- |
| 20-22 | 101-0 |
| 20-28 | 1-100 |
| 24-25 | 1100- |
| 24-26 | 110-0 |
| 24-28 | 11-00 |
| 14-30 | -1110 |
| 21-29 | 1-101 |
| 22-30 | 1-110 |
| 25-29 | 11-01 |
| 26-30 | 11-10 |
| 28-29 | 1110- |
| 28-30 | 111-0 |
| 29-31 | 111-1 |
| 30-31 | 1111- |

| **Column 3** | |
|---|---|
| 17-21-25-29 | 1--01 |
| 18-22-26-30 | 1--10 |
| 20-21-28-29 | 1-10- |
| 20-22-28-30 | 1-1-0 |
| 24-25-28-29 | 11-0- |
| 24-26-28-30 | 11--0 |
| 28-29-30-31 | 111-- |

**PI's**

$BCDE'$

$AD'E$

$ADE'$

$ACD'$

$ACE'$

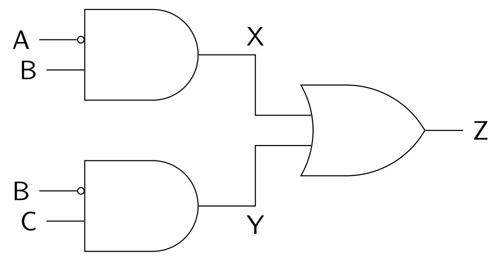
$ABD'$

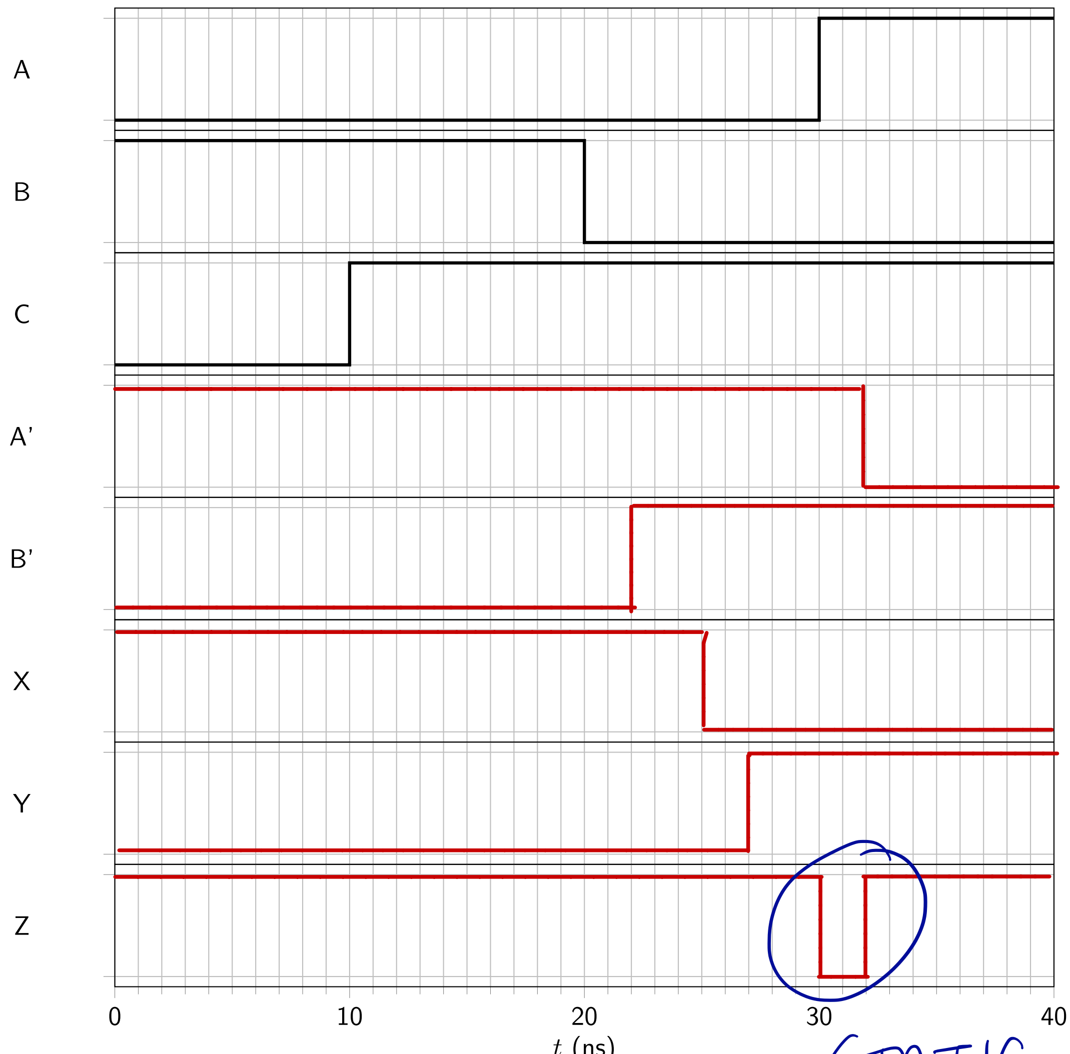
$ABE'$

$ABC$

V= $BCDE' + AD'E + ADE' + \left\{ \begin{array}{l} ACD' + ABD' \\ ACE' + ABD' \\ ABE' + ACD' \\ ABE' + ACE' \end{array} \right.$

4. (20 points) Draw a timing diagram for the following circuit, given gate delays of 2 ns for NOT gates, and 5 ns for AND and OR gates. Indicate any static hazards in the output signal




STATIC
ONE
HAZARD!

5. (25 points) Find the optimized implementation of the following two circuits. Show all work. How many gates and/or inputs do you save by implementing circuits together rather than individually?

$$X(A, B, C, D) = \Sigma m(2, 3, 4, 6, 7, 10, 12)$$

$$Y(A, B, C, D) = \Sigma m(4, 6, 7, 10, 12, 14, 15)$$

$BC'D$

| **X** | | ***AB*** | | |
|---|---|---|---|---|
| ***CD*** | **00** | **01** | **11** | **10** |
| **00** | 0 | 1 | 1 | 0 |
| **01** | 0 | 0 | 0 | 0 |
| **11** | 1* | 1 | 0 | 0 |
| **10** | 1* | 1 | 0 | 1 |

| **Y** | | ***AB*** | | |
|---|---|---|---|---|
| ***CD*** | **00** | **01** | **11** | **10** |
| **00** | 0 | 1 | 1 | 0 |
| **01** | 0 | 0 | 0 | 0 |
| **11** | 0 | 1 | 1* | 0 |
| **10** | 0 | 1 | 1* | 1 |

$A'C$

$BC$

$AB'CD'$

OPTIMIZED

$X = A'C + BC'D + AB'CD'$

$Y = BC + BC'D + AB'CD'$

SAVES 2 GATES & 4 INPUTS