

Name: SOLUTIONS

73% CLASS AVG.

Read each question carefully before answering. Answer all parts. Show all work, calculations, and/or reasoning, otherwise no points will be awarded. Properly labeled loops **must be shown** on K-maps to receive credit. Assume that you have access to gates with as many inputs as you need. Point values are as indicated. Usage of XOR and XNOR gates is **not allowed** on this exam!

1. (5 points) Using the following K-map, find the minimum SOP expression for a hazard-free circuit.

	A	
BC	0	1
00	0	0
01	0	1
11	1	1
10	1	0

Handwritten annotations on the K-map:  
 - A vertical loop labeled "AC" encircling the 1s in the A=1 column (cells 01 and 11).  
 - A horizontal loop labeled "BC" encircling the 1s in the B=1 row (cells 11 and 10).  
 - A diagonal loop labeled "A'B" encircling the 1s in the B=0 column (cells 11 and 10).

$$F_{\text{hazard-free}} = \underline{AC + BC + A'B}$$

All prime implicants are required to implement a hazard-free circuit!

2.  $F(A, B, C, D) = \Sigma m(2, 3, 6, 7, 8, 11) + \Sigma d(0, 10, 12, 13)$

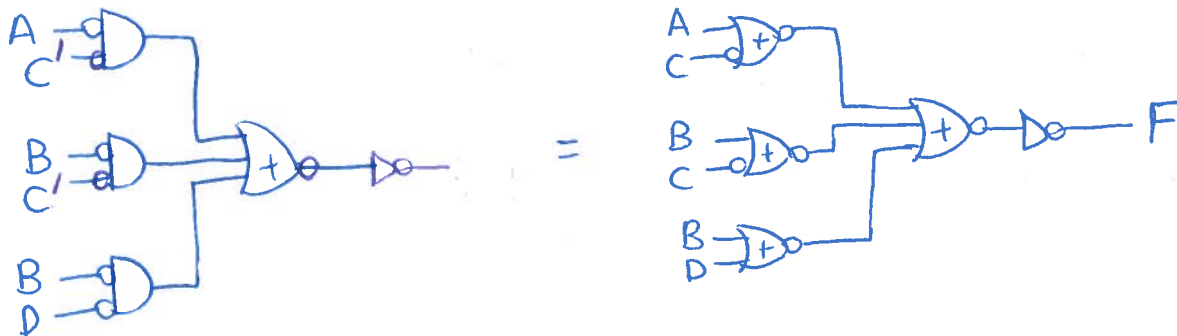
(a) (5 points) Using a K-map, find the minimum SOP expression.

	AB			
CD	00	01	11	10
00	X	0	X	1
01	0	0	X	0
11	1	1	0	1
10	1	1	0	X

Handwritten annotations on the K-map:  
 - A group of four 1s in the middle row (CD=11) is circled and labeled  $A'C$ .  
 - A group of four 1s in the middle column (AB=01) is circled and labeled  $B'C$ .  
 - A group of four 1s in the first and third columns (CD=00 and 10) is circled and labeled  $B'D'$ .

$$F_{SOP} = A'C + B'C + B'D'$$

(b) (5 points) Draw the circuit diagram as a NOR-only circuit. (Use either the double prime or bubble method and show your work.)



3.  $F(A, B, C, D) = \Pi M(1, 5, 8, 9, 11, 13) + \Pi D(2, 3, 6, 10, 14, 15)$

(a) (5 points) Using a K-map, find the minimum POS expression.

	AB			
CD	00	01	11	10
00	1	1	1	0
01	0	0	0	0
11	X	1	X	0
10	X	X	X	X

Handwritten annotations on the K-map:  
 - A vertical oval around the 0s in the 00 and 01 rows is labeled  $(A'+B)$ .  
 - A horizontal oval around the 0s in the 01 and 11 rows is labeled  $(C+D')$ .

$$F_{POS} = (C+D')(A'+B)$$

(b) (5 points) Draw the circuit diagram as a NAND-only circuit. (Use either the double prime or bubble method and show your work.)



3. Use the Quine-McCluskey method to find the minimum SOP expression for the following expression. Each column containing implicants is worth 5 points. Circle all of the prime implicants in each column. This question continues on the next page.

$$F(A, B, C, D, E) = \sum m(0, 1, 9, 11, 16, 17, 23, 29, 31) + \sum d(4, 6, 13, 15, 25, 27)$$

	Column 1	Column 2	Column 3	Column 4
zero	0. 00000 ✓	0-1 0000- ✓	0-1-16-17 -000- ✓	9-11-13-15-25-27-29-31 ✓
one	1. 00001 ✓	0-4 00-00 ✓	<del>0-16-17</del>	-1--1
	4. 00100 ✓	0-16 -0000 ✓	1-9-17-25 --001 ✓	all others are redundant
two	16. 10000 ✓	1-9 0-001 ✓	<del>17-9-25</del>	
	6. 00110 ✓	1-17 -0001 ✓	9-11-13-15 01--1 ✓	
three	9. 01001 ✓	4-6 001-0 ✓	9-11-25-27 -10-1 ✓	
	17. 10001 ✓	16-17 1000- ✓	9-13-11-15	
four	11. 01011 ✓	9-11 010-1 ✓	9-13-25-29 -1-01 ✓	
	13. 01101 ✓	9-13 01-01 ✓	<del>9-25-11-29</del>	
five	25. 11001 ✓	9-25 -1001 ✓	9-25-13-29	
	15. 01111 ✓	17-25 1-001 ✓	11-15-27-31 -1-11 ✓	
six	23. 10111 ✓	11-15 01-11 ✓	<del>11-27-15-31</del>	
	27. 11011 ✓	11-27 -1011 ✓	13-15-29-31 -11-1 ✓	
seven	29. 11101 ✓	13-15 011-1 ✓	<del>13-29-15-31</del>	
	31. 11111 ✓	13-29 -1101 ✓	25-27-29-31 11--1 ✓	
		25-27 110-1 ✓	<del>25-29-27-31</del>	
		25-29 11-01 ✓		
		15-31 -1111 ✓		
		23-31 1-111 ✓		
		27-31 11-11 ✓		
		29-31 111-1 ✓		

(a) (5 points) Fill out the prime implicant table.

Prime Implicants	0	1	9	11	16	17	23	29	31
0-4	X								
4-6									
* 23-31							X		X
* 0-1-16-17	X	X			X	X			
1-9-17-25		X	X			X			
* 9-11-13-15- 25-27-29-31			X	X				X	X

(b) (5 points) Identify all of the essential prime implicants.

ACDE (23-31)

B'C'D' (0-1-16-17)

BE (9-11-13-15-25-27-29-31)

(c) (5 points) Write the minimum SOP expression.

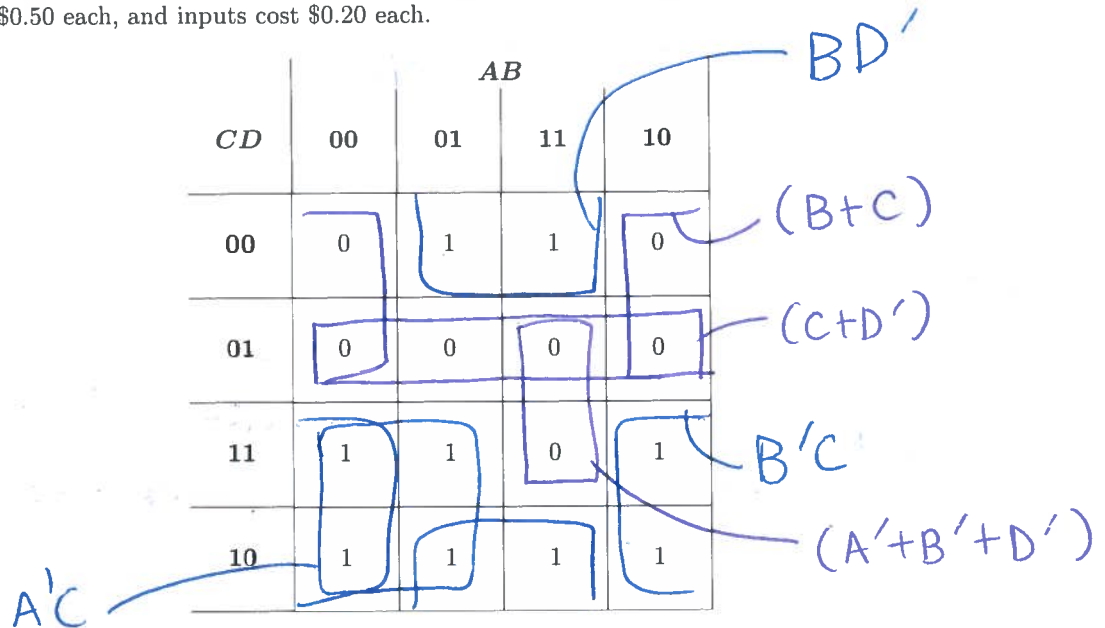
$$F_{SOP} = \underline{ACDE + B'C'D' + BE}$$

(d) (5 points) If this had to be implemented as a hazard-free circuit, write the corresponding circuit equation.

$$* F_{\text{hazard-free}} = \underline{A'B'D'E' + ACDE + B'C'D' + C'D'E + BE}$$

→ 4-6 is not necessary because it consists only of don't care terms!

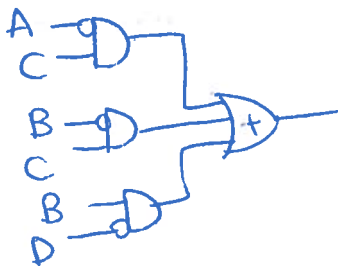
4. Use the following K-map to answer the following questions. In addition, assume that logic gates cost \$0.50 each, and inputs cost \$0.20 each.



- (a) (5 points) Find the minimum SOP expression.

$$A'C + B'C + BD'$$

- (b) (5 points) What is the cost of this circuit? (Show how many gates and inputs are needed, then calculate the cost.)



$$4 \text{ gates} * \$0.50 = \$2.00$$

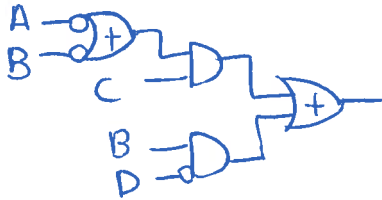
$$9 \text{ inputs} * \$0.20 = \$1.80$$

$$\boxed{\$3.80}$$

- (c) (5 points) Find the factored SOP expression.

$$C(A' + B') + BD'$$

- (d) (5 points) What is the cost of this circuit? (Show how many gates and inputs are needed, then calculate the cost.)



$$4 \text{ gates} * \$0.50 = \$2.00$$

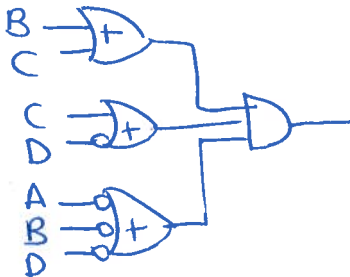
$$8 \text{ inputs} * \$0.20 = \$1.60$$

$$\boxed{\$3.60}$$

- (e) (5 points) Find the minimum POS expression.

$$(B + C)(C + D')(A' + B' + D')$$

- (f) (5 points) What is the cost of this circuit? (Show how many gates and inputs are needed, then calculate the cost.)



$$4 \text{ gates} * \$0.50 = \$2.00$$

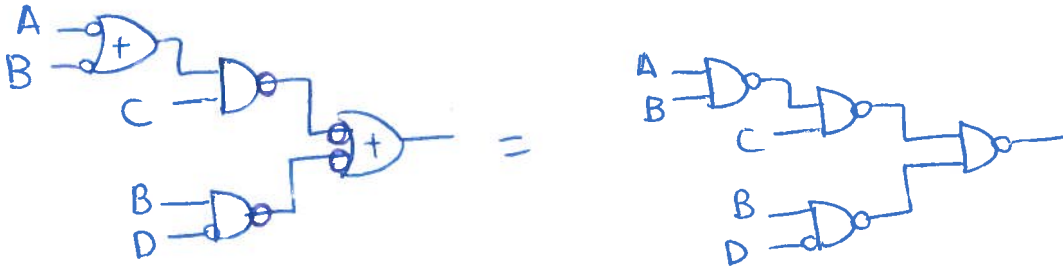
$$10 \text{ inputs} * \$0.20 = \$2.00$$

$$\boxed{\$4.00}$$

(g) (5 points) Identify the lowest cost implementation of this circuit.

Factored SOP

(h) (5 points) Draw the lowest cost circuit diagram as a NAND-only circuit. (Use either the double prime or bubble method and show your work.)



(i) (5 points) Draw the lowest cost circuit diagram as a NOR-only circuit. (Use either the double prime or bubble method and show your work.)

