Name: SOLUTIONS

Read each question carefully before answering. Answer all parts. Show all work, calculations, and/or reasoning, otherwise no points will be awarded. K-maps may be used to double check your work, but may NOT be used as your actual work. Point values are as indicated.

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1. Express the following as 5-bit signed binary numbers and **add the binary numbers together** (i.e. do not add the two decimal numbers together and convert the result to binary). Use 2's complement to represent negative numbers. Indicate if there is an overflow in any of the answers, and, if so, how you know there was an overflow situation.

   (a) (5 points) $-6_{10} + 8_{10}$

+6 = 0 0 1 1 0

1 1 0 0 1 FLIP BITS

-6 = 1 1 0 1 0 ADD ONE

$$\begin{array}{r} {\scriptstyle 1\ \ \ \ \ } \\ 1\ 1\ 0\ 1\ 0 \\ +\ 0\ 1\ 0\ 0\ 0 \\ \hline 0\ 0\ 0\ 1\ 0 \end{array}$$

(b) (5 points) $12_{10} + 10_{10}$

* THIS IS AN OVERFLOW: ADDING 2 POSITIVE NUMBERS HAD A NEGATIVE RESULT

$$\begin{array}{r} {\scriptstyle 1\ \ \ \ \ } \\ 0\ 1\ 1\ 0\ 0 \\ +\ 0\ 1\ 0\ 1\ 0 \\ \hline *\ 1\ 0\ 1\ 1\ 0 \end{array}$$

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2. Express the following as 5-bit signed binary numbers and **multiply the binary numbers together with all overflow bits present from the start** (i.e. do not multiply the two decimal numbers together and convert the result to binary). Use 2's complement to represent negative numbers. Indicate if there is an overflow in any of the answers, and, if so, how you know there was an overflow situation.

(a) (5 points) $9_{10} \times -1_{10}$

```
  1 1 1 1 1 1 1 1 1 1
x 0 0 0 0 0 0 1 0 0 1
-------------------------
  1 1 1 1 1 1 1 1 1 1
+ 1 1 1 1 1 1 1 x x x
```

NO OVERFLOW

1 1 1 1 1 1 0 1 1 1

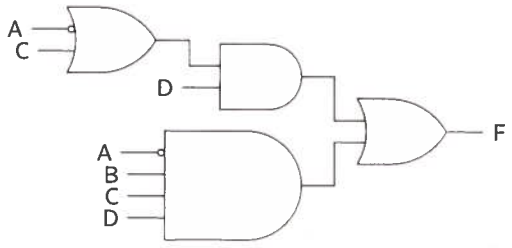
(b) (5 points) $-8_{10} \times 3_{10}$

```
    1 1 1 1 1 1 1 0 0 0
x   0 0 0 0 0 0 0 0 1 1
-------------------------
    1 1 1 1 1 1 1 0 0 0
+ 1 1 1 1 1 1 0 0 0 x
```

OVERFLOW

1 1 1 1 1 0 1 0 0 0

3. (10 points) Express F as a minterm expansion of four variables.



AKA

$\Sigma m(1,3,5,7,11,15)$

$(A'+C)D+A'BCD$

$A'D+CD+\cancel{A'BCD}$ (ABSORPTION)

$A'(BC+BC'+B'C+B'C')D+(AB+AB'+A'B+A'B')CD$

$A'BCD+A'BC'D+A'B'CD+A'B'C'D+\cancel{ABCD}+\cancel{AB'CD}+\cancel{A'BCD}+\cancel{A'B'CD}$

(IDEMPOTENT)

$A'BCD+A'BC'D+A'B'CD+A'B'C'D+ABCD+\cancel{AB'CD}$

4. (10 points) Determine if the following expressions are equivalent. Explain your reasoning.

$$F = A \oplus BC$$
$$G = A'BC + AB' + AC'$$

$G = A'(BC) + A(B'+C')$

$= A'(BC) + A(BC)'$

$= A \oplus BC$

F IS EQUIVALENT TO G.

5. (20 points) Express F as a minimum product-of-sums equation. Show and label all steps in your minimization process.

$A \equiv B' = AB' + A'B$

$CD \oplus B' = BCD + B'C' + B'D'$

$$F = (A \equiv B')(CD \oplus B') + ABCD$$

$$F = (AB' + A'B)(BCD + B'C' + B'D') + ABCD$$

$$= AB'C' + AB'D' + \underbrace{A'BCD + ABCD}_{\text{UNITING}}$$

$$= AB'C' + AB'D' + BCD$$

CONVERT TO P.O.S.

$$F' = \underbrace{(A' + B + C)(A' + B + D)}_{\text{2ND DISTRIB}}(B' + C' + D')$$

$$= (A' + B + CD)(B' + C' + D')$$

$$= A'B' + \underbrace{\cancel{A'C'} + \cancel{A'D'}}_{\text{CONSENSUS}} + BC' + BD' + B'CD$$

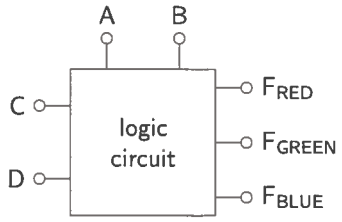
$$F' = A'B' + BC' + BD' + B'CD$$

CONSENSUS CHECK:

AA'	NO TERMS
BB'	$A'B', B'CD, BC', BD' : (A'C', A'D')$
CC'	$BC', B'CD : \emptyset$
DD'	$BD', B'CD : \emptyset$

$$F = (A+B)(B'+C)(B'+D)(B+C'+D')$$

6. (25 points) You receive two 2-bit numbers designated as AB and CD. If $AB > CD$, a red LED should turn on with all other LEDs off. If $AB < CD$, a green LED should turn on with all other LEDs off. If $AB = CD$, a blue LED should turn on with all other LEDs off. Derive minimized Boolean expressions (you may choose SOP or POS) to control each LED. Show and label all steps in your minimization process.



A B C D	AB CD	FRED	FGRN	FBLU
0 0 0 0	0 = 0	0	0	1
0 0 0 1	0 < 1	0	1	0
0 0 1 0	0 < 2	0	1	0
0 0 1 1	0 < 3	0	1	0
0 1 0 0	1 > 0	1	0	0
0 1 0 1	1 = 1	0	0	1
0 1 1 0	1 < 2	0	1	0
0 1 1 1	1 < 3	0	1	0
1 0 0 0	2 > 0	1	0	0
1 0 0 1	2 > 1	1	0	0
1 0 1 0	2 = 2	0	0	1
1 0 1 1	2 < 3	0	1	0
1 1 0 0	3 > 0	1	0	0
1 1 0 1	3 > 1	1	0	0
1 1 1 0	3 > 2	1	0	0
1 1 1 1	3 = 3	0	0	1

$F_B = A'B'C'D' + A'BC'D + AB'CD' + ABCD$

$$F_R = A'BC'D' + \underbrace{AB'C'D' + AB'C'D}_{\text{UNITING}} + \underbrace{ABC'D' + ABC'D}_{\text{UNITING}} + ABCD'$$

$$= A'BC'D' + \underbrace{AB'C' + ABC'}_{\text{UNITING}} + ABCD'$$

$$= \underbrace{A'BC'D' + AC'}_{\text{ELIMINATION}} + ABCD'$$

$$= BC'D' + \underbrace{AC' + ABCD'}_{\text{ELIMINATION}}$$

$$\boxed{F_R = BC'D' + AC' + ABD'}$$

$$F_G = A'B'C'D + \underbrace{A'B'CD' + A'B'CD}_{\text{UNITING}} + \underbrace{A'BCD' + A'BCD}_{\text{UNITING}} + AB'CD$$

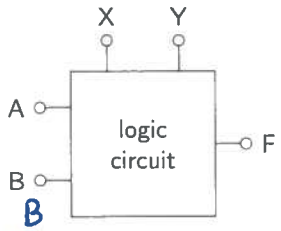
$$= A'B'CD + \underbrace{A'B'C + A'BC}_{\text{UNITING}} + AB'CD$$

$$= \underbrace{A'B'C'D + A'C}_{\text{ELIMINATION}} + AB'CD$$

$$= B'C'D + \underbrace{A'C + AB'CD}_{\text{ELIMINATION}}$$

$$\boxed{F_G = B'C'D + A'C + AB'D}$$

7. (25 points) Use a truth table to derive a minimum sum-of-products expression for the following programmable logic gate. Show and label all steps in your minimization process.



If X=0 and Y=0, F=AB
If X=1 and Y=1, F=A+B
If X=1 and Y=0, F=0
X=0 and Y=1 will never occur

X	Y	A	B	F	
0	0	0	0	0	AND
0	0	0	1	0	
0	0	1	0	0	
0	0	1	1	1	
0	1	0	0	X	X
0	1	0	1	X	
0	1	1	0	X	
0	1	1	1	X	
1	0	0	0	0	0
1	0	0	1	0	
1	0	1	0	0	
1	0	1	1	0	
1	1	0	0	0	OR
1	1	0	1	1	
1	1	1	0	1	
1	1	1	1	1	

$F = x'y'AB + xyA'B + xyAB' + xyAB$ (UNITING)

$= x'y'AB + xyA'B + xyA$ (ELIMINATION)

$= x'y'AB + xyB + xyA$

CHECK IF ANY X TERMS WILL HELP

$x'yAB'$ WILL ALLOW A UNITING STEP

$\boxed{F = x'y'A + xyB + xyA}$