Pre-Lab 14

Carefully read the entirety of Lab 14, then answer the following questions. Attach a separate sheet of paper, if necessary, to show all work and calculations.

1. The following questions pertain to lab Circuit 1.

   (a) Calculate the impedance of the load. Assume that each inductor has a parasitic resistance of 50 Ω.

   (b) Calculate V_a and express the value **in polar form**. (V_a is depicted in figure 14.1.) You will use this calculation to get an idea of what you should expect to see on the oscilloscope when you build the circuit in lab.

   (c) Calculate the RMS value of the load current ($\mathbf{I_{LOAD,RMS}}$).

   (d) Use equation 14.2 to calculate the complex power consumed by the load.

   (e) Calculate the value of the capacitor that should be placed in parallel with the load in order to increase the power factor of the circuit to one. (The placement of the compensating capacitor is depicted schematically in figure 14.2.)

2. The following questions pertain to lab Circuit 3. (Note: the only difference between Circuits 1 and 3 is the frequency of operation.)

   (a) Calculate the impedance of the load. Assume that each inductor has a parasitic resistance of 50 Ω.

   (b) Calculate V_a and express the value **in polar form**. (V_a is depicted in figure 14.1.) You will use this calculation to get an idea of what you should expect to see on the oscilloscope when you build the circuit in lab.

   (c) Calculate the RMS value of the load current ($\mathbf{I_{LOAD,RMS}}$).

   (d) Use equation 14.2 to calculate the complex power consumed by the load.

   (e) Calculate the value of the capacitor that should be placed in parallel with the load in order to increase the power factor of the circuit to one. (The placement of the compensating capacitor is depicted schematically in figure 14.2.)

3. The following questions pertain to lab Circuit 7.

   (a) Calculate the Thévenin impedance ($\mathbf{Z_{TH}}$) of the circuit shown in figure 14.6. (Assume that the parasitic resistance of the inductor is 50 Ω.)

   (b) Calculate the load impedance ($\mathbf{Z_L}$) of the circuit that would lead to maximum power transferred to the load.

   (c) What component values will need to be placed on the load to obtain the maximum transfer of power? Use the inventory of parts to determine what values are available. You may use series and parallel combinations to get as close as possible to the desired values.

   (d) Based on the component values you chose above, calculate the average power that will be consumed by the load.

Lab 14: AC Power

In this lab, we will explore the concept of complex power, which can occur when the voltage and current through a load are not in phase.

For lab resources and information, go to the following URL or scan the QR code. doctor-pasquale.com/circuit-analysis-lab-14

14.1 Complex Power

Complex power refers to both the real and imaginary components of power. This occurs due to a phase difference between current and voltage in AC circuits with complex impedances. Because impedance is frequency dependent, the real and imaginary components of AC power are also frequency dependent.

The components of AC power include the quantities described in table 14.1. Each quantity can be distinguished by its symbol and also by the units. While all units of AC power are technically the same, the units are differentiated to make each type of power more easily identifiable.

Quantity	Symbol	Units
Average power	P	watts (W)
Reactive power	Q	volt-amp reactive (VAR)
Complex power	$\mathbf{S}$	volt-amp (VA)
Apparent power	$\lvert\mathbf{S}\rvert$	volt-amp (VA)
Power factor	pf	(none)

Table 14.1: Components of AC power.

The equation used to calculate the power consumed by the load of a complex circuit element is defined by equation 14.1. From this value, all other quantities related to power can be derived.

$$\mathbf{S} = \mathbf{V_{LOAD,RMS}} \mathbf{I^*_{LOAD,RMS}} \tag{14.1}$$

In the first four circuits you will build in this lab, it is not possible measure both the load voltage and a voltage that is directly proportional to the current. Instead, you will measure only a voltage value that is proportional to current by measuring the voltage dropped over a small resistor in series with the load. Using Ohm's law, it will be possible to calculate the load current. Because the load voltage is not known, equation 14.1 can be rewritten in terms of current and impedance. This relationship is defined by equation 14.2.

$$\mathbf{S} = \lvert\mathbf{I_{LOAD,RMS}}\rvert^2 \mathbf{Z_{LOAD}} \tag{14.2}$$

14.2 Power Factor Correction

The power factor of a load relates to the ratio of average power to reactive power, as defined in equation 14.3.

$$pf = \frac{P}{\lvert\mathbf{S}\rvert} \tag{14.3}$$

When the power factor of a load is one, the current and voltage are in phase with each other and all of the power consumed by the load is real. Real power is power that is capable of doing useful work. Reactive power is simply power that oscillates back and forth from load to source. Reactive power is therefore not useful, but still costs money when electric power is billed by a power utility.

For this reason, it is generally desirable to increase the power factor of a load as much as possible. Circuits containing an inductive load (which exists in most consumer appliances that are powered by a motor) can have the reactance cancelled out or reduced by a capacitor placed in parallel. A circuit containing a capacitive load can have the reactance reduced by an inductor placed in parallel.

14.2.1 Correcting an Inductive Load

To correct an inductive load and obtain a power factor of one, equation 14.4 will be used to determine the value of the compensating capacitor to be placed in parallel with the load, where Q_i is the initial reactive power consumed by the load of the uncompensated circuit, ω is the angular frequency of the circuit, and $|\mathbf{V_{LOAD,RMS}}|$ is the magnitude of the RMS value of the load voltage.

$$C = \frac{Q_i}{|\mathbf{V_{LOAD,RMS}}|^2 \omega} \tag{14.4}$$

In the first four circuits, you will not be able to measure the load voltage. For this reason, you will use Ohm's law to determine another way to calculate the value of the compensating capacitor, described in equation 14.5, where the load current and load impedance are used instead of load voltage.

$$C = \frac{Q_i}{|\mathbf{I_{LOAD,RMS}}|^2 |\mathbf{Z_{LOAD}}|^2 \omega} \tag{14.5}$$

14.2.2 Correcting a Capacitive Load

To correct a capacitive load and obtain a power factor of one, equation 14.6 will be used to determine the value of the compensating inductor to be placed in parallel with the load, where Q_i is the initial reactive power consumed by the load of the uncompensated circuit, ω is the angular frequency of the circuit, and $|\mathbf{V_{LOAD,RMS}}|$ is the magnitude of the RMS value of the load voltage.

$$L = \frac{|\mathbf{V_{LOAD,RMS}}|^2}{-Q_i \omega} \tag{14.6}$$

You will use this equation in circuits 5 and 6, where you will know the value of the load voltage. Therefore, there is no need to use Ohm's law to calculate this in terms of current and impedance.

Circuit 1: Collect components for the circuit shown in figure 14.1. Measure and record all component values (including the parasitic resistance of the inductor), and record these values in table 14.2.

Parameter	**Value**	**Parameter**	**Value**
L		R150	
RL		R10	

Table 14.2: Circuit 1 data table.

Build the circuit shown in figure 14.1. You will use the oscilloscope to monitor the input voltage (V_s) and ensure that its magnitude and frequency are correct. Because current cannot be directly measured using the oscilloscope, a 10 Ω resistor will be used to indirectly measure it. You will measure the voltage dropped over the 10 Ω resistor (labeled as V_a in figure 14.1) and use Ohm's law in the lab homework to calculate the current through the load. Because the oscilloscope and function generator both contain an Earth ground connection, the voltage dropped over the load cannot be measured in this circuit. (Fortunately, it does not need to be.)

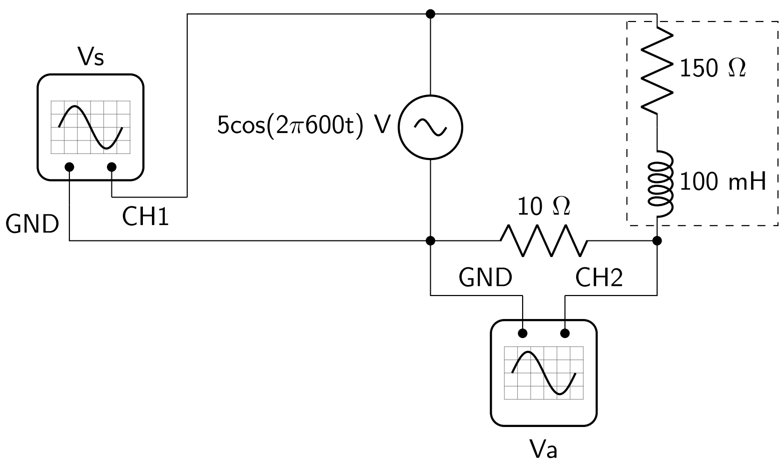

Figure 14.1: Inductive load circuit diagram. The load is indicated with dashed lines. The placement of oscilloscope probes is shown.

Using the oscilloscope, measure the amplitude of V_a and **convert to an RMS value**. Then use the cursors to measure the phase angle between V_a and V_s. (V_s and V_{load} will have nearly identical phase angles, so this is a reasonable measurement to take to determine the phase difference between load voltage and load current.) Record these values in table 14.3.

Parameter	**Magnitude (RMS)**	**Phase**
Va		

Table 14.3: Circuit 1 output data table.

When you have finished gathering data, show it to your instructor to receive a stamp.

Instructor Stamp: ______

Circuit 2: In the pre-lab, you should have calculated the value of the capacitor that would have to be placed in parallel with the load, as shown in figure 14.2, to compensate for the phase angle generated by the inductor. You may need to recalculate this value if assumptions made in the pre-lab about the parasitic resistance of the inductor are incorrect. Connect capacitors in series and parallel as needed to create this value. Wire the capacitor in parallel with the load, as shown in figure 14.2.

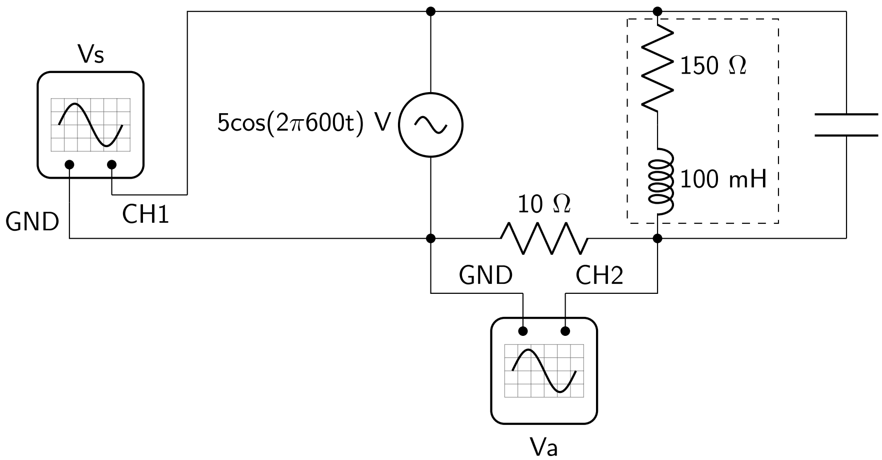

Figure 14.2: Inductive load circuit diagram with capacitive load correction. The load is indicated with dashed lines. The placement of oscilloscope probes is shown.

Record the amplitude and phase of V_a (**convert the amplitude to an RMS value**). Record this data in table 14.4. You may need to slightly modify your capacitance value in order to obtain a phase angle that is as close to 0 as possible. In table 14.4, record the final measured value of the capacitance that you use as C_{final}.

Parameter	**Value**
Va (RMS)	
Phase	
Cfinal	

Table 14.4: Circuit 2 output data table.

When you have finished gathering data, show it to your instructor to receive a stamp.

Instructor Stamp: ______

Circuit 3: Repeat the Circuit 1 activity (no compensating capacitance should be placed in parallel with the load) using a frequency of 1500 Hz. Measure the amplitude and phase of V_a (**convert the amplitude to an RMS value**) and record this data in table 14.5.

Parameter	Magnitude (RMS)	Phase
Va		

Table 14.5: Circuit 3 output data table.

When you have completed recording this data, demonstrate it to your instructor to receive a stamp.

Instructor Stamp: ______

Circuit 4: In the pre-lab, you should have calculated the value of the capacitor that would have to be placed in parallel with the load, as shown in figure 14.2, to compensate for the phase angle generated by the inductor. You may need to recalculate this value if assumptions made in the pre-lab about the parasitic resistance of the inductor are incorrect. Connect capacitors in series and parallel as needed to create this value. Wire the capacitor in parallel with the load, as shown in figure 14.2.

Record the amplitude and phase of V_a (**convert the amplitude to an RMS value**). Record this data in table 14.6. You may need to slightly modify your capacitance value in order to obtain a phase angle that is as close to 0 as possible. In table 14.6, record the final measured value of the capacitance that you use as C_{final}.

Parameter	Value
Va (RMS)	
Phase	
Cfinal	

Table 14.6: Circuit 4 output data table.

When you have finished gathering data, show it to your instructor to receive a stamp.

Instructor Stamp: ______

14.3 Mystery Complex Circuits

There are two mystery circuits that contain unknown complex loads, consisting of a resistance in series with either an inductance or a capacitance. Each was connected to current and voltage probes as shown in figure 14.3. (Note: the Vernier current and voltage probes are capable of measuring both current and voltage directly. They are not as good at measuring AC circuits as an oscilloscope, however, and will only be used for the next two circuits in this lab.)

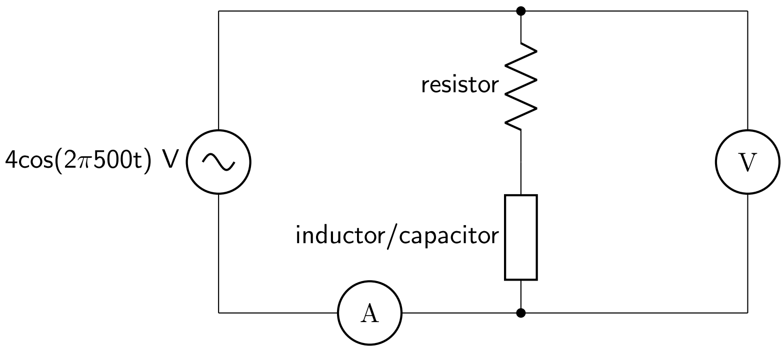

Figure 14.3: Circuit diagram for complex mystery boxes, including both components necessary to create a complex load. The placement of the current and voltage probes is indicated.

At the conclusion of the previous lab, you determined the component values that were used in each mystery box and recorded them in tables 14.7 and 14.9.

For the two following circuits, you will determine the value of capacitance or inductance to place in parallel with the load to compensate for the reactive power and increase the power factor to one. This compensation circuit will be connected as shown in figure 14.4.

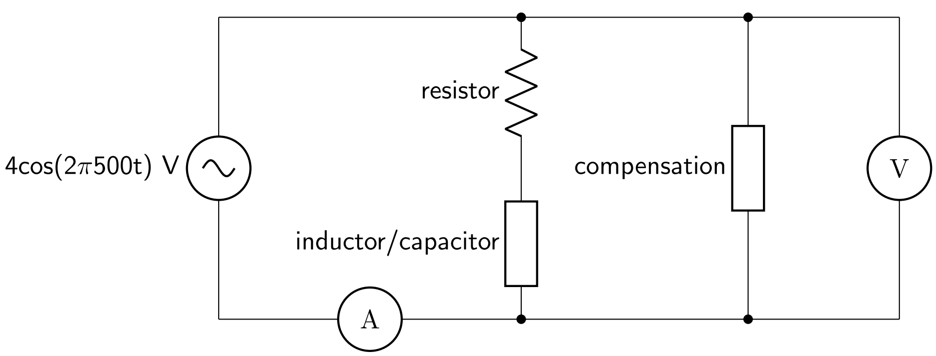

Figure 14.4: Circuit diagram for complex mystery boxes, including both components necessary to create a complex load. The placement of the compensating circuitry required to increase the power factor is shown. The placement of the current and voltage probes is indicated.

14.3.1 Vernier Current and Voltage Probes

Because the frequency of operation is low enough, the Vernier current and voltage probes can be used to directly measure both load current and load voltage simultaneously in the following two circuits. Using the LabQuest Mini box, a sample rate of 10,000 samples/second can be obtained, which will be adequate for the following two circuits that operate at 500 Hz. Following is the procedure for using the Vernier probes and LoggerPro software.

1. Connect the voltage probe to channel 1 and the current probe to channel 2.

2. Open LoggerPro.

3. With nothing connected to the probes, zero both of them.
4. Build the circuit (figures 14.3 and 14.4) and connect the probes as shown in the circuit diagrams.
5. In LoggerPro, go to Experiment > Data Collection.
   (a) Duration: enter 0.02 seconds
   (b) Sampling rate: enter 10,000 samples/second
   (c) Click on OK
6. Collect data.
7. Ensure that the voltage has the same value as the original circuit, otherwise the compensation values will be incorrect.
8. Autoscale the current data so that you can see it.
9. Use the curve fitting function and fit both voltage and current to $A\cos(Bt + C) + D$. (Note that the units of C are radians.)

Circuit 5: Record the measured values of voltage and current for mystery box A (**convert to RMS values**), as well as the calculated impedance value in table 14.7.

Parameter	Magnitude	Phase
V (VRMS)		
I (ARMS)		
Z (Ω)		

Table 14.7: Mystery box A measurements prior to load compensation.

Calculate the component values used for the resistance and capacitance/inductance in the original mystery box, and record them below.

Build the circuit using the component values you determined and the schematic given in figure 14.3. You may want to use the Vernier current and voltage probes to verify that your resistance and capacitance/inductance calculations were correct before continuing.

Determine the circuit element(s) that must be placed in parallel with the load (as shown in figure 14.4) in order to increase the power factor of the load to one. Record the value of this circuit element below. (Be clear if it is a capacitor or inductor.)

Build that circuit. Use the Vernier current and voltage probes to measure the new values of voltage and current (**convert to RMS values**). Record these values in table 14.8. Verify that the phase difference is as close to $0°$ as possible.

Parameter	Magnitude	Phase
V (VRMS)		
I (ARMS)		

Table 14.8: Mystery box A measurements after load compensation.

When you have completed this circuit, demonstrate your results to your instructor to receive a stamp.

Instructor Stamp: ______

Circuit 6: Record the measured values of voltage and current for mystery box B (**convert to RMS values**), as well as the calculated impedance value in table 14.9.

Parameter	Magnitude	Phase
V (VRMS)		
I (ARMS)		
Z (Ω)		

Table 14.9: Mystery box B measurements prior to load compensation.

Calculate the component values used for the resistance and capacitance/inductance in the original mystery box, and record them below.

Build the circuit using the component values you determined and the schematic given in figure 14.3. You may want to use the Vernier current and voltage probes to verify that your resistance and capacitance/inductance calculations were correct before continuing.

Determine the circuit element(s) that must be placed in parallel with the load (as shown in figure 14.4) in order to increase the power factor of the load to one. Record the value of this circuit element below. (Be clear if it is a capacitor or inductor.)

Build that circuit. Use the Vernier current and voltage probes to measure the new values of voltage and current (**convert to RMS values**). Record these values in table 14.10. Verify that the phase difference is as close to 0° as possible.

Parameter	Magnitude	Phase
V (VRMS)		
I (ARMS)		

Table 14.10: Mystery box B measurements after load compensation.

When you have completed this circuit, demonstrate your results to your instructor to receive a stamp.

Instructor Stamp: ______

14.4 Maximum Power Transfer

Any linear complex circuit can be modeled as a Thévenin equivalent circuit, with a Thévenin equivalent voltage ($\mathbf{V_{TH}}$) and Thévenin equivalent impedance ($\mathbf{Z_{TH}}$), as shown in figure 14.5.

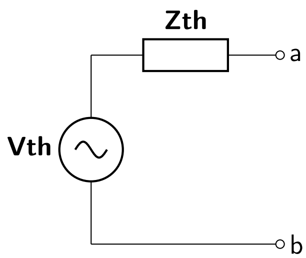

Figure 14.5: Thévenin equivalent circuit for a linear complex circuit.

Maximum power will be transferred to any load placed between nodes a and b in the circuit when the load impedance is equal to the complex conjugate of the Thévenin equivalent impedance, as defined in equation 14.7.

$$\mathbf{Z_{LOAD}} = \mathbf{Z_{TH}}^* \tag{14.7}$$

The value of the maximum average power consumed by the load when equation 14.7 is satisfied is defined by equation 14.8.

$$P_{LOAD,MAX} = \frac{|\mathbf{V_{TH,RMS}}|^2}{4R_{TH}} \tag{14.8}$$

Circuit 7: Using the circuit diagram in figure 14.6 as your guide, determine what load circuit must be placed in the circuit to lead to maximum power transferred by the load. **Be sure that you consider the parasitic resistance of the inductor in these values!** Measure the component values that you choose and record these values in table 14.11.

Parameter	Value
Rload	
Xload	

Table 14.11: Circuit 4 component data table.

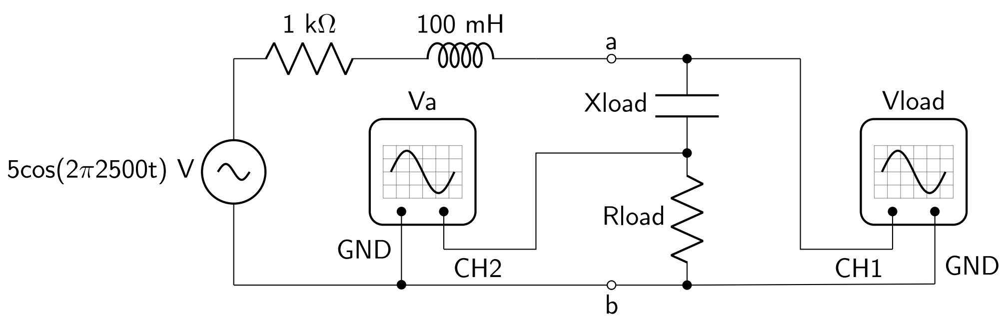

Figure 14.6: Complex circuit diagram for maximum power transfer to the load. The placement of oscilloscope probes is shown.

Build the circuit (shown in figure 14.6) with the load components that you measured. Use the oscilloscope to measure the voltage dropped over the load (depicted as V_{load}), as well as V_a, which is proportional to the current through the load. **Convert these values to RMS** and record them in table 14.12. Use the cursors to determine the phase difference between the two voltages and record that value in table 14.12 as well.

Parameter	Value
Vload (RMS)	
Va (RMS)	
Phase	

Table 14.12: Circuit 7 output data table.

When you have completed this circuit, demonstrate your results to your instructor to receive a stamp.

Instructor Stamp: ______

Lab 14 Homework

Carefully read each question before answering. Show all work or justify your answers to receive credit. Attach a separate sheet of paper, if necessary, to show all work and calculations.

1. Use component values that were measured and recorded in table 14.2 to answer the following questions about lab Circuit 1. Use the output voltage data that you collected in table 14.3.

   (a) Use Ohm's law to calculate the RMS value of the current flowing through the load.

   (b) Calculate the impedance of the load.

   (c) Use equation 14.2 to calculate the complex power consumed by the load. Express the value in Cartesian form so that it is in terms of $P + jQ$.

   (d) Calculate the power factor.

2. Use component values that were measured and recorded in table 14.2 to answer the following questions about lab Circuit 2. Use the output voltage data that you collected in table 14.4.

   (a) Use Ohm's law to calculate the RMS value of the current flowing through the load.

   (b) Calculate the impedance of the load.

   (c) Use equation 14.2 to calculate the complex power consumed by the load. Express the value in Cartesian form so that it is in terms of $P + jQ$.

   (d) Calculate the power factor. (As this circuit contained a compensating capacitor, this value should be close to one. If it is not, the most likely issue is a math error in part a or b of this question.)

3. Use component values that were measured and recorded in table 14.2 to answer the following questions about lab Circuit 3. Use the output voltage data that you collected in table 14.5.

   (a) Use Ohm's law to calculate the RMS value of the current flowing through the load.

   (b) Calculate the impedance of the load.

   (c) Use equation 14.2 to calculate the complex power consumed by the load. Express the value in Cartesian form so that it is in terms of $P + jQ$.

   (d) Calculate the power factor.

4. Use component values that were measured and recorded in table 14.2 to answer the following questions about lab Circuit 4. Use the output voltage data that you collected in table 14.6.

   (a) Use Ohm's law to calculate the RMS value of the current flowing through the load.

   (b) Calculate the impedance of the load.

   (c) Use equation 14.2 to calculate the complex power consumed by the load. Express the value in Cartesian form so that it is in terms of $P + jQ$.

   (d) Calculate the power factor. (As this circuit contained a compensating capacitor, this value should be close to one. If it is not, the most likely issue is a math error in part a or b of this question.)

5. Assuming that there is an inductive load, how does the frequency affect the value of the capacitor that would need to be used to cancel out the phase difference?

6. The following questions pertain to lab Circuit 5, mystery box A, **before** the power factor correction. Use the data that you collected in table 14.7.

   (a) Use equation 14.1 to calculate the complex power consumed by the load. Express the value in Cartesian form so that it is in terms of $P + jQ$.

   (b) Calculate the power factor.

7. The following questions pertain to lab Circuit 5, mystery box A, **after** the power factor correction. Use the data that you collected in table 14.8.

   (a) Use equation 14.1 to calculate the complex power consumed by the load. Express the value in Cartesian form so that it is in terms of $P + jQ$.

   (b) Calculate the power factor. (As this circuit was compensated, this value should be close to one. If it is not, the most likely issue is a math error in part a or b of this question.)

8. The following questions pertain to lab Circuit 6, mystery box B, **before** the power factor correction. Use the data that you collected in table 14.9.

   (a) Use equation 14.1 to calculate the complex power consumed by the load. Express the value in Cartesian form so that it is in terms of $P + jQ$.

   (b) Calculate the power factor.

9. The following questions pertain to lab Circuit 6, mystery box B, **after** the power factor correction. Use the data that you collected in table 14.10.

   (a) Use equation 14.1 to calculate the complex power consumed by the load. Express the value in Cartesian form so that it is in terms of $P + jQ$.

   (b) Calculate the power factor. (As this circuit was compensated, this value should be close to one. If it is not, the most likely issue is a math error in part a or b of this question.)

10. Use component values that were measured and recorded in table 14.11 to answer the following questions about lab Circuit 7. Use the output voltage data that you collected in table 14.12.

   (a) Use Ohm's law to calculate the RMS value of the current flowing through the load.

   (b) Use equation 14.1 to calculate the complex power consumed by the load. Express the value in Cartesian form so that it is in terms of $P + jQ$.

   (c) Based on the theoretical maximum average power calculated in the pre-lab, what percentage of maximum load power did your circuit obtain? (You may need to recalculate the pre-lab value if the parasitic resistance is much different from the assumption made in the pre-lab.)

   (d) If your percentage is less than 90%, first check that your calculations in parts a and b of this question are correct. If they are indeed correct, explain the most likely reasons why the average power consumed by the load was not as high as expected. (Human error is not a reason.)

   (e) If your percentage is less than 90%, what could be done to the load circuit to increase the percentage?