Circuit Analysis Solutions

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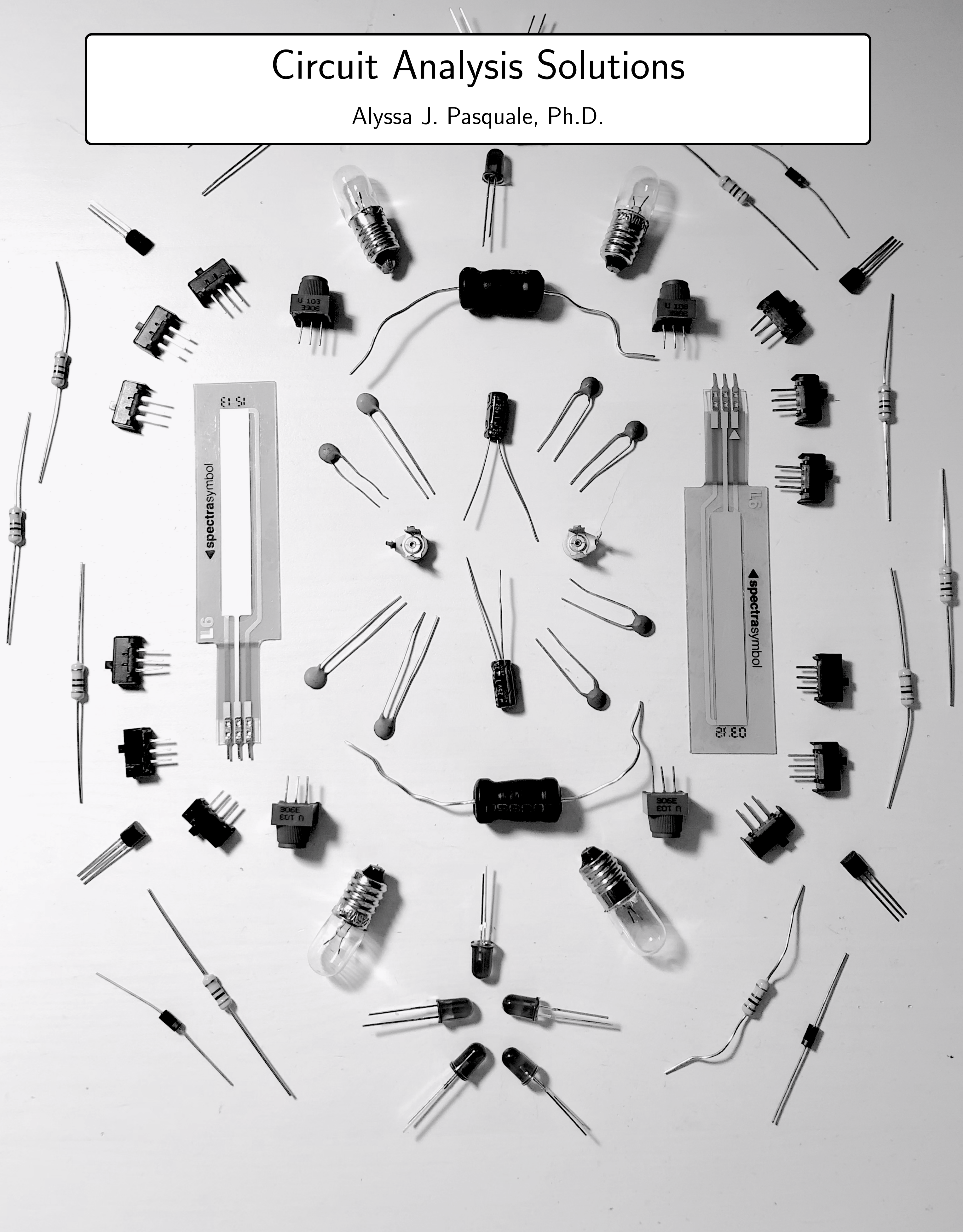

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Author Note

This book is the companion to Circuit Analysis and contains step-by-step solutions to each of the end of chapter questions. The intent behind this solution manual is for students to check their work in detail, and to provide help in case a student doesn't know how to either start or continue working out an example.

I cannot guarantee that this resource is free from typos. I will, however, do my best to implement your feedback if you find any issues with the text. Feel free to e-mail me at pasqualea185@cod.edu with your notes.

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The entirety of this work was created by Alyssa J. Pasquale, Ph.D. The cover photograph is by the author and is a collection of circuit components (capacitors, trimmer pots, trimmer capacitors, soft potentiometers, inductors, toggle switches, transistors, resistors, diodes, light-emitting diodes, and incandescent lamps). All circuit diagrams, equations, and figures in this text were created by the author using LaTeX libraries.

Changelog

Date	Chapter(s)	Description of Change(s)
2021-06-02	all	First edition of this book published online
2021-06-03	11	Changed terminology to inverse Laplace transform
2025-03-17	1	Corrected typo in question 13 solution

1 Chapter 1 Solutions

1.1 Power

1. How much power is supplied by the unknown element shown in figure 1.1?

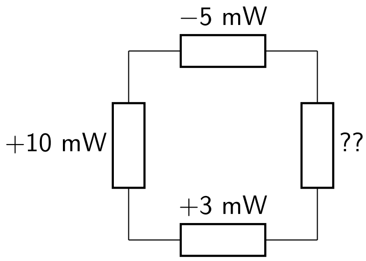


Figure 1.1: Circuit schematic for power question 1.

Use conservation of power to find the unknown quantity.

$$0 = 10 \text{ mW} - 5 \text{ mW} + P + 3 \text{ mW}$$
$$P = -8 \text{ mW}$$

The unknown element supplies 8 mW of power. (The negative sign on the power means that power is supplied. A positive sign means that power is absorbed by the circuit element.)

2. A voltage source supplies 12 V and has a power consumption of 5 W. How much current is the voltage source supplying to the circuit?

Use the instantaneous power equation $P = IV$ to solve for current.

$$I = \frac{5 \text{ W}}{12 \text{ V}}$$
$$= 0.416 \text{ A}$$
$$= 416.67 \text{ mA}$$

3. Which of the circuit elements shown in figure 1.2 (a) absorb power and (b) deliver power? Current flows clockwise through the circuit.

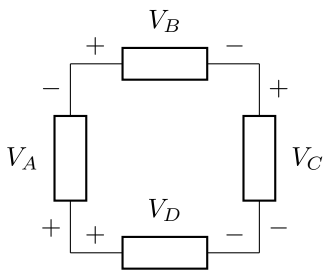


Figure 1.2: Circuit schematic for power question 3.

Analyze the direction of current flow relative to the voltage drops. Current flows from high to low potential in elements A, B, and C. Therefore elements A, B, and C absorb power. Current flows from low to high potential in element D. Therefore element D supplies power.

4. Calculate the power consumed by a circuit when the voltage is $v(t) = 5\cos(2\pi 50t)$ V and the current is $i(t) = 0.1\cos(2\pi 50t)$ A.

Use the instantaneous power equation $p(t) = i(t)v(t)$.

$$
\begin{aligned}
p(t) &= (0.1\cos(2\pi 50t)\ \text{A})\,(5\cos(2\pi 50t)\ \text{V}) \\
&= 0.5\cos^2(2\pi 50t)\ \text{W}
\end{aligned}
$$

5. Calculate the power consumed by a circuit when $v(t) = 3t$ V and $i(t) = 40\delta(t-5)$ mA.

Use the instantaneous power equation $p(t) = i(t)v(t)$.

$$
\begin{aligned}
p(t) &= (40\delta(t-5)\ \text{mA})(3t\ \text{V}) \\
&= 120t\delta(t-5)\ \text{mW}
\end{aligned}
$$

1.2 Sinusoidal Waves

6. What are V_m, V_{DC}, f, and ϕ of the function $v(t) = 3.8\cos(2\pi 20t + 25\pi/180) + 3$ V?

A sinusoidal wave takes the form of $V_m \cos(2\pi ft + \phi\pi/180) + V_{DC}$ (given f in units of Hz and ϕ in units of degrees). Therefore carefully analyze the function to determine the values of each property.

$$V_m = 3.8 \text{ V}$$
$$V_{DC} = 3 \text{ V}$$
$$f = 20 \text{ Hz}$$
$$\phi = 25°$$

7. What are V_m, V_{PP}, and T of the function $v(t) = 2.19\cos(2\pi 40t - 20\pi/180) + 6$ V?

A sinusoidal wave takes the form of $V_m \cos(\frac{2\pi t}{T} + \phi\pi/180) + V_{DC}$ (given ϕ in units of degrees). V_{PP} is equal to twice the amplitude. Therefore carefully analyze the function to determine the values of each property.

$$V_m = 2.19 \text{ V}$$
$$V_{PP} = 4.38 \text{ V}$$
$$T = 0.025 \text{ s}$$

8. What are V_m, V_{DC}, and f of the function shown in figure 1.3?

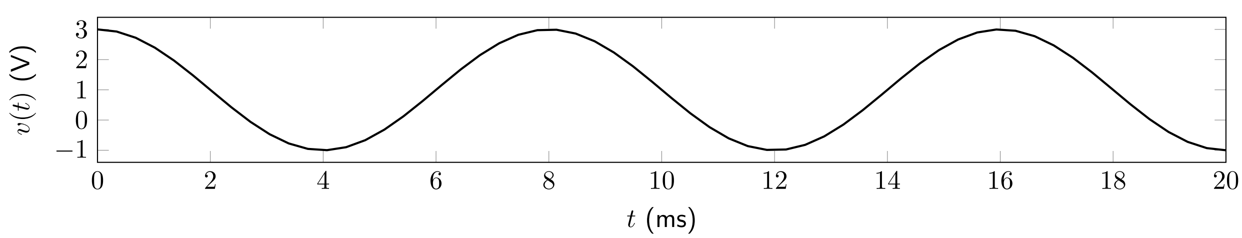


Figure 1.3: Waveform for sinusoidal waves question 8.

The sinusoidal wave voltages oscillate between –1 V and 3 V with an average value of 1 V. The amplitude is equal to the distance between equilibrium (1 V) and peak displacement (3 V).

$$V_m = 3 \text{ V} - 1 \text{ V}$$
$$= 2 \text{ V}$$

The DC offset is equal to the average value of the waveform.

$$V_{DC} = 1 \text{ V}$$

The first peak shown on the graph occurs at 0 s with the second peak occurring at 8 ms. Use the relationship between period and frequency $f = 1/T$ to calculate the frequency of the wave.

$$\begin{aligned} f &= \frac{1}{T} \\ &= \frac{1}{0.008 \text{ s}} \\ &= 125 \text{ Hz} \end{aligned}$$

9. Sketch the function $i(t) = 400\cos(2\pi 100t + 45\pi/180) + 200$ mA.

Identify the waveform properties.

$$\begin{aligned} I_m &= 400 \text{ mA} \\ I_{DC} &= 200 \text{ mA} \\ f &= 100 \text{ Hz} \\ T &= 10 \text{ ms} \\ \phi &= 45^\circ \end{aligned}$$

The minimum and maximum currents can be calculated based on the values of I_m and I_{DC}.

$$\begin{aligned} I_{MAX} &= I_{DC} + I_m = 600 \text{ mA} \\ I_{MIN} &= I_{DC} - I_m = -200 \text{ mA} \end{aligned}$$

The first minimum will occur when the argument of the cosine function is equal to π. 5 ms later the maximum will occur.

$$\begin{aligned} \pi &= 2\pi 100 t_{MIN} + \frac{45\pi}{180} \\ 200 t_{MIN} &= 1 - \frac{45}{180} \\ t_{MIN} &= \frac{1 - \frac{45}{180}}{200} = 0.00375 \text{ s} = 3.75 \text{ ms} \end{aligned}$$

The waveform is shown in figure 1.4.

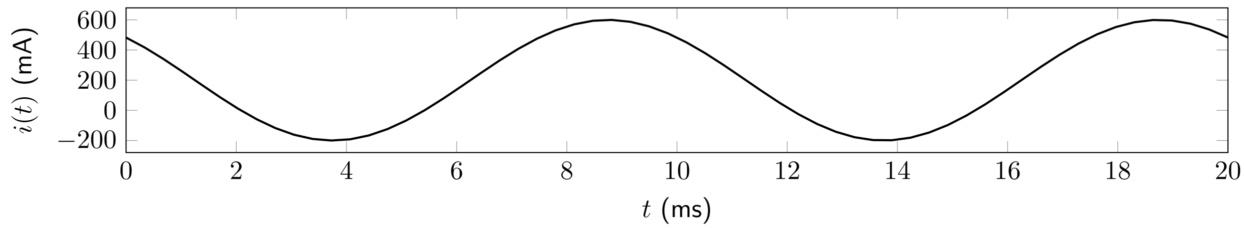


Figure 1.4: Waveform for sinusoidal waves answer 9.

10. What minimum DC offset should be added to $i(t) = 30\cos(2\pi 200t - 50\pi/180)$ mA to prevent any negative values from occurring on the output?

For $i(t)$ to remain positive, the minimum allowable current will be 0 mA. Use the relationship between minimum value, DC offset, and amplitude to calculate this value.

$$I_{MIN} = I_{DC} - I_m$$
$$I_{DC} = I_{MIN} + I_m$$
$$= 0 \text{ mA} + 30 \text{ mA}$$
$$= 30 \text{ mA}$$

1.3 Sources

11. What kind of dependent source is shown in figure 1.5? How do you know?

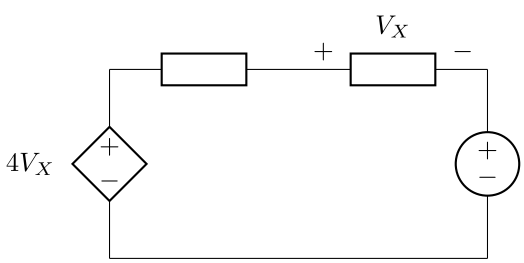


Figure 1.5: Circuit schematic for sources question 11.

The dependent source is a VCVS. The controlling value is V_X, a voltage drop. The symbol on the source is a + and − sign indicating that it is a voltage source.

12. What kind of dependent source is shown in figure 1.6? How do you know?

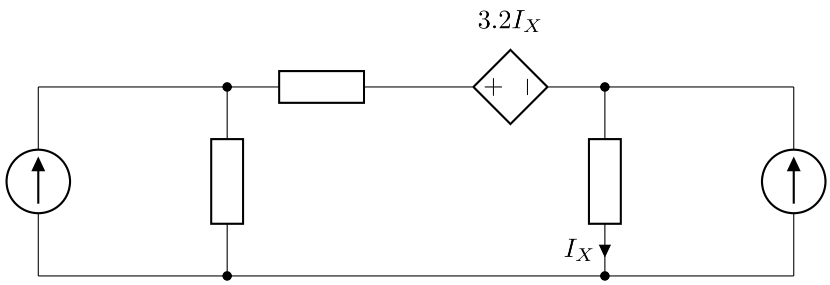


Figure 1.6: Circuit schematic for sources question 12.

The dependent source is a CCVS. The controlling value is I_X, a current flow. The symbol on the source is a + and − sign indicating that it is a voltage source.

13. Calculate the current supplied by the dependent source as shown in figure 1.7.

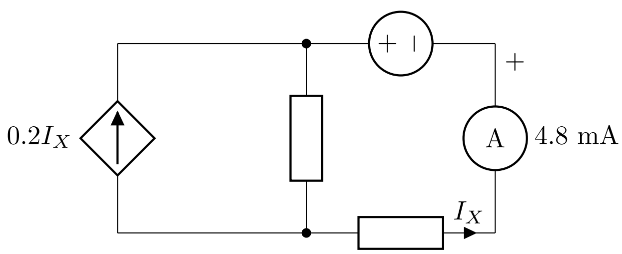


Figure 1.7: Circuit schematic for sources question 13.

The ammeter reads a value of 4.8 mA, and the high potential end of the ammeter is depicted with a + sign. The controlling current (I_X) is therefore in the opposite direction as the measured current and has a value of -4.8 mA. Use the relationship on the CCCS to determine the value of the source current.

$$\begin{aligned} I_S &= 0.2 I_X \\ &= 0.2(-4.8 \text{ mA}) \\ &= -0.96 \text{ mA} \end{aligned}$$

14. Reduce figure 1.8 to a circuit that contains only one independent source.

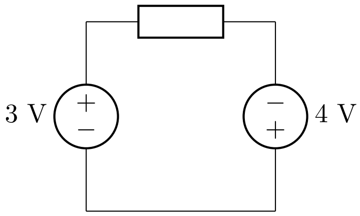


Figure 1.8: Circuit schematic for sources question 14.

The two voltage sources can be added because they are in series with each other. The reduced circuit diagram is shown in figure 1.9.

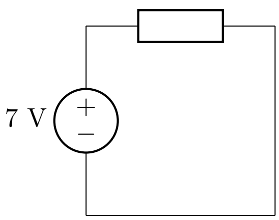


Figure 1.9: Reduced circuit schematic for sources question 14.

15. Reduce the circuit given in figure 1.10 as much as possible.

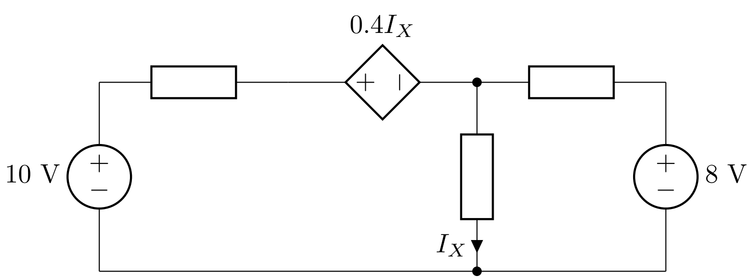


Figure 1.10: Circuit schematic for sources question 15.

Combine the 10 V independent source with the dependent source. The 8 V source cannot be combined because it is not in series with the other sources. The reduced circuit diagram is shown in figure 1.11.

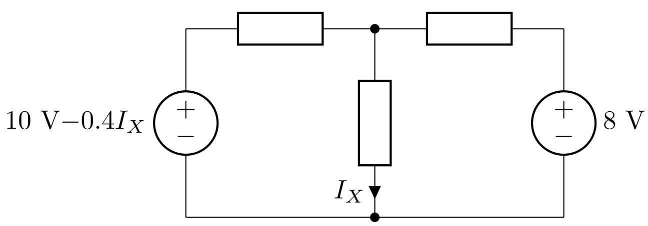


Figure 1.11: Reduced circuit schematic for sources question 15.

1.4 Elementary Signals

16. Find an equation for the signal shown in figure 1.12.

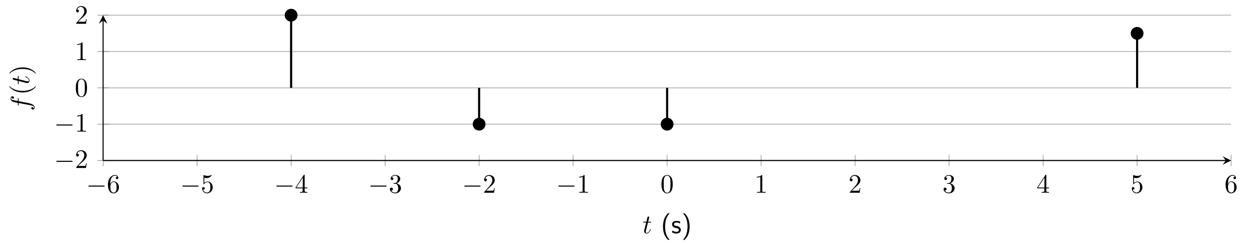


Figure 1.12: Signal for elementary signals question 16.

The left-most delta function occurs at $t = -4$ s and has an amplitude of 2. The next occurs at $t = -2$ s and has an amplitude of -1. The next occurs at $t = 0$ s and has an amplitude of -1. The right-most delta function occurs at $t = 5$ s and has an amplitude of 1.5. Add each of the delta functions together.

$$f(t) = 2\delta(t+4) - \delta(t+2) - \delta(t) + 1.5\delta(t-5)$$

17. Find an equation for the signal shown in figure 1.13.

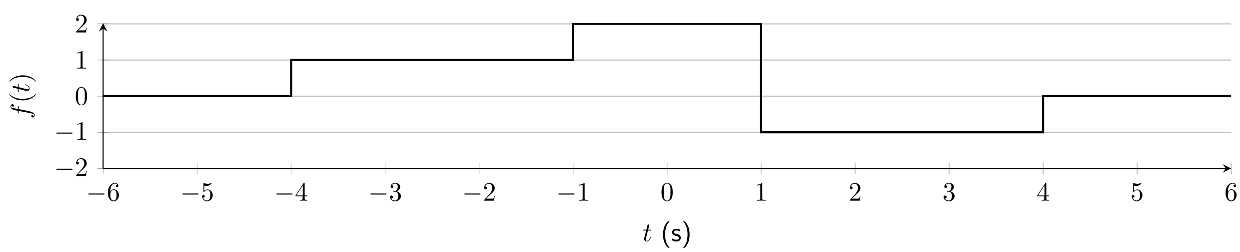


Figure 1.13: Signal for elementary signals question 17.

Option one is to use step functions to create this signal. The first step function occurs at $t = -4$ s with an amplitude of 1. At $t = -1$ s another step function with amplitude of 1 is added. At $t = 1$ s another step function of amplitude -3 is added. At $t = 4$ s a step function with amplitude of 1 is added. After that point, the signal remains constant at zero.

$$f(t) = u(t+4) + u(t+1) - 3u(t-1) + u(t-4)$$

Option two is to use the rectangular pulse function to create this signal. The first pulse is centered at $t = -2.5$ s, has a width of 3 s, and an amplitude of 1. The second pulse is centered at $t = 0$ s, has a width of 2 s, and an amplitude of 2. The third pulse is centered at $t = 2.5$ s, has a width of 3 s, and an amplitude of −1.

$$f(t) = \text{rect}\left(\frac{t+2.5}{3}\right) + 2\ \text{rect}\left(\frac{t}{2}\right) - \text{rect}\left(\frac{t-2.5}{3}\right)$$

18. Sketch the function $f(t) = 3\ \mathbf{tri}(t+2) - 2\ \mathbf{tri}\left(\frac{t-5}{5}\right)$.

The first term in the expression is a triangle function centered at $t = -2$ s, has a half-width of 1 s, and an amplitude of 3. The second term in the expression is centered at $t = 5$ s, has a half-width of 5 s, and an amplitude of −2. The sketch is shown in figure 1.14.

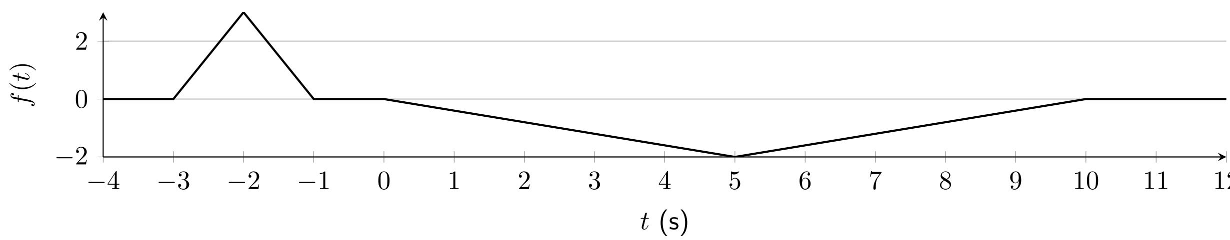


Figure 1.14: Signal function for elementary signals question 18.

19. Sketch the function $f(t) = 3u(t+5) + \frac{t}{2}u(t)$.

The signal has a value of zero until $t = -5$ s when a step function with amplitude 3. The function will have a constant amplitude of 3 until $t = 0$, at which time a ramp function with a slope of one-half is added. The sketch is shown in figure 1.15.

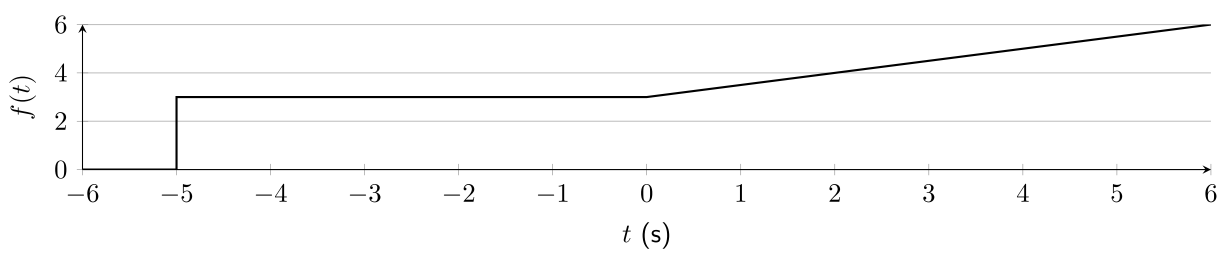


Figure 1.15: Signal function for elementary signals question 19.

20. If the charge in a circuit is measured as $q(t) = 2\ \mathbf{tri}\left(\frac{t+6}{2}\right) + 0.5tu(t) - 1.5(t-4)u(t-4) + 1.5(t-8)u(t-8) - 0.5(t-12)u(t-12)$ C, sketch the current.

Current is equal to the derivative of charge. Charge is plotted below in figure 1.16 to make it easier to visualize.

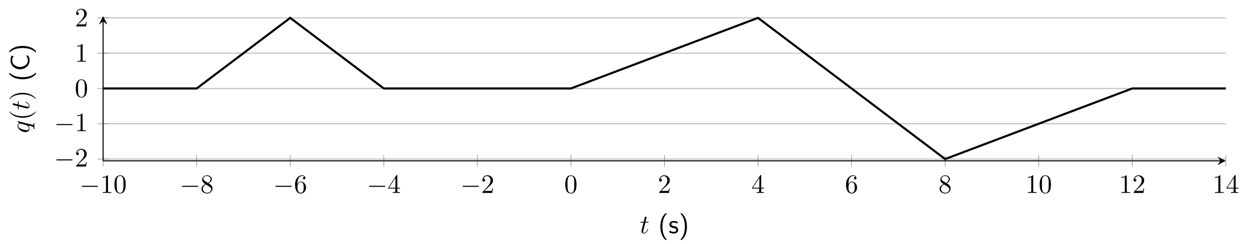


Figure 1.16: Signal function (charge) for elementary signals question 20.

Calculate the slope at each interval of time. Current is shown below in figure 1.17.

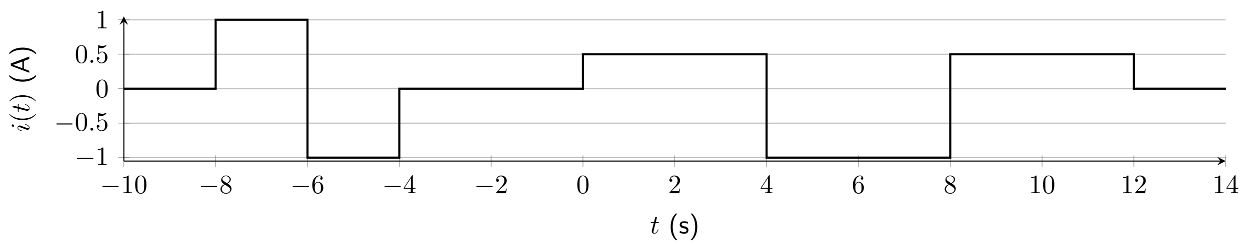


Figure 1.17: Signal function (current) for elementary signals question 20.

2 Chapter 2 Solutions

2.1 Equivalent Resistance

1. Calculate the equivalent resistance of the resistors shown in figure 2.1.

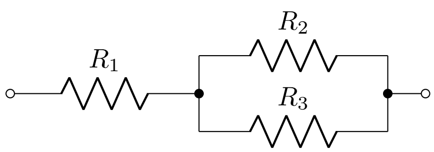


Figure 2.1: Circuit diagram for equivalent resistance question 1.

Combine R_2 and R_3 in parallel. Then combine that result in series with R_1.

$$R_{EQ} = R_1 + \frac{R_2 R_3}{R_2 + R_3}$$

2. Minimize the circuit diagram shown in figure 2.2 as much as possible.

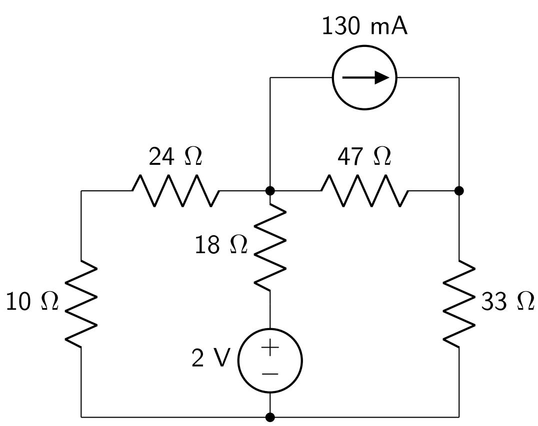


Figure 2.2: Circuit diagram for equivalent resistance question 2.

The only things that can be combined in this circuit is the series combination of 10 Ω and 24 Ω. The reduced circuit diagram is shown in figure 2.3.

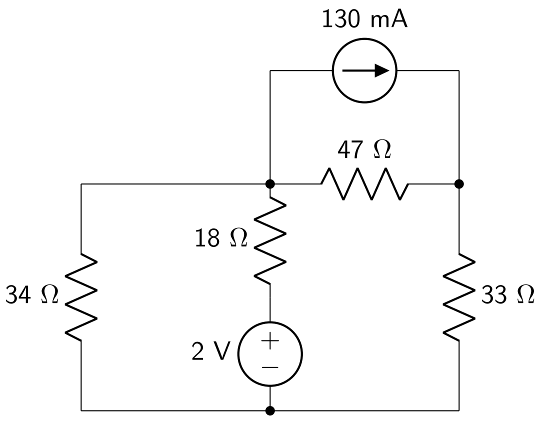


Figure 2.3: Circuit diagram for equivalent resistance answer 2.

3. Use the circuit diagram shown in figure 2.4 to calculate the equivalent resistance between nodes a and b. Each resistor has a value of 1 kΩ.

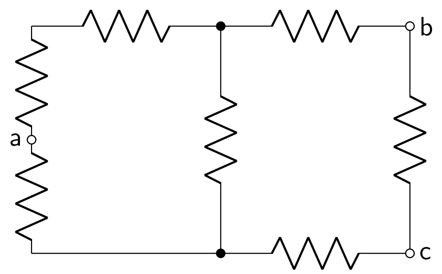
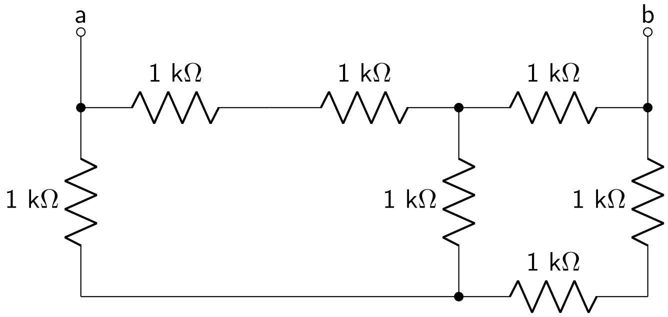
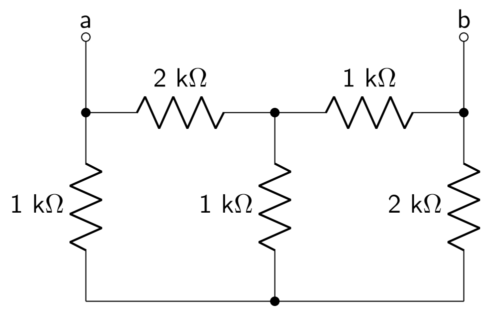


Figure 2.4: Circuit diagram for equivalent resistance questions 3.

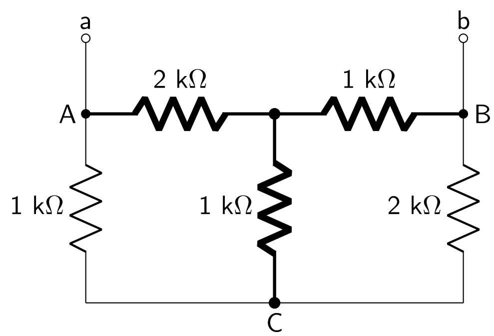
Re-draw the circuit.



Combine all series resistors.



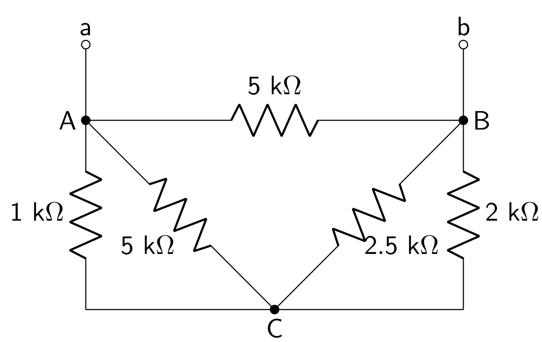
Either a wye-delta transform or a delta-wye transform can be performed to solve this circuit. The following solution utilizes a wye-delta transform.



Calculate each of the delta resistances.

$$R_1 = \frac{(2\text{ k}\Omega)(1\text{ k}\Omega) + (1\text{ k}\Omega)(1\text{ k}\Omega) + (2\text{ k}\Omega)(1\text{ k}\Omega)}{1\text{ k}\Omega} = 5\text{ k}\Omega$$
$$R_2 = \frac{(2\text{ k}\Omega)(1\text{ k}\Omega) + (1\text{ k}\Omega)(1\text{ k}\Omega) + (2\text{ k}\Omega)(1\text{ k}\Omega)}{2\text{ k}\Omega} = 2.5\text{ k}\Omega$$
$$R_3 = \frac{(2\text{ k}\Omega)(1\text{ k}\Omega) + (1\text{ k}\Omega)(1\text{ k}\Omega) + (2\text{ k}\Omega)(1\text{ k}\Omega)}{1\text{ k}\Omega} = 5\text{ k}\Omega$$

Re-draw the circuit.



The equivalent resistance can be calculated using parallel and series combinations.

$$\begin{aligned} R_{EQ} &= ((2\ \text{k}\Omega)//(2.5\ \text{k}\Omega) + (1\ \text{k}\Omega)//(5\ \text{k}\Omega))//5\ \text{k}\Omega \\ &= (1.11\ \text{k}\Omega + 0.83\ \text{k}\Omega)//5\ \text{k}\Omega \\ &= 1.94\ \text{k}\Omega//5\ \text{k}\Omega \\ &= 1.4\ \text{k}\Omega \end{aligned}$$

4. Use the circuit diagram shown in figure 2.5 to calculate the equivalent resistance between nodes b and c. Each resistor has a value of 1 kΩ.

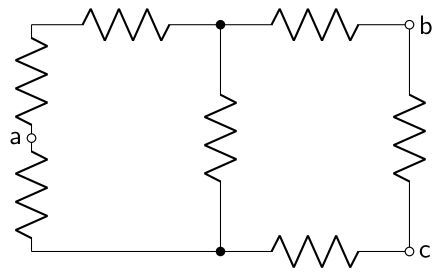
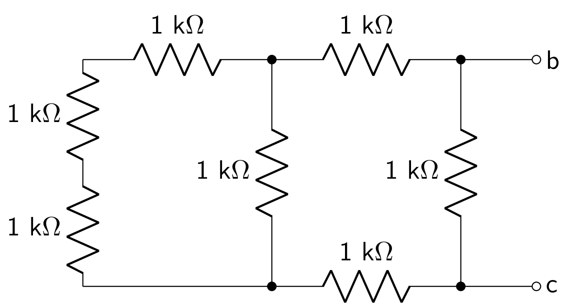


Figure 2.5: Circuit diagram for equivalent resistance questions 4.

Re-draw the circuit.



The equivalent resistance can be calculated using parallel and series combinations.

$$\begin{aligned} R_{EQ} &= ((3\ \text{k}\Omega//1\ \text{k}\Omega) + 2\ \text{k}\Omega)//1\ \text{k}\Omega \\ &= (0.75\ \text{k}\Omega + 2\ \text{k}\Omega)//1\ \text{k}\Omega \\ &= 2.75\ \text{k}\Omega//1\ \text{k}\Omega \\ &= 0.73\ \text{k}\Omega \end{aligned}$$

5. Calculate the equivalent resistance of the resistors shown in figure 2.6.

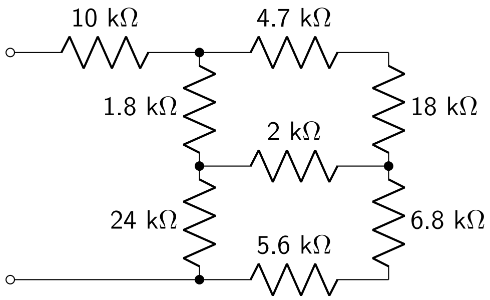
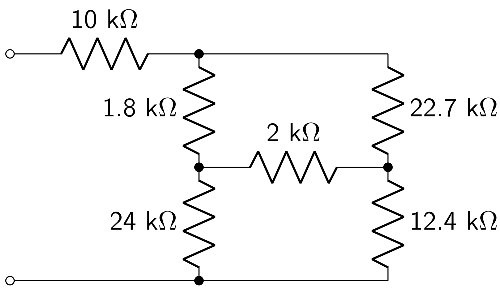
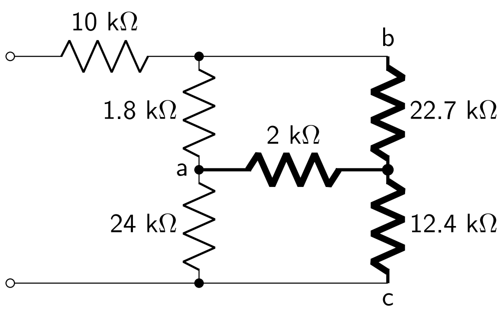


Figure 2.6: Circuit diagram for equivalent resistance question 5.

Combine all series resistances and re-draw.



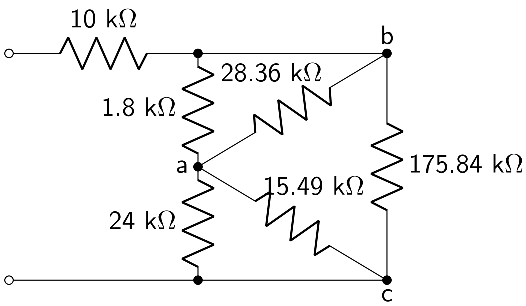
Either a wye-delta transform or a delta-wye transform can be done to reduce the circuit. The following solution utilizes a wye-delta transform.



Calculate each of the delta resistances.

$$R_1 = \frac{(2\text{ k}\Omega)(12.4\text{ k}\Omega) + (12.4\text{ k}\Omega)(22.7\text{ k}\Omega) + (2\text{ k}\Omega)(22.7\text{ k}\Omega)}{12.4\text{ k}\Omega} = 28.36\text{ k}\Omega$$
$$R_2 = \frac{(2\text{ k}\Omega)(12.4\text{ k}\Omega) + (12.4\text{ k}\Omega)(22.7\text{ k}\Omega) + (2\text{ k}\Omega)(22.7\text{ k}\Omega)}{2\text{ k}\Omega} = 175.84\text{ k}\Omega$$
$$R_3 = \frac{(2\text{ k}\Omega)(12.4\text{ k}\Omega) + (12.4\text{ k}\Omega)(22.7\text{ k}\Omega) + (2\text{ k}\Omega)(22.7\text{ k}\Omega)}{22.7\text{ k}\Omega} = 15.49\text{ k}\Omega$$

Re-draw the circuit.



The equivalent resistance can be calculated using series and parallel combinations.

$$\begin{aligned}
R_{EQ} &= 10\text{ k}\Omega + ((1.8\text{ k}\Omega//28.36\text{ k}\Omega) + (24\text{ k}\Omega//15.49\text{ k}\Omega))//157.84\text{ k}\Omega \\
&= 10\text{ k}\Omega + (1.69\text{ k}\Omega + 9.41\text{ k}\Omega)//175.84\text{ k}\Omega \\
&= 10\text{ k}\Omega + 11.11\text{ k}\Omega//175.84\text{ k}\Omega \\
&= 10\text{ k}\Omega + 10.45\text{ k}\Omega \\
&= 20.45\text{ k}\Omega
\end{aligned}$$

2.2 Ohm's Law

6. If the voltage source can supply a maximum current of 2 A, what is the minimum value of R_X that can be used in the circuit shown in figure 2.7.

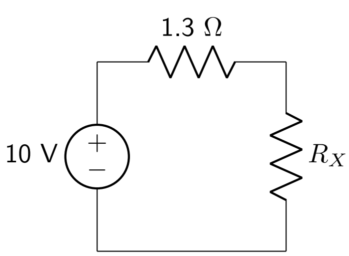


Figure 2.7: Circuit diagram for Ohm's law question 6.

Use Ohm's law.

$$\begin{aligned}
R &= \frac{V}{I} \\
1.3\ \Omega + R_X &= \frac{10\text{ V}}{2\text{ A}} \\
&= 5\ \Omega \\
R_X &= 3.7\ \Omega
\end{aligned}$$

7. Calculate I_X in the circuit shown in figure 2.8.

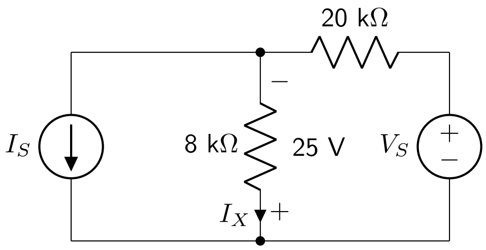


Figure 2.8: Circuit diagram for Ohm's law question 7.

Use Ohm's law. Pay attention to the direction of current flow, which is defined opposite to the direction of the voltage.

$$I_X = \frac{-25 \text{ V}}{8 \text{ k}\Omega} = -3.125 \text{ mA}$$

8. Calculate the amount of power consumed by the 20 Ω resistor in the circuit shown in figure 2.9.

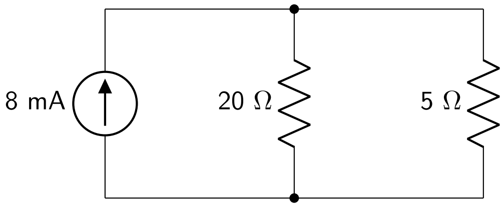


Figure 2.9: Circuit diagram for Ohm's law question 8.

Calculate the equivalent resistance of the circuit to determine the voltage dropped over each resistor.

$$R_{EQ} = \frac{(20 \ \Omega)(5 \ \Omega)}{20 \ \Omega + 5 \ \Omega)} = 4 \ \Omega$$
$$V = (0.008 \text{ A})(4 \ \Omega) = 0.032 \text{ V}$$

Use the power equation to determine the power consumed by the 20 Ω resistor.

$$\begin{aligned} P &= \frac{V^2}{R} \\ &= \frac{(0.032 \text{ V})^2}{20 \ \Omega} \\ &= \frac{0.001024 \text{ V}^2}{20 \ \Omega} \\ &= 51.2 \times 10^{-6} \text{ W} \\ &= 51.2 \ \mu\text{W} \end{aligned}$$

9. Calculate the amount of power supplied by the load in the circuit shown in figure 2.10.

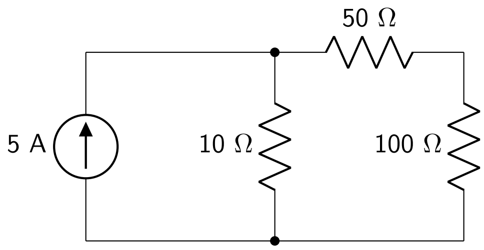


Figure 2.10: Circuit diagram for Ohm's law question 9.

Calculate the equivalent resistance of the circuit.

$$\begin{aligned} R_{EQ} &= (10\ \Omega)//(50\ \Omega + 100\ \Omega) \\ &= (10\ \Omega)//(150\ \Omega) \\ &= 9.375\ \Omega \end{aligned}$$

Use the power equation to calculate the power supplied by the load.

$$\begin{aligned} P &= I^2R \\ &= (5\ \text{A})^2(9.375\ \Omega) \\ &= 234.375\ \text{W} \end{aligned}$$

10. Calculate the minimum value of R that can be used to keep the power consumed by either resistor to less than or equal to 250 mW in the circuit shown in figure 2.11.

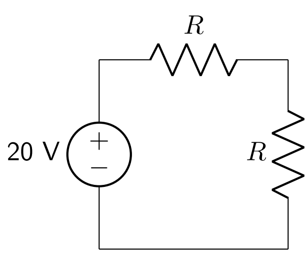


Figure 2.11: Circuit diagram for Ohm's law question 10.

Each resistor will experience a voltage drop of 10 V. Use the power equation to solve for R.

$$\begin{aligned} R &= \frac{V^2}{P} \\ &= \frac{(10\text{ V})^2}{0.25\text{ W}} \\ &= \frac{100\text{ V}^2}{0.25\text{ W}} \\ &= 400\ \Omega \end{aligned}$$

2.3 Voltage and Current Divider

11. Use the voltage divider rule to calculate V_X in the circuit shown in figure 2.12.

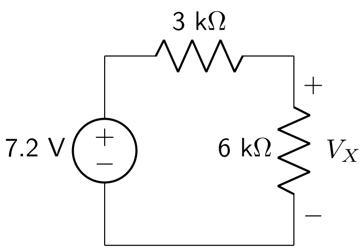


Figure 2.12: Circuit diagram for voltage and current divider question 11.

Use the voltage divider rule.

$$\begin{aligned} V_X &= 7.2\text{ V}\left(\frac{6\text{ k}\Omega}{3\text{ k}\Omega + 6\text{ k}\Omega}\right) \\ &= 7.2\text{ V}(0.67) \\ &= 4.8\text{ V} \end{aligned}$$

12. Use the current divider rule to calculate I_X in the circuit shown in figure 2.13.

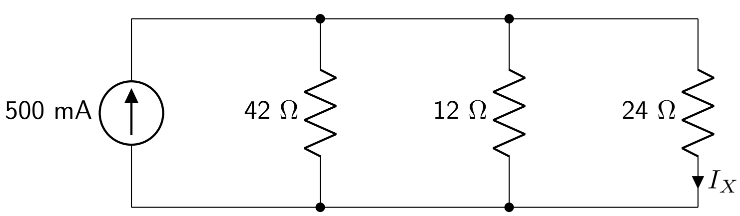


Figure 2.13: Circuit diagram for voltage and current divider question 12.

Calculate the equivalent resistance of the circuit.

$$R_{EQ} = \frac{1}{(\frac{1}{42\ \Omega}) + (\frac{1}{12\ \Omega}) + (\frac{1}{24\ \Omega})}$$
$$= 6.72\ \Omega$$

Use the current divider rule.

$$I_X = 500\ \text{mA}\left(\frac{6.72\ \Omega}{24\ \Omega}\right)$$
$$= 140\ \text{mA}$$

13. Use the voltage divider rule to calculate V_X in the circuit shown in figure 2.14.

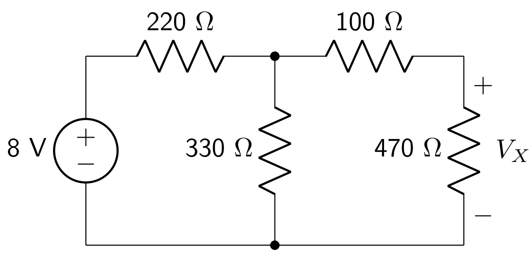
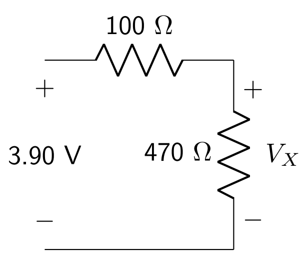


Figure 2.14: Circuit diagram for voltage and current divider question 13.

Use a voltage divider to calculate the voltage dropped over the 300 Ω resistor.

$$V_A = 8\ \text{V}\left(\frac{(330\ \Omega)//(100\ \Omega + 470\ \Omega)}{220\ \Omega + (330\ \Omega)//(100\ \Omega + 470\ \Omega)}\right)$$
$$= 8\ \text{V}\left(\frac{209\ \Omega}{220\ \Omega + 209\ \Omega}\right)$$
$$= 3.90\ \text{V}$$

The circuit can now be re-drawn.



Use a voltage divider to calculate V_X.

$$V_X = 3.90\ \text{V}\left(\frac{470\ \Omega}{100\ \Omega + 470\ \Omega}\right)$$
$$= 3.21\ \text{V}$$

14. Use the current divider rule to calculate I_X in the circuit shown in figure 2.15.

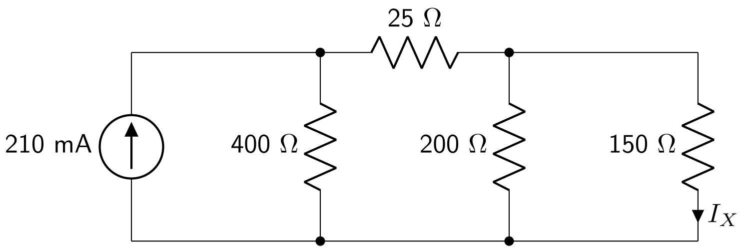
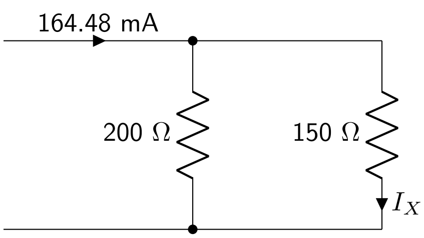


Figure 2.15: Circuit diagram for voltage and current divider question 14.

Use the current divider rule to calculate the current flowing through the 25 Ω resistor.

$$I_A = 210\ \text{mA}\left(\frac{400\ \Omega//(200\ \Omega//150\ \Omega + 25\ \Omega)}{200\ \Omega//150\ \Omega + 25\ \Omega}\right)$$
$$= 210\ \text{mA}\left(\frac{400\ \Omega//110.71\ \Omega}{110.71\ \Omega}\right)$$
$$= 210\ \text{mA}\left(\frac{86.71\ \Omega}{110.71\ \Omega}\right)$$
$$= 164.48\ \text{mA}$$

The circuit can be re-drawn.



Use a current divider to calculate I_X.

$$I_A = 164.48\ \text{mA}\left(\frac{200\ \Omega//150\ \Omega}{150\ \Omega}\right)$$
$$= 164.48\ \text{mA}\left(\frac{85.71\ \Omega}{110.71\ \Omega}\right)$$
$$= 93.99\ \text{mA}$$

15. Use the voltage divider rule to calculate V_{X1} and V_{X2} in the circuit shown in figure 2.16.

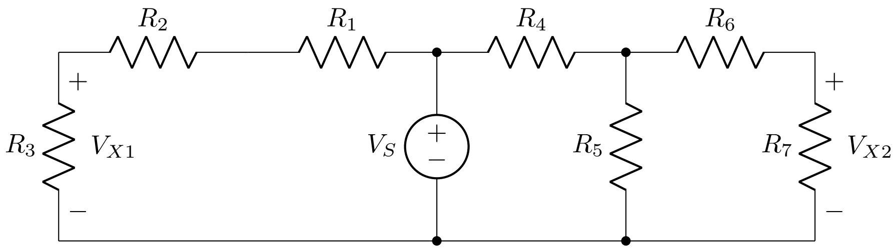
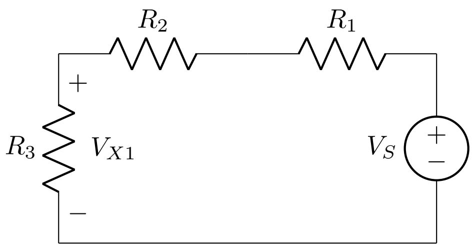


Figure 2.16: Circuit diagram for voltage and current divider question 15.

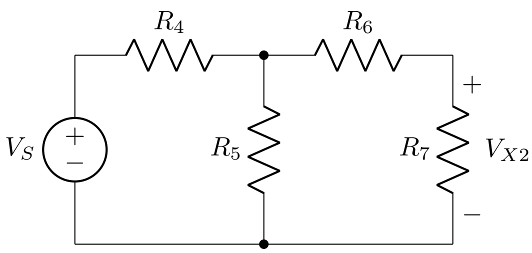
Both "halves" of the circuit are in parallel with each other. From a perspective of voltage, they can be treated independently. Re-draw the left-half of the circuit to calculate V_{X1}.



Use a voltage divider to solve for V_{X1}.

$$V_{X1} = V_S \left(\frac{R_3}{R_1 + R_2 + R_3} \right)$$

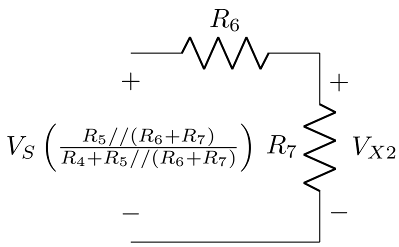
Re-draw the right-half of the circuit to calculate V_{X2}.



Use a voltage divider to calculate the voltage dropped over resistor R_5.

$$V_A = V_S \left(\frac{R_5//(R_6 + R_7)}{R_4 + R_5//(R_6 + R_7)} \right)$$

The circuit can be re-drawn.



Use a voltage divider to calculate V_{X2}.

$$V_{X2} = V_S\left(\frac{R_5//(R_6+R_7)}{R_4+R_5//(R_6+R_7)}\right)\left(\frac{R_7}{R_6+R_7}\right)$$

2.4 Kirchhoff's Laws

16. Calculate V_X in the circuit shown in figure 2.17.

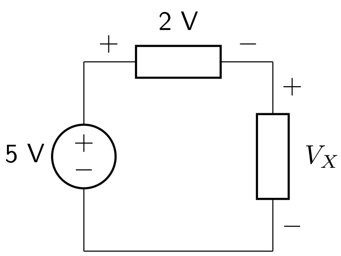


Figure 2.17: Circuit diagram for Kirchhoff's laws question 16.

Use KVL to solve for V_X.

$$5\ \text{V} = 2\ \text{V} + V_X$$
$$3\ \text{V} = V_X$$

17. Calculate I_X in the circuit shown in figure 2.18.

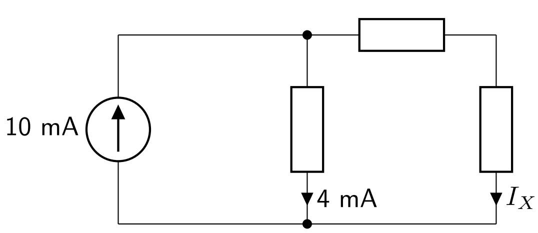


Figure 2.18: Circuit diagram for Kirchhoff's laws question 17.

Use KCL to solve for I_X.

$$10 \text{ mA} = 4 \text{ mA} + I_X$$
$$6 \text{ mA} = I_X$$

18. Calculate I_X in the circuit shown in figure 2.19.

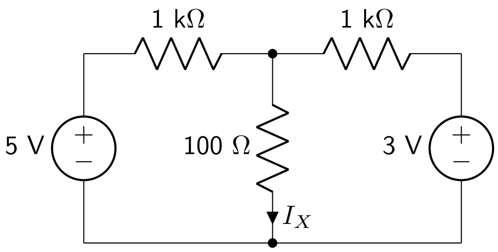
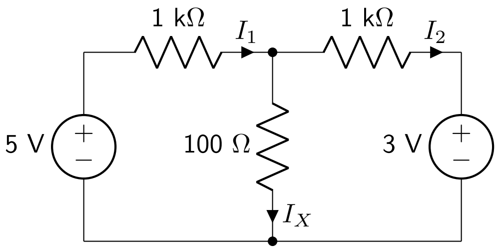


Figure 2.19: Circuit diagram for Kirchhoff's laws question 18.

Re-draw the circuit to define each branch current.



Perform KCL at the node connecting each of the three resistors.

$$I_1 - I_X - I_2 = 0$$

Perform KVL around the left loop, and then perform KVL around the right loop. (All units are in V, mA, and kΩ.)

$$5 = I_1 + 0.1I_x$$
$$-3 = -0.1I_X + I_2$$

Place all three equations into form $\alpha I_1 + \beta I_2 + \gamma I_X = c$, and then place each coefficient into a matrix.

$$\begin{bmatrix} 1 & -1 & -1 & 0 \\ 1 & 0 & 0.1 & 5 \\ 0 & 1 & -0.1 & -3 \end{bmatrix}$$

Solve the matrix. $I_X = 6.67$ mA.

19. Calculate V_X in the circuit shown in figure 2.20.

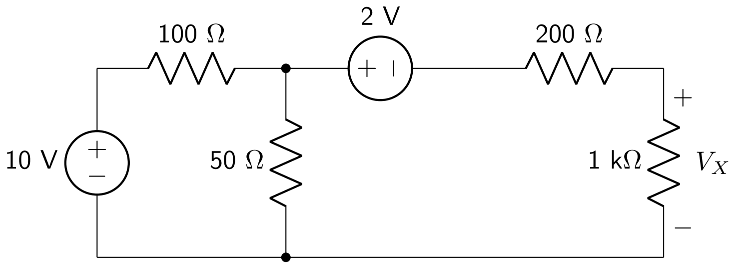
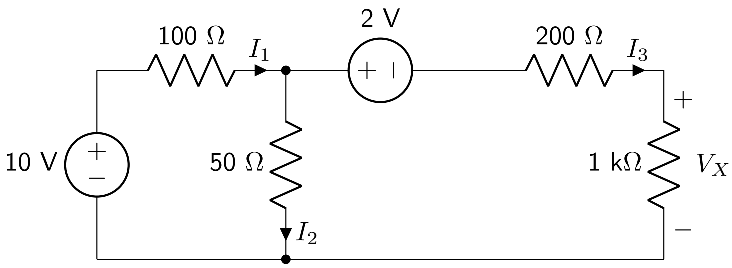


Figure 2.20: Circuit diagram for Kirchhoff's laws question 19.

Re-draw the circuit to define each branch current



Perform KCL at the node connecting the 100 Ω resistor, the 50 Ω resistor, and the 2 V source.

$$I_1 - I_2 - I_3 = 0$$

Perform KVL around the left loop, and then perform KVL around the right loop. (All units are in V, mA, and kΩ.)

$$10 = 0.1I_1 + 0.05I_2$$

$$-2 = -0.05I_2 + 1.2I_3$$

Place all three equations into form $\alpha I_1 + \beta I_2 + \gamma I_3 = c$, and then place each coefficient into a matrix.

$$\begin{bmatrix} 1 & -1 & -1 & 0 \\ 0.1 & 0.05 & 0 & 10 \\ 0 & -0.05 & 1.2 & -2 \end{bmatrix}$$

Solve the matrix for I_3, which is 1.08 mA. Then use Ohm's law to calculate V_X.

$$V_X = (1.08 \text{ mA})(1 \text{ k}\Omega)$$
$$= 1.08 \text{ V}$$

20. Calculate V_X in the circuit shown in figure 2.21.

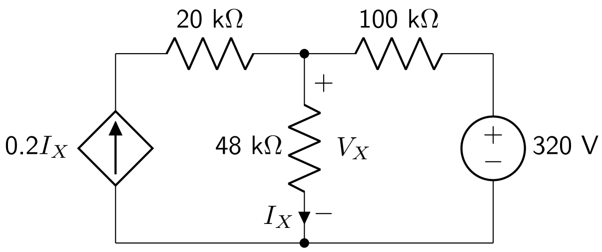


Figure 2.21: Circuit diagram for Kirchhoff's laws question 20.

Note that the branch current through the 20 kΩ resistor is equal to $0.2I_X$. The only branch current that needs to be defined is the branch current through the 100 kΩ resistor, defined here as I_2. Perform KCL at the node connecting the three resistors.

$$0.2I_X - I_X - I_2 = 0$$

The only perfect loop is the right-hand loop. Perform KVL around the loop. (All units are in V, mA, and kΩ.)

$$-48I_X + 100I_2 = -320$$

Place all equations into form $\alpha I_X + \beta I_2 = c$, and then place each coefficient into a matrix.

$$\begin{bmatrix} -0.8 & -1 & 0 \\ -48 & 100 & -320 \end{bmatrix}$$

Solve the matrix for I_X, which is 2.5 mA. Then use Ohm's law to calculate V_X.

$$V_X = (2.5 \text{ mA})(48 \text{ k}\Omega)$$
$$= 120 \text{ V}$$

2.5 Mesh Analysis

21. Calculate mesh currents I_A and I_B in the circuit shown in figure 2.22.

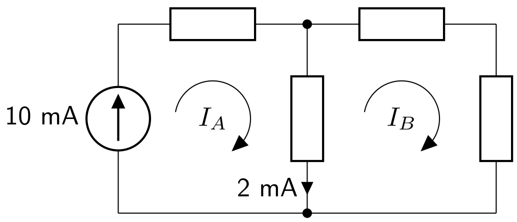


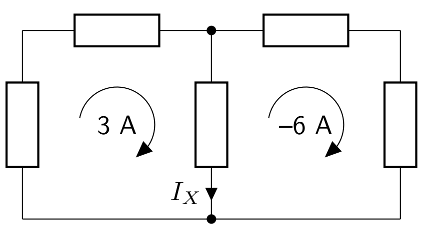
Figure 2.22: Circuit diagram for mesh analysis question 21.

Because mesh I_A contains a current source, $I_A = 10$ mA. Use the relationship between branch and mesh currents to calculate I_B.

$$I_A - I_B = 2 \text{ mA}$$
$$10 \text{ mA} - I_B = 2 \text{ mA}$$
$$-I_B = -8 \text{ mA}$$
$$I_B = 8 \text{ mA}$$

22. A branch is shared by two clockwise meshes. The left mesh current is 3 A and the right mesh current is –6 A. Calculate the branch current.

Draw the circuit to define the meshes and branch.



Use the relationship between branch and mesh currents to calculate I_X.

$$I_X = 3 \text{ A} - (-6 \text{ A})$$
$$= 9 \text{ A}$$

23. Calculate mesh current I_X in the circuit shown in figure 2.23. Assume that each mesh contains at least one linear circuit element.

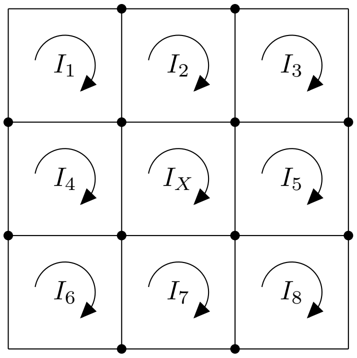


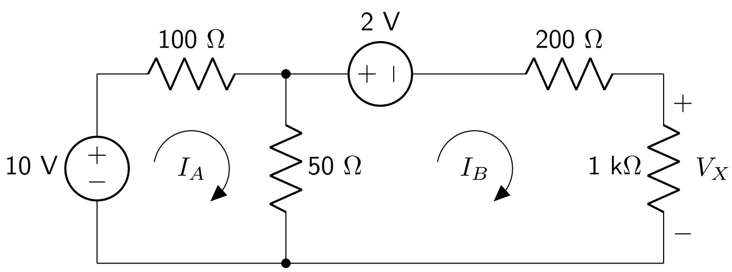
Figure 2.23: Circuit diagram for mesh analysis question 23.

Note that all of the numbered mesh currents are also branch currents. The only mesh current that isn't a branch current is in the center (I_X). Use the relationship between branch and mesh currents to calculate I_X.

$$I_X = -I_2 - I_4 - I_5 - I_7$$

24. Use mesh analysis to calculate V_X in the circuit shown in figure 2.20 (in the Kirchhoff's laws section).

Draw the circuit to define the mesh currents.



Derive the mesh equations. (All units are in V, mA, and kΩ.)

$$10 = 0.1I_A + 0.05(I_A - I_B)$$
$$-2 = 0.05(I_B - I_A) + 1.2I_B$$

Place all equations into form $\alpha I_A + \beta I_B = c$, and then place each coefficient into a matrix.

$$\begin{bmatrix} 0.15 & -0.05 & 10 \\ -0.05 & 1.25 & -2 \end{bmatrix}$$

Solve the matrix for I_B, which is 1.08 mA. Then use Ohm's law to calculate V_X.

$$V_X = (1.08 \text{ mA})(1 \text{ k}\Omega)$$
$$= 1.08 \text{ V}$$

Note that this should be the same answer as question 19, and it is.

25. Use mesh analysis to calculate V_X in the circuit shown in figure 2.24.

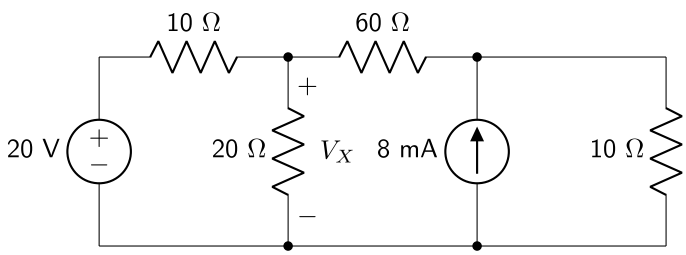
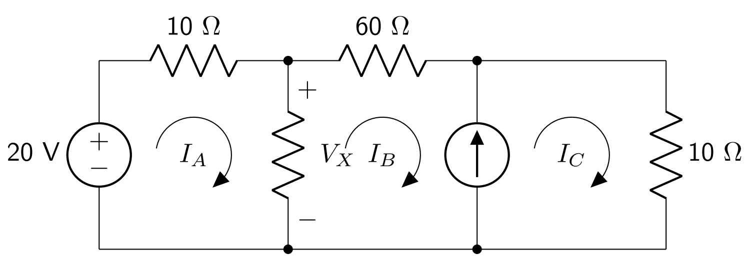


Figure 2.24: Circuit diagram for mesh analysis question 25.

Redraw the circuit to define the mesh currents. (Note that some parameter values have been removed to make it easier to read the mesh current labels.)



Derive equations for the left mesh and the supermesh. (All units are in V, A, and Ω.)

$$20 = 10I_A + 20(I_A - I_B)$$
$$0 = 20(I_B - I_A) + 60I_B + 10I_C$$

Derive a KCL equation at the intersecting node of the supermesh.

$$I_B + 0.008 - I_C = 0$$

Place all three equations into form $\alpha I_A + \beta I_B + \gamma I_C = c$, and then place each coefficient into a matrix.

$$\begin{bmatrix} 30 & -20 & 0 & 20 \\ -20 & 80 & 10 & 0 \\ 0 & 1 & -1 & -0.008 \end{bmatrix}$$

Solve the matrix for I_A and I_B.

$$I_A = 0.782 \text{ A}$$
$$I_B = 0.173 \text{ A}$$

Use Ohm's law to calculate V_X.

$$V_X = (0.782 \text{ A} - 0.173 \text{ A})(20\ \Omega)$$
$$= 12.18 \text{ V}$$

3 Chapter 3 Solutions

3.1 Superposition

1. Use superposition to calculate V_X in the circuit shown in figure 3.1.

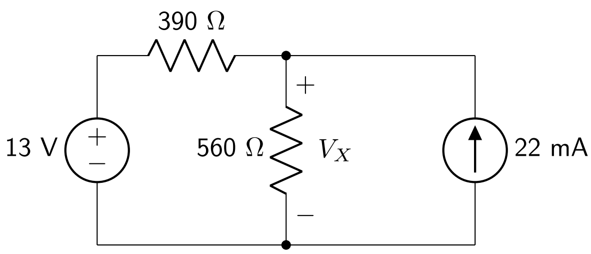
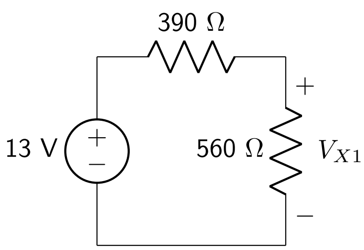


Figure 3.1: Circuit diagram for superposition question 1.

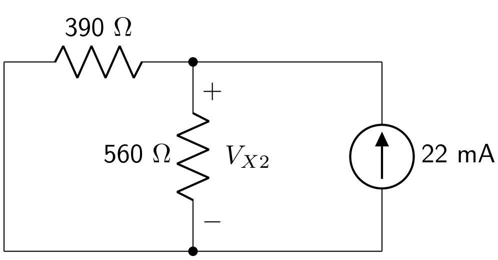
Draw the circuit with only the voltage source activated.



Use the voltage divider rule to calculate V_{X1}.

$$V_{X1} = 13\text{ V}\left(\frac{560\ \Omega}{390\ \Omega + 560\ \Omega}\right)$$
$$= 7.66\text{ V}$$

Draw the circuit with only the current source activated.



Combine the resistors in parallel and use Ohm's law to calculate V_{X2}.

$$\begin{aligned} V_{X2} &= (560\ \Omega//390\ \Omega)(0.022\ \text{A}) \\ &= (229.89\ \Omega)(0.022\ \text{mA}) \\ &= 5.06\ \text{V} \end{aligned}$$

Add the two values to find the voltage V_X.

$$\begin{aligned} V_X &= V_{X1} + V_{X2} \\ &= 7.66\ \text{V} + 5.06\ \text{V} \\ &= 12.72\ \text{V} \end{aligned}$$

2. Use superposition to calculate V_X in the circuit shown in figure 3.2.

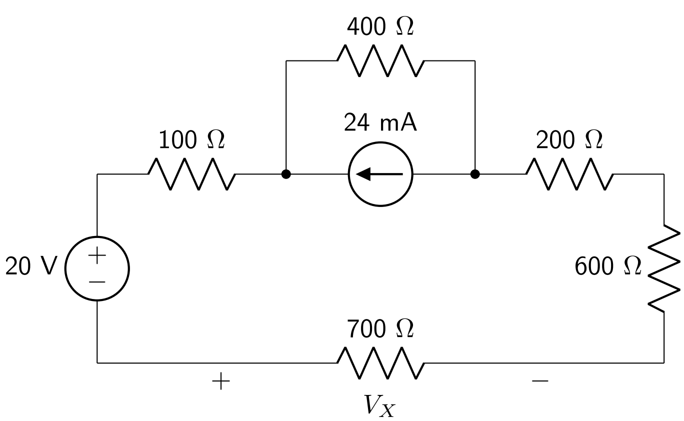
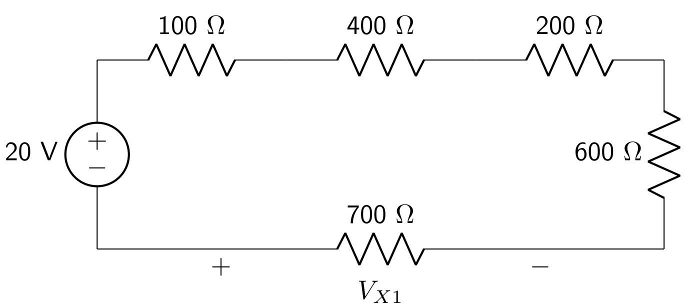
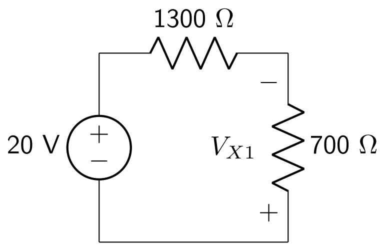


Figure 3.2: Circuit diagram for superposition question 2.

Draw the circuit with only the voltage source activated.



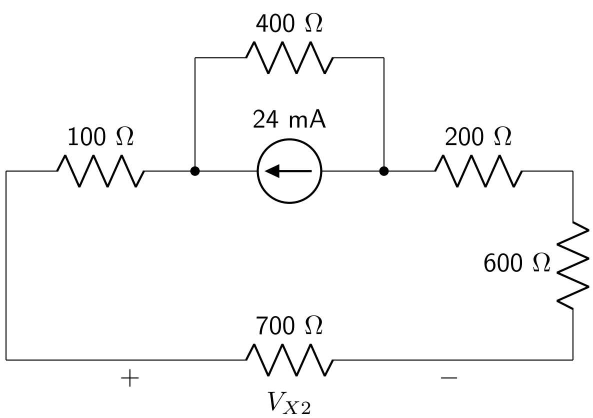
Combine resistors in series and re-draw.



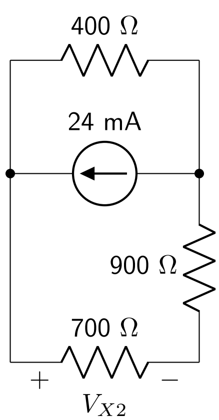
Use the voltage divider rule to calculate V_{X1}.

$$V_{X1} = -20 \text{ V} \left(\frac{700 \ \Omega}{1300 \ \Omega + 700 \ \Omega} \right)$$
$$= -7 \text{ V}$$

Draw the circuit with only the current source activated.



Combine resistors in series and re-draw.



Use the current divider rule to calculate the current flowing through the 700 Ω resistor.

$$\begin{aligned} I &= 24\text{ mA}\left(\frac{400\ \Omega // 1600\ \Omega}{1600\ \Omega}\right) \\ &= 24\text{ mA}\left(\frac{320\ \Omega}{1600\ \Omega}\right) \\ &= 4.8\text{ mA} \end{aligned}$$

Use Ohm's law to calculate V_{X2}.

$$\begin{aligned} V_{X2} &= (4.8\text{ mA})(0.7\text{ k}\Omega) \\ &= 3.36\text{ V} \end{aligned}$$

Add the two values to find the voltage V_X.

$$\begin{aligned} V_X &= V_{X1} + V_{X2} \\ &= -7\text{ V} + 3.36\text{ V} \\ &= -3.64\text{ V} \end{aligned}$$

3. Use superposition to calculate I_X in the circuit shown in figure 3.3.

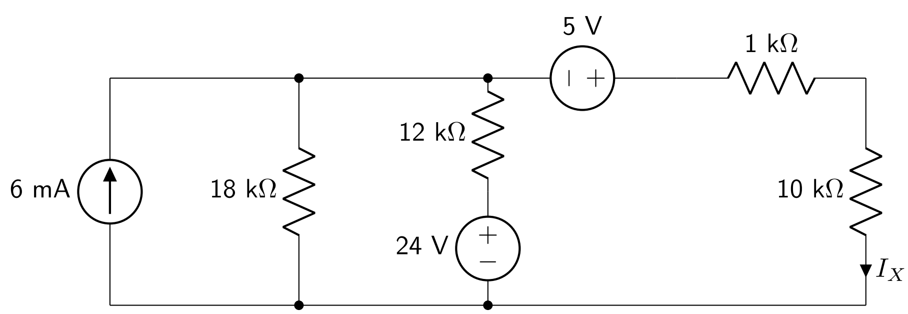
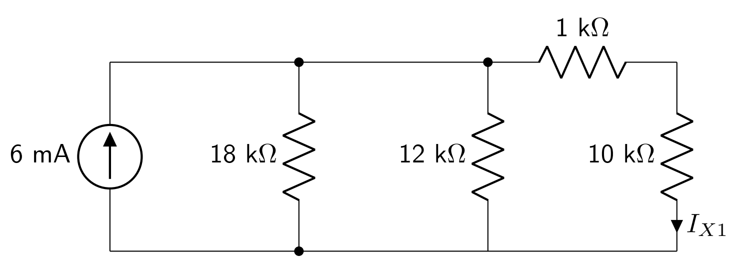


Figure 3.3: Circuit diagram for superposition question 3.

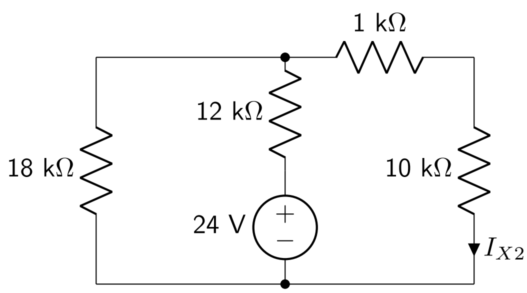
Draw the circuit with only the current source activated.



Use the current divider rule to calculate I_{X1}.

$$
\begin{aligned}
I_{X1} &= 6\ \text{mA}\left(\frac{18\ \text{k}\Omega//12\ \text{k}\Omega//11\ \text{k}\Omega}{11\ \text{k}\Omega}\right)\\
&= 6\ \text{mA}\left(\frac{4.35\ \text{k}\Omega}{11\ \text{k}\Omega}\right)\\
&= 2.37\ \text{mA}
\end{aligned}
$$

Draw the circuit with only the 24 V source activated.



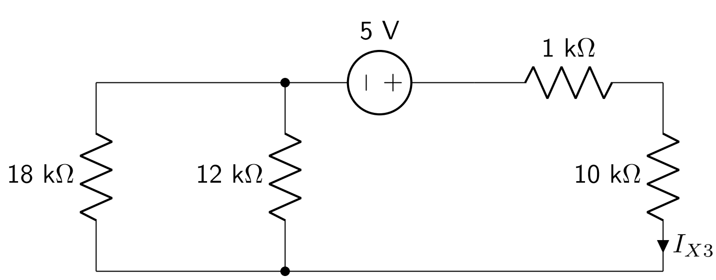
Use the voltage divider rule to calculate the voltage dropped over the 18 kΩ resistor.

$$
\begin{aligned}
V &= 24\ \text{V}\left(\frac{18\ \text{k}\Omega//11\ \text{k}\Omega}{12\ \text{k}\Omega + 18\ \text{k}\Omega//11\ \text{k}\Omega}\right)\\
&= 24\ \text{V}\left(\frac{6.83\ \text{k}\Omega}{12\ \text{k}\Omega + 6.83\ \text{k}\Omega}\right)\\
&= 8.70\ \text{V}
\end{aligned}
$$

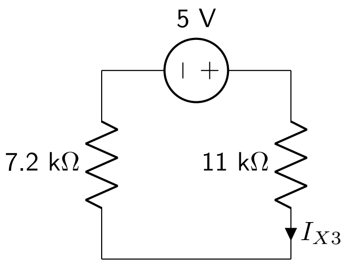
Use Ohm's law to calculate I_{X2}.

$$
\begin{aligned}
I_{X2} &= \frac{8.70\ \text{V}}{11\ \text{k}\Omega}\\
&= 0.79\ \text{mA}
\end{aligned}
$$

Draw the circuit with only the 5 V source activated.



Combine resistors in series and parallel, and re-draw.



Use the Ohm's law to calculate I_{X3}.

$$\begin{aligned} I_{X3} &= \frac{5 \text{ V}}{11 \text{ k}\Omega + 7.2 \text{ k}\Omega} \\ &= 0.27 \text{ mA} \end{aligned}$$

Add the three values to find the current I_X.

$$\begin{aligned} I_X &= I_{X1} + I_{X2} + I_{X3} \\ &= 2.37 \text{ mA} + 0.79 \text{ mA} + 0.27 \text{ mA} \\ &= 3.43 \text{ mA} \end{aligned}$$

4. Use superposition to calculate V_X in the circuit shown in figure 3.4.

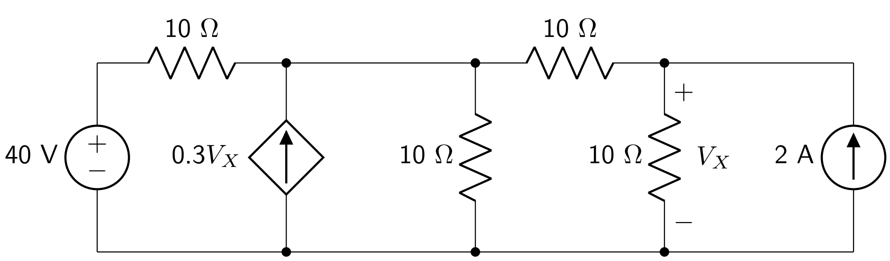
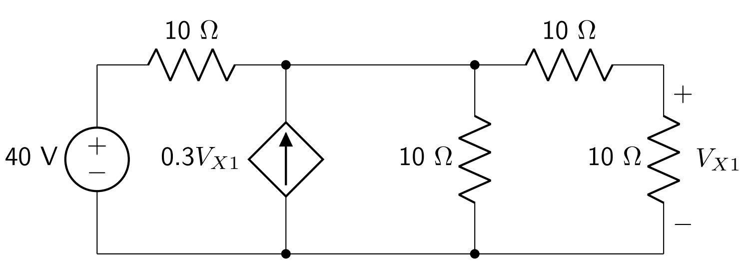


Figure 3.4: Circuit diagram for superposition question 4.

Draw the circuit with only the voltage source activated.



Use mesh analysis. Define the left mesh as I_A, the center mesh as I_B, and the right mesh as I_C. Calculate the supermesh and mesh equations. (All units are in V, A, and Ω.)

$$40 = 10I_A + 10(I_B - I_C)$$
$$0 = 10(I_C - I_B) + 20I_C$$

Perform KCL at the node between the supermesh.

$$0 = I_A + 0.3V_{X1} - I_B$$

Derive a dependent source equation.

$$V_{X1} = 10I_C$$

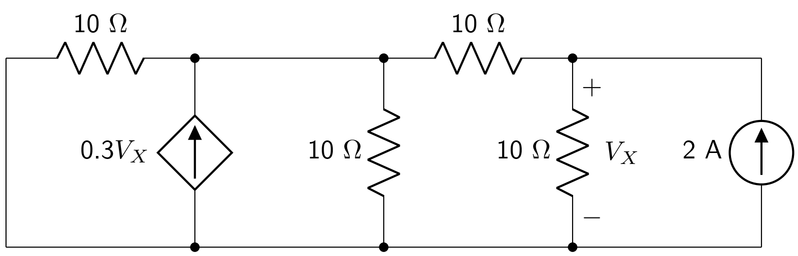
Place all four equations into form $\alpha I_A + \beta I_B + \gamma I_C + \delta V_{X1} = c$, and then place each coefficient into a matrix.

$$\begin{bmatrix} 10 & 10 & -10 & 0 & 40 \\ 0 & -10 & 30 & 0 & 0 \\ 1 & -1 & 0 & 0.3 & 0 \\ 0 & 0 & -10 & 1 & 0 \end{bmatrix}$$

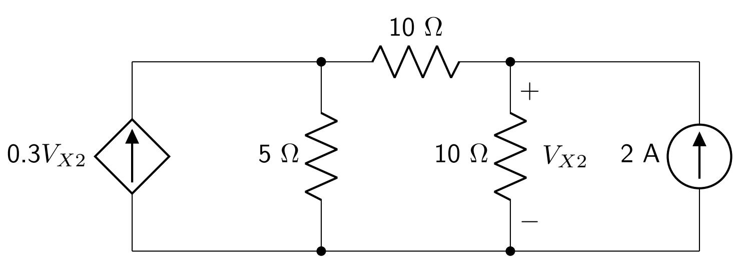
Solve the matrix for V_{X1}.

$$V_{X1} = 20 \text{ V}$$

Draw the circuit with only the current source activated.



Combine resistors in parallel.



Use mesh analysis. Define the center mesh as I_A. Both other meshes contain a current source. Calculate the mesh equation for the center mesh. (All units are in V, A, and Ω.)

$$0 = 5(I_A - 0.3V_{X2}) + 10I_A + 10(I_A + 2)$$

Derive a dependent source equation.

$$V_{X2} = 10(I_A + 2)$$

Place both equations into form $\alpha I_A + \beta V_{X2} = c$, and then place each coefficient into a matrix.

$$\begin{bmatrix} 25 & -1.5 & -20 \\ -10 & 1 & 20 \end{bmatrix}$$

Solve the matrix for V_{X2}.

$$V_{X2} = 30 \text{ V}$$

Add the two values to find the current V_X.

$$\begin{aligned} V_X &= V_{X1} + V_{X2} \\ &= 20 \text{ V} + 30 \text{ V} \\ &= 50 \text{ V} \end{aligned}$$

5. Use superposition to calculate V_X in the circuit shown in figure 3.5.

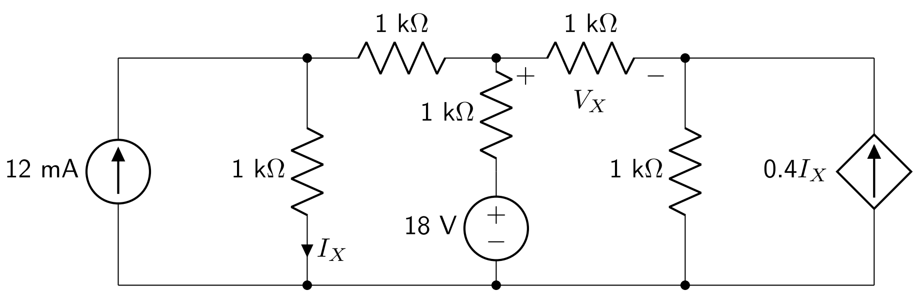
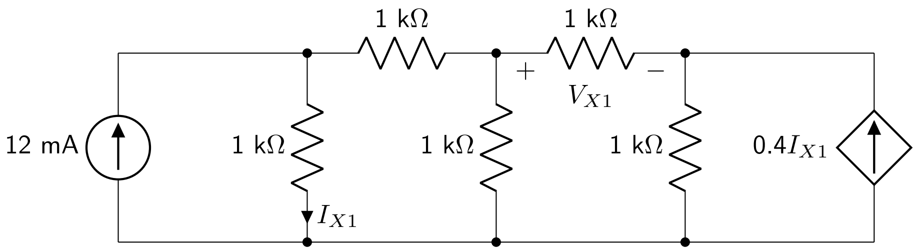
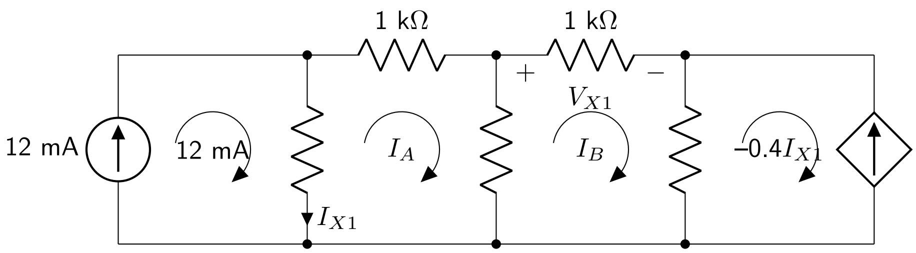


Figure 3.5: Circuit diagram for superposition question 5.

Draw the circuit with only the current source activated.



Use mesh analysis. Each mesh is defined below. (Note that some component values have been hidden so that the mesh labels can be read.)



Derive the mesh equations. (All units are in V, mA, and kΩ.)

$$0 = (I_A - 12) + I_A + (I_A - I_B)$$
$$0 = (I_B - I_A) + I_B + (I_B + 0.4I_{X1})$$

Derive a dependent source equation.

$$I_{X1} = 12 - I_A$$

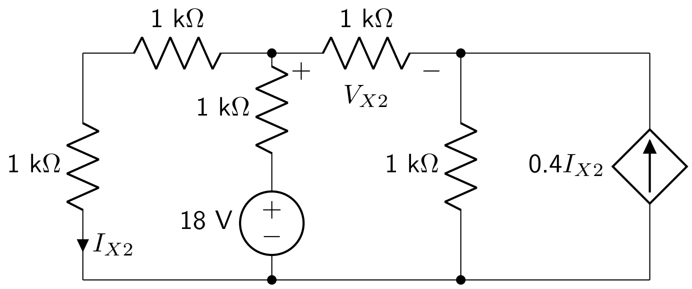
Place all three equations into form $\alpha I_A + \beta I_B + \gamma I_{X1} = c$, and then place each coefficient into a matrix.

$$\begin{bmatrix} 3 & -1 & 0 & 12 \\ -1 & 3 & 0.4 & 0 \\ 1 & 0 & 1 & 12 \end{bmatrix}$$

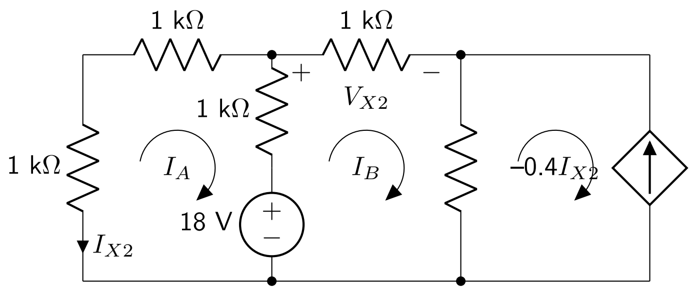
Solve the matrix for I_B and use Ohm's law to calculate V_{X1}.

$$\begin{aligned} V_{X1} &= (1\text{ k}\Omega)(I_B) \\ &= (1\text{ k}\Omega)(0.32\text{ mA}) \\ &= 0.32\text{ V} \end{aligned}$$

Draw the circuit with only the voltage source activated.



Use mesh analysis. Each mesh is defined below. (Note that some component values have been hidden so that the mesh labels can be read.)



Derive the mesh equations. (All units are in V, mA, and kΩ.)

$$-18 = I_A + I_A + (I_A - I_B)$$
$$18 = (I_B - I_A) + I_B + (I_B + 0.4I_{X2})$$

Derive a dependent source equation.

$$I_{X2} = -I_A$$

Place all three equations into form $\alpha I_A + \beta I_B + \gamma I_{X2} = c$, and then place each coefficient into a matrix.

$$\begin{bmatrix} 3 & -1 & 0 & -18 \\ -1 & 3 & 0.4 & 18 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

Solve the matrix for I_B and use Ohm's law to calculate V_{X2}.

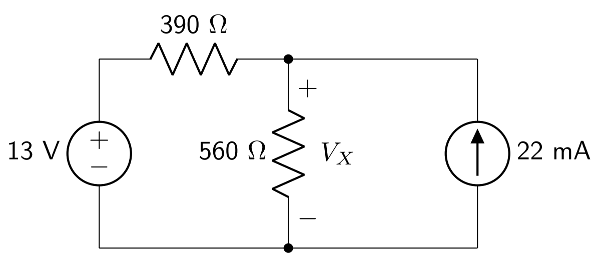
$$\begin{aligned} V_{X2} &= (1\ \text{k}\Omega)(I_B) \\ &= (1\ \text{k}\Omega)(3.79\ \text{mA}) \\ &= 3.79\ \text{V} \end{aligned}$$

Add the two values to find the current V_X.

$$
\begin{aligned}
V_X &= V_{X1} + V_{X2} \\
&= 0.32 \text{ V} + 3.79 \text{ V} \\
&= 4.11 \text{ V}
\end{aligned}
$$

3.2 Source Transformation

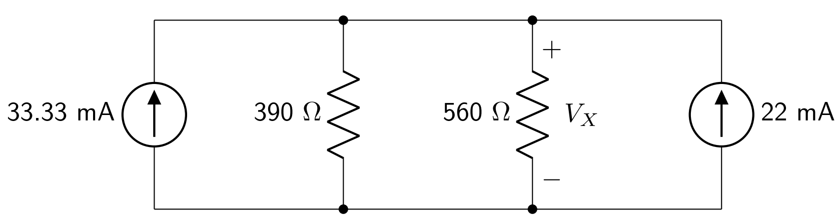
6. Use source transformation to calculate V_X in the circuit shown in figure 3.1 (in the superposition section).



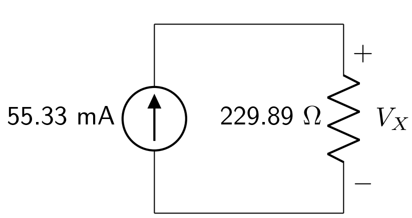
Convert the 13 V source to a current source.

$$
\begin{aligned}
I_S &= \frac{13 \text{ V}}{0.39\ k\Omega} \\
&= 33.33 \text{ mA}
\end{aligned}
$$

Re-draw the circuit.



Combine current sources and parallel resistors and re-draw.

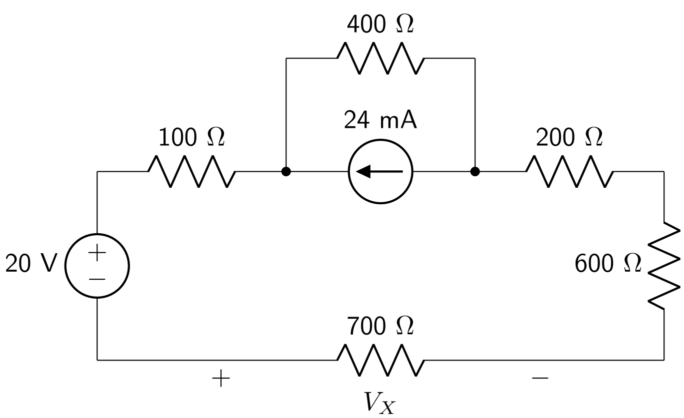


Use Ohm's law to calculate V_X.

$$\begin{aligned} V_X &= (55.33 \text{ mA})(0.22989 \text{ k}\Omega) \\ &= 12.72 \text{ V} \end{aligned}$$

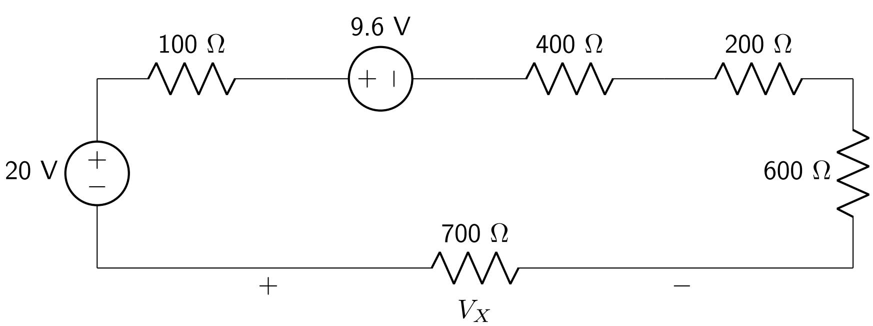
This answer should be and is identical to question 1 in the superposition section.

7. Use source transformation to calculate V_X in the circuit shown in figure 3.2 (in the superposition section).

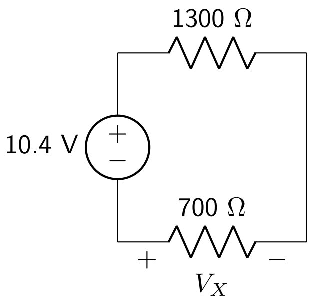


Convert the 24 mA source to a voltage source.

$$\begin{aligned} V_S &= (24 \text{ mA})(0.4 \text{ k}\Omega) \\ &= 9.6 \text{ V} \end{aligned}$$



Combine voltage sources and series resistors and re-draw.

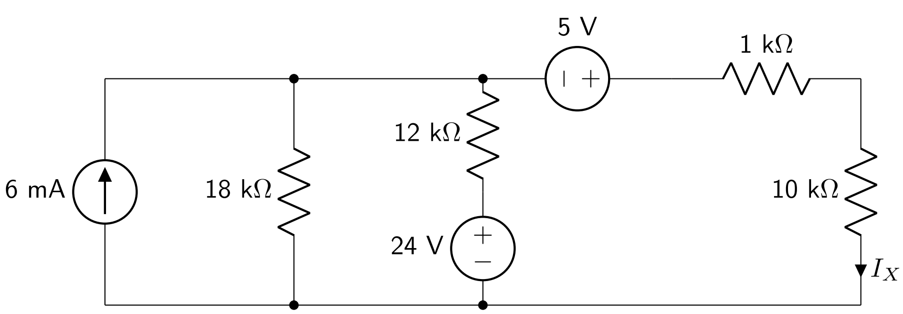


Use the voltage divider rule to solve for V_X.

$$V_X = -10.4\text{ V}\left(\frac{700\ \Omega}{1300\ \Omega + 700\ \Omega}\right)$$
$$= -3.64\text{ V}$$

This answer should be and is identical to question 2 in the superposition section.

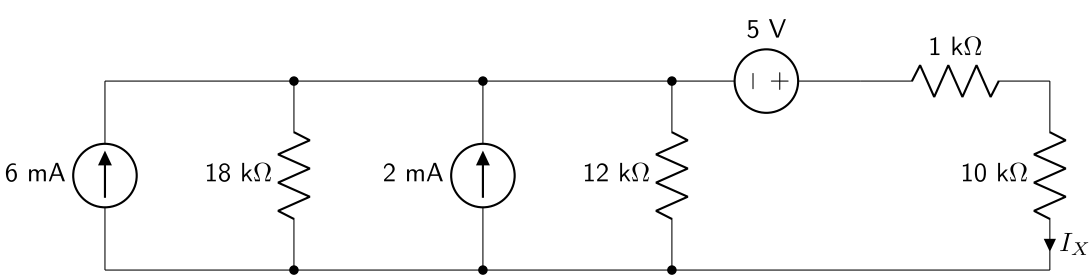
8. Use source transformation to calculate I_X in the circuit shown in figure 3.3 (in the superposition section).



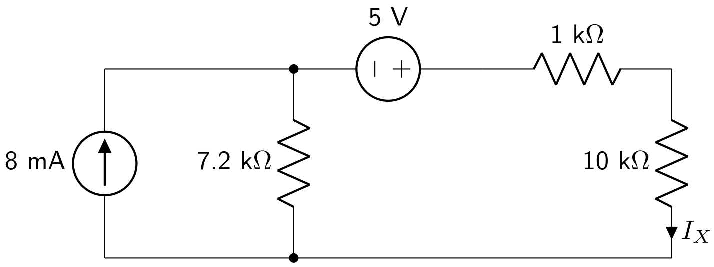
Convert the 24 V source into a current source.

$$I_S = \frac{24\text{ V}}{12\text{ k}\Omega}$$
$$= 2\text{ mA}$$

Re-draw the circuit.



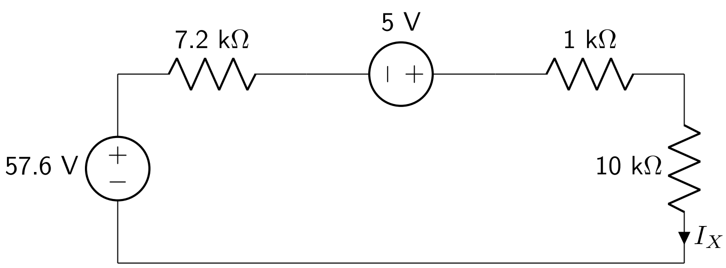
Combine current sources and parallel resistors.



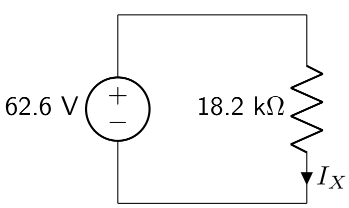
Convert the 8 mA source into a voltage source.

$$\begin{aligned} V_S &= (8 \text{ mA})(7.2 \text{ k}\Omega) \\ &= 57.6 \text{ V} \end{aligned}$$

Re-draw the circuit.



Combine voltage sources and series resistors.

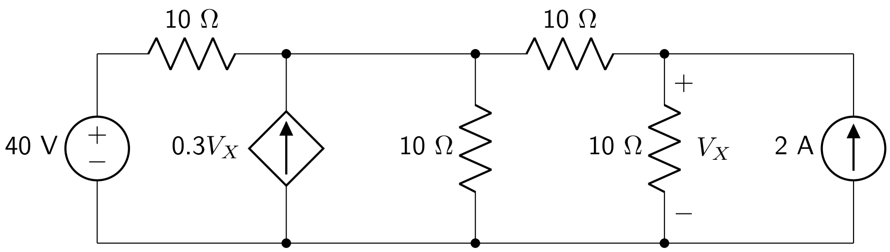


Use Ohm's law to calculate I_X.

$$\begin{aligned} I_X &= \frac{62.6 \text{ V}}{18.2 \text{ k}\Omega} \\ &= 3.44 \text{ mA} \end{aligned}$$

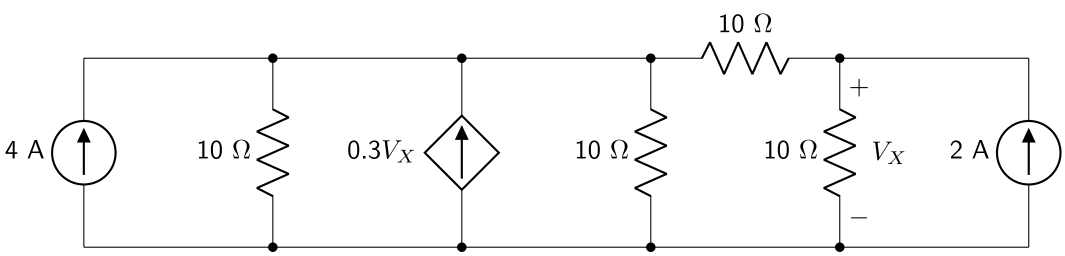
This answer should be and is identical to question 3 in the superposition section.

9. Use source transformation to calculate V_X in the circuit shown in figure 3.4 (in the superposition section).

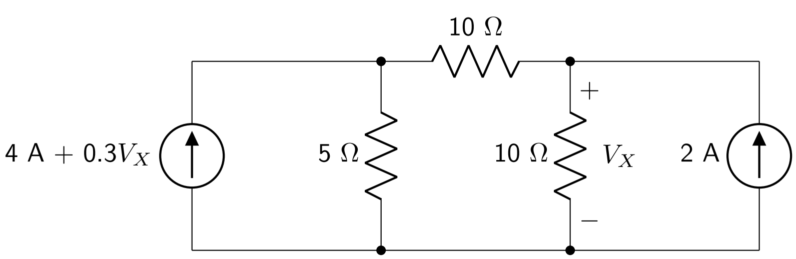


Convert the 40 V source to a current source.

$$I_S = \frac{40 \text{ V}}{10\ \Omega}$$
$$= 4 \text{ A}$$



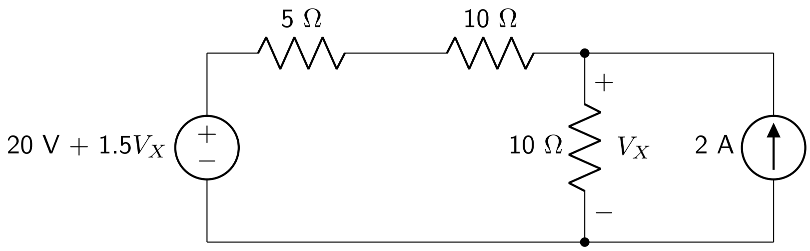
Combine sources and resistors.



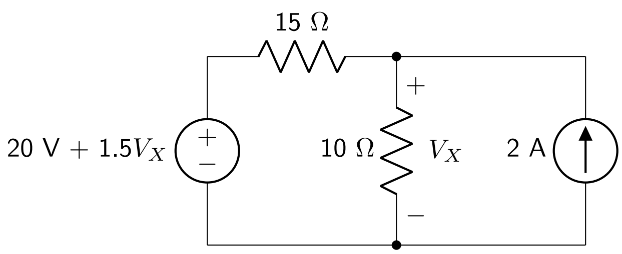
Convert the 4 A + 0.3V_X source into a voltage source.

$$V_S = (4 \text{ A} + 0.3V_X)(5\ \Omega)$$
$$= 20 \text{ V} + 1.5V_X$$

Re-draw the circuit.



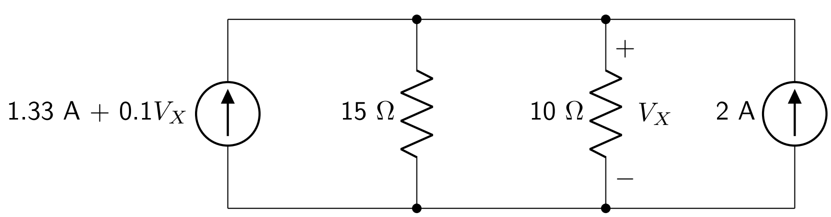
Combine resistors in series.



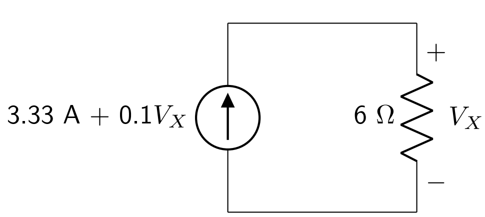
Convert the 20 V + 1.5V_X source to a current source.

$$\begin{aligned} I_S &= \frac{20\text{ V} + 1.5V_X}{15\ \Omega} \\ &= 1.33\text{ A} + 0.1V_X \end{aligned}$$

Re-draw the circuit.



Combine sources and resistors and re-draw the circuit.



Calculate V_X.

$$\begin{aligned} V_X &= (3.33 \text{ A} + 0.1V_X)(6\ \Omega) \\ &= 20 \text{ V} + 0.6V_X \\ 0.4V_X &= 20 \text{ V} \\ V_X &= \frac{20 \text{ V}}{0.4} \\ &= 50 \text{ V} \end{aligned}$$

This answer should be and is identical to question 4 in the superposition section.

10. Use source transformation to calculate V_X and V_Y in the circuit shown in figure 3.6.

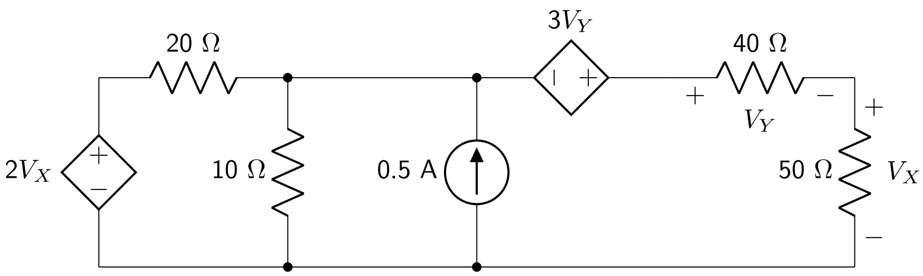
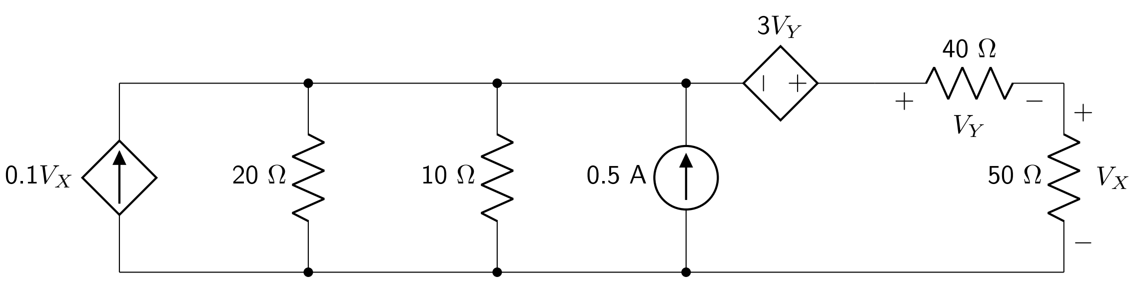


Figure 3.6: Circuit diagram for source transformation question 10.

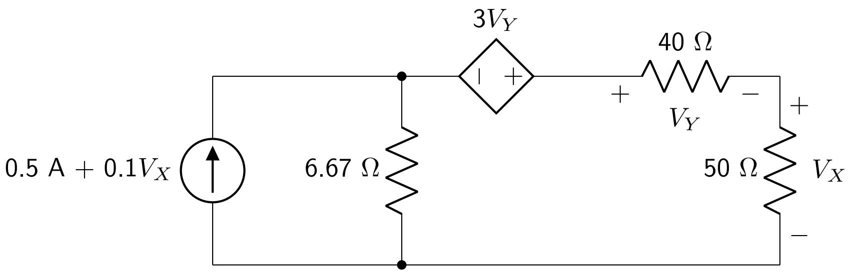
Convert the $2V_X$ source into a current source.

$$\begin{aligned} I_S &= \frac{2V_X}{20\ \Omega} \\ &= 0.1V_X \end{aligned}$$

Re-draw the circuit.



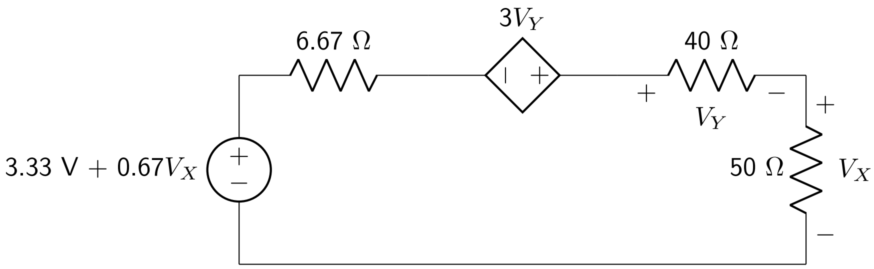
Combine sources and resistors and re-draw the circuit.



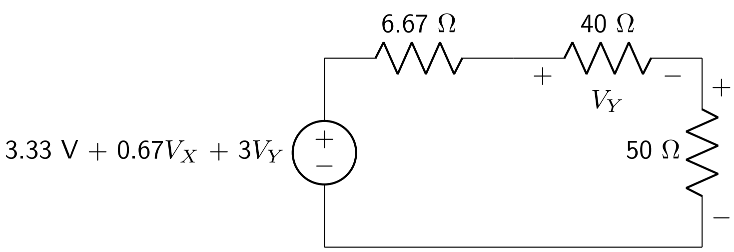
Convert the 0.5 A + 0.1V_X source into a voltage source.

$$V_S = (0.5\text{ A} + 0.1V_X)(6.67\ \Omega)$$
$$= 3.33\text{ V} + 0.67V_X$$

Re-draw the circuit.



Combine the voltage sources and re-draw.



Use the voltage divider rule to find equations for V_X and V_Y. (All units are in V, A, and Ω.)

$$V_X = (3.33 + 0.67V_X + 3V_Y)\left(\frac{40}{96.67}\right)$$
$$V_Y = (3.33 + 0.67V_X + 3V_Y)\left(\frac{50}{96.67}\right)$$

Place both equations into form $\alpha V_X + \beta V_Y = c$, and then place each coefficient into a matrix.

$$\begin{bmatrix} -0.28 & -0.24 & 1.38 \\ 0.66 & -1.55 & 1.72 \end{bmatrix}$$

Solve the matrix for V_X and V_Y.

$$V_X = -2.94 \text{ V}$$
$$V_Y = -2.35 \text{ V}$$

3.3 Thévenin and Norton's Theorems

11. Derive the Thévenin equivalent circuit between nodes a and b in the circuit shown in figure 3.7.

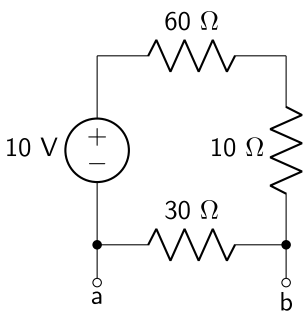


Figure 3.7: Circuit diagram for Thévenin and Norton's theorems question 11.

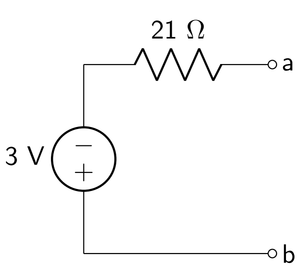
Use the voltage divider rule to calculate the Thévenin equivalent voltage.

$$\begin{aligned} V_{TH} &= -10 \text{ V} \left(\frac{30\ \Omega}{100\ \Omega} \right) \\ &= -3 \text{ V} \end{aligned}$$

Deactivate the voltage source and calculate the equivalent resistance as seen between nodes a and b.

$$\begin{aligned} R_{TH} &= 30\ \Omega // (10\ \Omega + 60\ \Omega) \\ &= 30\ \Omega // 70\ \Omega \\ &= 21\ \Omega \end{aligned}$$

Draw the Thévenin equivalent circuit.



12. Derive the Norton equivalent circuit between nodes a and b in the circuit shown in figure 3.8.

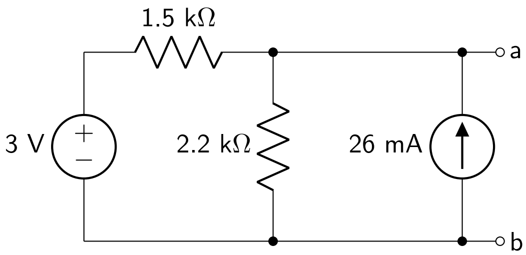
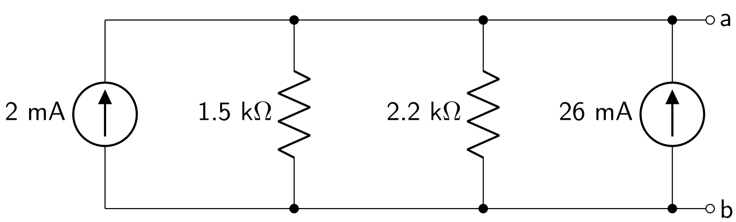


Figure 3.8: Circuit diagram for Thévenin and Norton's theorems question 12.

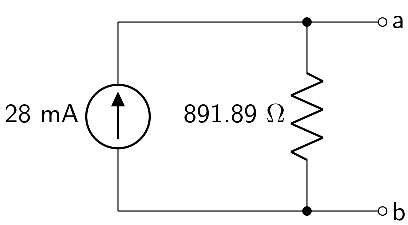
Use Source transformation to convert the 3 V source into a current source.

$$I_S = \frac{3\text{ V}}{1.5\text{ k}\Omega}$$
$$= 2\text{ mA}$$

Re-draw the circuit.



Combine both resistors and both current sources and re-draw the circuit, which is the Norton equivalent circuit.



13. Derive the Thévenin equivalent circuit between nodes a and b in the circuit shown in figure 3.9.

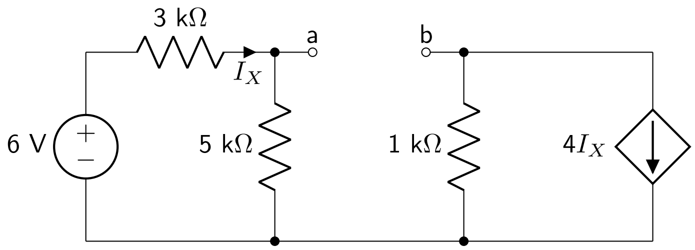


Figure 3.9: Circuit diagram for Thévenin and Norton's theorems question 13.

Use Ohm's law to calculate I_X.

$$\begin{aligned} I_X &= \frac{6 \text{ V}}{8 \text{ k}\Omega} \\ &= 0.75 \text{ mA} \end{aligned}$$

Use the voltage divider rule to calculate V_a.

$$\begin{aligned} V_a &= 6 \text{ V} \left(\frac{5 \text{ k}\Omega}{8 \text{ k}\Omega} \right) \\ &= 3.75 \text{ V} \end{aligned}$$

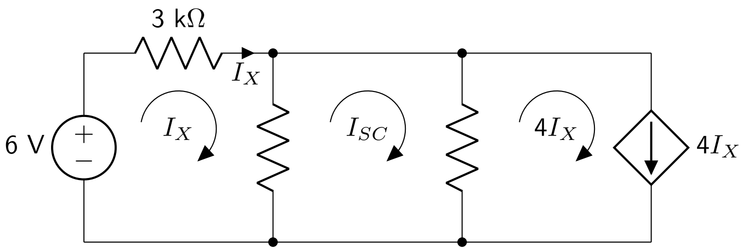
Use Ohm's law to calculate V_b.

$$\begin{aligned} V_b &= (1 \text{ k}\Omega)(-4 I_X) \\ &= (1 \text{ k}\Omega)(-4(0.75 \text{ mA})) \\ &= (1 \text{ k}\Omega)(-3 \text{ mA})) \\ &= -3 \text{ V} \end{aligned}$$

The Thévenin equivalent voltage is equal to $V_a - V_b$.

$$\begin{aligned} V_{TH} &= 3.75 \text{ V} - (-3 \text{ V}) \\ &= 6.75 \text{ V} \end{aligned}$$

Short nodes a and b together and use mesh analysis to solve for I_{SC}. (Note: some of the component labels have been hidden so that the mesh current labels can be read.)



Use the left and middle meshes to derive mesh equations. (All units are in V, mA, and kΩ.)

$$6 = 3I_X + 5(I_X - I_{SC})$$
$$0 = 5(I_{SC} - I_X) + (I_{SC} - 4I_X)$$

Place both equations into form $\alpha I_{SC} + \beta I_X = c$, and then place each coefficient into a matrix.

$$\begin{bmatrix} -5 & 8 & 6 \\ 6 & -9 & 0 \end{bmatrix}$$

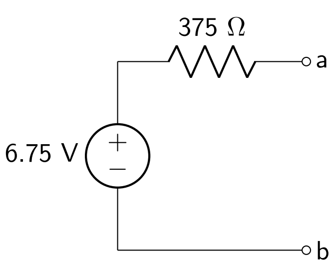
Solve the matrix for I_{SC}.

$$I_{SC} = 18 \text{ mA}$$

Use Ohm's law to calculate the Thévenin equivalent resistance.

$$R_{TH} = \frac{6.75 \text{ V}}{18 \text{ mA}}$$
$$= 375 \ \Omega$$

Draw the Thévenin equivalent circuit.



14. Derive the Thévenin equivalent circuit between nodes a and b in the circuit shown in figure 3.10.

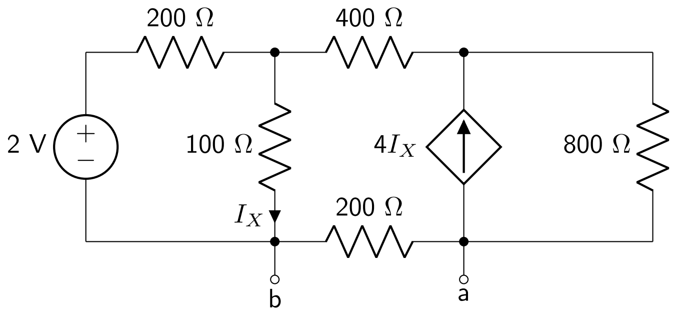
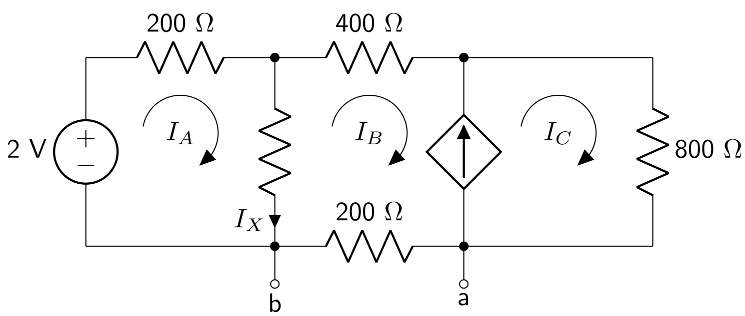


Figure 3.10: Circuit diagram for Thévenin and Norton's theorems question 14.

Use mesh analysis to calculate the Thévenin equivalent voltage. (Note: some of the component labels have been hidden so that the mesh current labels can be read.)



Derive mesh equations. (All units are in V, mA, and kΩ.)

$$2 = 0.2I_A + 0.1(I_A - I_B)$$
$$0 = 0.1(I_B - I_A) + 0.6I_B + 0.8I_C$$

Perform KCL at the node connecting the supermesh.

$$0 = I_B + 4I_X - I_C$$

Derive a dependent source equation.

$$I_X = I_A - I_B$$

Place all four equations into form $\alpha I_A + \beta I_B + \gamma I_C + \delta I_X = c$, and then place each coefficient into a

matrix.

$$\begin{bmatrix} 0.3 & -0.1 & 0 & 0 & 2 \\ -0.1 & 0.7 & 0.8 & 0 & 0 \\ 0 & 1 & -1 & 4 & 0 \\ -1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

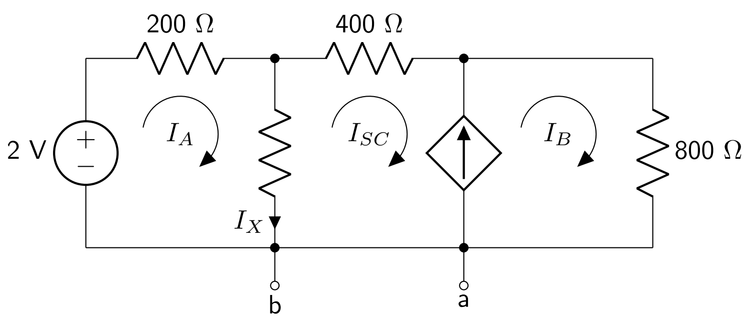
Solve the matrix for I_B.

$$I_B = 31 \text{ mA}$$

Use Ohm's law to calculate V_{TH}.

$$V_{TH} = (31 \text{ mA})(0.2 \text{ k}\Omega)$$
$$= 6.2 \text{ V}$$

Short nodes a and b together and use mesh analysis to calculate I_{SC}. (Note: some of the component labels have been hidden so that the mesh current labels can be read.)



Derive mesh equations. (All units are in V, mA, and kΩ.)

$$2 = 0.2I_A + 0.1(I_A - I_{SC})$$
$$0 = 0.1(I_{SC} - I_A) + 0.4I_{SC} + 0.8I_B$$

Perform KCL at the node connecting the supermesh.

$$0 = I_{SC} + 4I_X - I_B$$

Derive a dependent source equation.

$$I_X = I_A - I_{SC}$$

Place all four equations into form $\alpha I_A + \beta I_{SC} + \gamma I_B + \delta I_X = c$, and then place each coefficient into a

matrix.

$$\begin{bmatrix} 0.3 & -0.1 & 0 & 0 & 2 \\ -0.1 & 0.5 & 0.8 & 0 & 0 \\ 0 & 1 & -1 & 4 & 0 \\ -1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

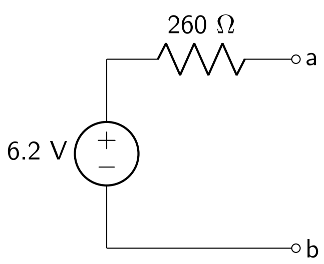
Solve the matrix for I_{SC}.

$$I_{SC} = 23.851 \text{ mA}$$

Use Ohm's law to calculate R_{TH}.

$$R_{TH} = \frac{6.2 \text{ V}}{23.85 \text{ mA}}$$
$$= 260\ \Omega$$

Draw the Thévenin equivalent circuit.



15. Derive the Norton equivalent circuit between nodes a and b in the circuit shown in figure 3.11.

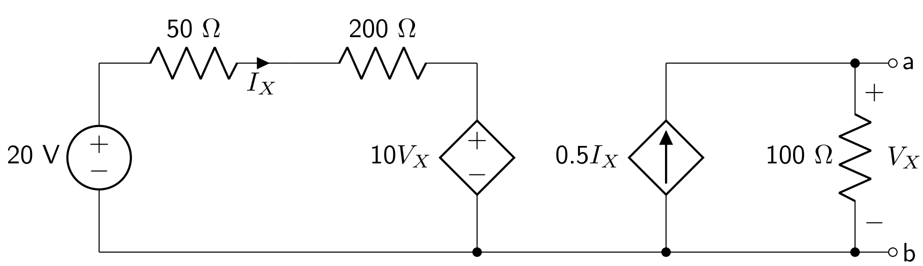
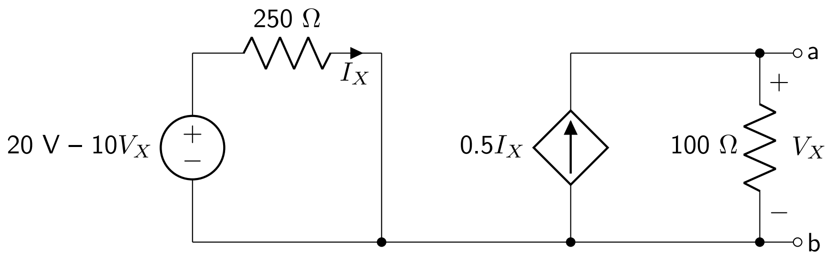


Figure 3.11: Circuit diagram for Thévenin and Norton's theorems question 15.

Combine resistors and voltage sources in series and re-draw.



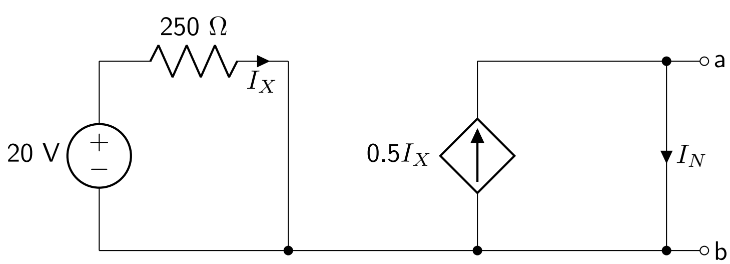
Use Ohm's law to calculate I_X.

$$I_X = \frac{20 \text{ V} - 10V_X}{250 \ \Omega}$$
$$= 0.08 \text{ A} - 0.04V_X$$

Use Ohm's law to calculate V_X, which is the open-circuit voltage.

$$V_X = (100 \ \Omega)(0.5I_X)$$
$$= (100 \ \Omega)(0.5)(0.08 \text{ A} - 0.04V_X)$$
$$= 4 \text{ V} - 2V_X$$
$$3V_X = 4 \text{ V}$$
$$V_X = 1.33 \text{ V}$$

Short the nodes between a and b and re-draw the circuit. Because V_X is zero, the VCVS contributes no voltage to the circuit.



Use Ohm's law to calculate I_X.

$$I_X = \frac{20 \text{ V}}{250 \ \Omega}$$
$$= 0.08 \text{ A}$$

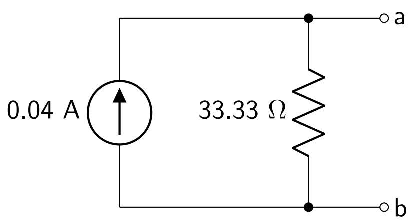
Use KCL to calculate I_N.

$$\begin{aligned} I_N &= 0.5I_X \\ &= 0.5(0.08\text{ A}) \\ &= 0.04\text{ A} \end{aligned}$$

Use Ohm's law to calculate R_N.

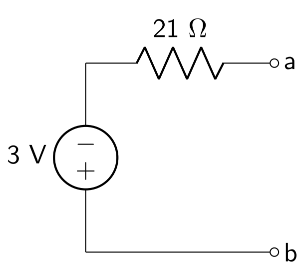
$$\begin{aligned} R_N &= \frac{1.33\text{ V}}{0.04\text{ A}} \\ &= 33.33\ \Omega \end{aligned}$$

Draw the Norton equivalent circuit.



3.4 Maximum Power Transfer

16. Calculate the resistance for maximum power transfer, and the maximum amount of power transferred to the load under that condition, for the circuit shown in figure 3.7 (in the Thévenin and Norton's theorem section).

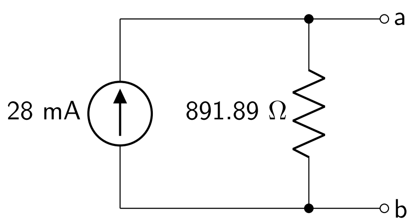


R_{LOAD} will be equal to the Thévenin equivalent resistance of 21 Ω. Calculate the maximum power

transferred to the load.

$$P_{MAX} = \frac{V_{TH}^2}{4R_{TH}} = \frac{(-3\ \text{V})^2}{4(21\ \Omega)} = 0.107\ \text{W}$$

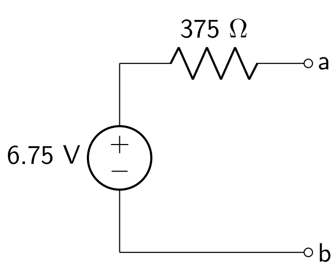
17. Calculate the resistance for maximum power transfer, and the maximum amount of power transferred to the load under that condition, for the circuit shown in figure 3.8 (in the Thévenin and Norton's theorem section).



R_{LOAD} will be equal to the Norton equivalent resistance of 891.89 Ω. Calculate the maximum power transferred to the load.

$$P_{MAX} = \frac{I_N^2 R_N}{4} = \frac{(0.028\ \text{A})^2(891.89\ \Omega)}{4} = 0.175\ \text{W}$$

18. Calculate the resistance for maximum power transfer, and the maximum amount of power transferred to the load under that condition, for the circuit shown in figure 3.9 (in the Thévenin and Norton's theorem section).

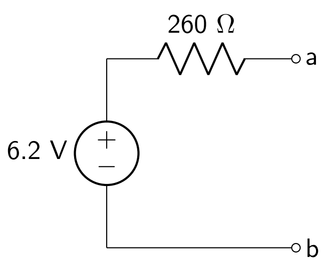


R_{LOAD} will be equal to the Thévenin equivalent resistance of 375 Ω. Calculate the maximum power

transferred to the load.

$$P_{MAX} = \frac{V_{TH}^2}{4R_{TH}} = \frac{(6.75 \text{ V})^2}{4(375 \ \Omega)} = 0.03 \text{ W}$$

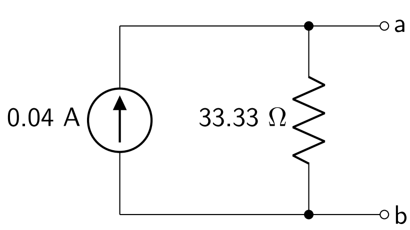
19. Calculate the resistance for maximum power transfer, and the maximum amount of power transferred to the load under that condition, for the circuit shown in figure 3.10 (in the Thévenin and Norton's theorem section).



R_{LOAD} will be equal to the Thévenin equivalent resistance of 260 Ω. Calculate the maximum power transferred to the load.

$$P_{MAX} = \frac{V_{TH}^2}{4R_{TH}} = \frac{(6.2 \text{ V})^2}{4(260 \ \Omega)} = 0.37 \text{ W}$$

Calculate the resistance for maximum power transfer, and the maximum amount of power transferred to the load under that condition, for the circuit shown in figure 3.11 (in the Thévenin and Norton's theorem section).



R_{LOAD} will be equal to the Norton equivalent resistance of 33.33 Ω. Calculate the maximum power

transferred to the load.

$$\begin{aligned}
P_{MAX} &= \frac{I_N^2 R_N}{4} \\
&= \frac{(0.04\ \text{A})^2(33.33\ \Omega)}{4} \\
&= 0.013\ \text{W}
\end{aligned}$$

4 Chapter 4 Solutions

1. Calculate the output voltage and gain of the circuit shown in figure 4.1. (Assume that the supply voltage is sufficient to generate any output value.)

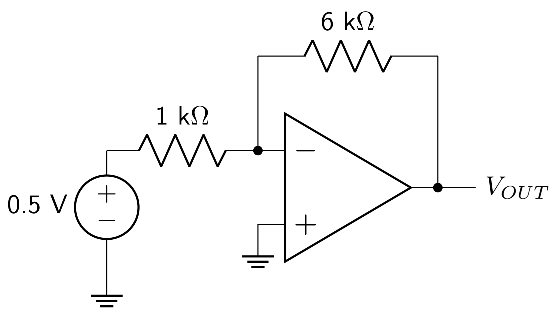


Figure 4.1: Circuit diagram for op-amps question 1.

This circuit is an inverting op-amp. Calculate the gain.

$$\begin{aligned} A &= -\frac{6\ \text{k}\Omega}{1\ \text{k}\Omega} \\ &= -6 \end{aligned}$$

Calculate V_{OUT}.

$$\begin{aligned} V_{OUT} &= AV_{IN} \\ &= (-6)(0.5\ \text{V}) \\ &= -3\ \text{V} \end{aligned}$$

2. Calculate the output voltage of the circuit shown in figure 4.2. (Assume that the supply voltage is sufficient to generate any output value.)

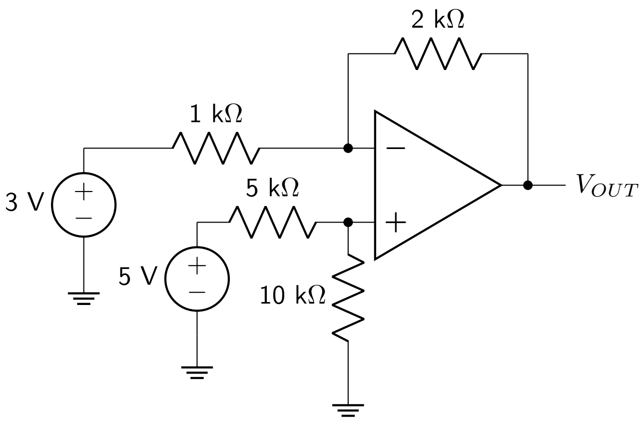


Figure 4.2: Circuit diagram for op-amps question 2.

Use the voltage divider rule to calculate the voltage at the non-inverting input.

$$\begin{aligned} V_P &= 5\text{ V}\left(\frac{10\text{ k}\Omega}{15\text{ k}\Omega}\right) \\ &= 3.33\text{ V} \end{aligned}$$

Use KCL, Ohm's law, and the virtual node property to calculate V_{OUT}.

$$\begin{aligned} \frac{3\text{ V} - V_P}{1\text{ k}\Omega} &= \frac{V_P - V_{OUT}}{2\text{ k}\Omega} \\ \frac{3\text{ V} - 3.33\text{ V}}{1\text{ k}\Omega} &= \frac{3.33\text{ V} - V_{OUT}}{2\text{ k}\Omega} \\ -0.67\text{ V} &= 3.33\text{ V} - V_{OUT} \\ 4\text{ V} &= V_{OUT} \end{aligned}$$

3. Calculate the output voltage and gain of the circuit shown in figure 4.3. (Assume that the supply voltage is sufficient to generate any output value.)

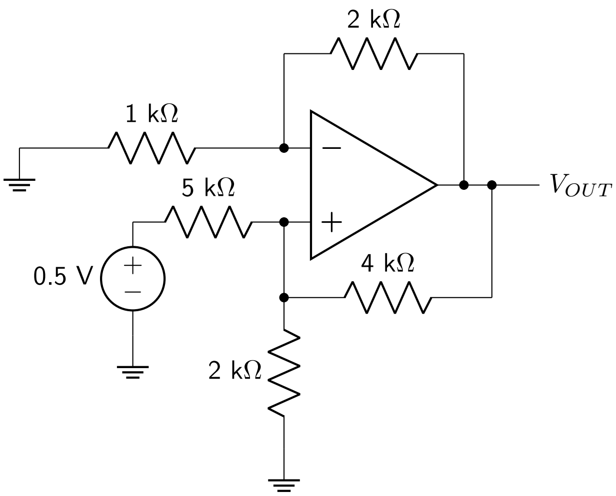


Figure 4.3: Circuit diagram for op-amps question 3.

Use KCL at both the inverting and non-inverting nodes to find two equations for two unknowns. The virtual node voltage is labeled V_X in the equations below.

$$\frac{0 - V_X}{1\ \text{k}\Omega} = \frac{V_X - V_{OUT}}{2\ \text{k}\Omega}$$
$$\frac{0.5\ \text{V} - V_X}{5\ \text{k}\Omega} = \frac{V_X}{2\ \text{k}\Omega} + \frac{V_X - V_{OUT}}{4\ \text{k}\Omega}$$

Place both equations into form $\alpha V_X + \beta V_{OUT} = c$, and then place each coefficient into a matrix. (All units are in V, mA, and kΩ.)

$$\begin{bmatrix} 1.5 & -0.5 & 0 \\ 0.95 & -0.25 & 0.1 \end{bmatrix}$$

Solve the matrix for V_{OUT}.

$$V_{OUT} = 1.5\ \text{V}$$

Calculate the gain.

$$\begin{aligned} A &= \frac{V_{OUT}}{V_{IN}} \\ &= \frac{1.5\ \text{V}}{0.5\ \text{V}} \\ &= 3 \end{aligned}$$

4. Calculate the gain of the circuit shown in figure 4.4.

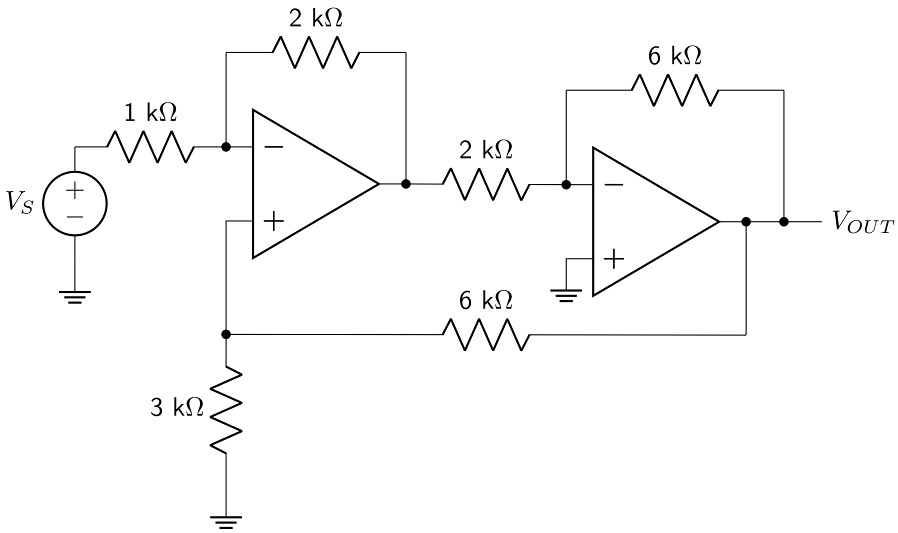


Figure 4.4: Circuit diagram for op-amps question 4.

Perform KCL at the non-inverting node of the left op-amp. The virtual node voltage will be labeled V_X in the equations below, and solve for V_X.

$$\frac{0 - V_X}{3\ \text{k}\Omega} = \frac{V_X - V_{OUT}}{6\ \text{k}\Omega}$$
$$-V_X = 0.5V_X - 0.5V_{OUT}$$
$$-1.5V_X = -0.5V_{OUT}$$
$$V_X = \frac{1}{3}V_{OUT}$$

Perform KCL at the inverting node of the right op-amp. The output voltage of the left op-amp will be labeled V_Y in the equations below. Solve for V_Y.

$$\frac{V_Y}{2\ \text{k}\Omega} = \frac{0 - V_{OUT}}{6\ \text{k}\Omega}$$
$$V_Y = -\frac{1}{3}V_{OUT}$$

Perform KCL at the inverting node of the left op-amp. Then, plug in the values of V_X and V_Y defined above, and solve for V_{OUT}/V_S.

$$\begin{aligned}
\frac{V_S - V_X}{1\text{ k}\Omega} &= \frac{V_X - V_Y}{2\text{ k}\Omega} \\
V_S - V_X &= 0.5V_X - 0.5V_Y \\
V_S - \frac{1}{3}V_{OUT} &= 0.5(\frac{1}{3}V_{OUT}) - 0.5(-\frac{1}{3}V_{OUT}) \\
V_S &= \frac{1}{3}V_{OUT} + \frac{1}{6}V_{OUT} + \frac{1}{6}V_{OUT} \\
V_S &= \frac{4}{6}V_{OUT} \\
\frac{V_{OUT}}{V_S} &= 1.5
\end{aligned}$$

5. Calculate the output voltage of the circuit shown in figure 4.5. (Assume that the supply voltage is sufficient to generate any output value.)

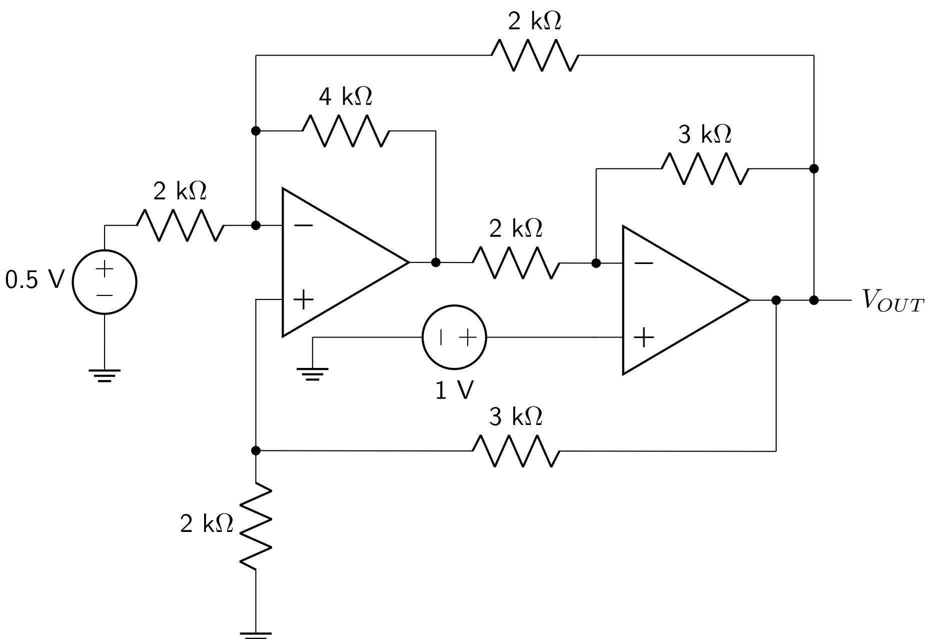


Figure 4.5: Circuit diagram for op-amps question 5.

Perform KCL at the inverting input of the left op-amp. The virtual node voltage will be labeled as V_X. The op-amp output voltage will be labeled as V_Y.

$$\frac{0.5\text{ V} - V_X}{2\text{ k}\Omega} = \frac{V_X - V_Y}{4\text{ k}\Omega} + \frac{V_X - V_{OUT}}{2\text{ k}\Omega}$$

Perform KCL at the non-inverting input of the left op-amp.

$$\frac{0 - V_X}{2\text{ k}\Omega} = \frac{V_X - V_{OUT}}{3\text{ k}\Omega}$$

Perform KCL at the inverting input of the right op-amp.

$$\frac{V_Y - 1\text{ V}}{2\text{ k}\Omega} = \frac{1\text{ V} - V_{OUT}}{3\text{ k}\Omega}$$

Place all three equations into form $\alpha I_X + \beta V_Y + \gamma V_{OUT} = c$, and then place each coefficient into a matrix.

$$\begin{bmatrix} 1.25 & -0.25 & -0.5 & 0.25 \\ 0.83 & 0 & -0.33 & 0 \\ 0 & 0.5 & 0.33 & 0.83 \end{bmatrix}$$

Solve the matrix for V_{OUT}.

$$V_{OUT} = 4\text{ V}$$

5 Chapter 5 Solutions

5.1 Capacitance and Equivalent Capacitance

1. The current flowing through a 25 nF capacitor is $i(t) = \mathrm{e}^{-500t}u(t)$ mA. Derive equations for the voltage dropped over the capacitor and the instantaneous power consumed by the capacitor.

Calculate the voltage drop. Convert the current to amps before inserting into the integral.

$$\begin{aligned}
v(t) &= \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt \\
&= \frac{1}{25\times10^{-9}}\int_{-\infty}^{t}\frac{1}{1000}\mathrm{e}^{-500\tau}u(\tau)\,d\tau \\
&= \frac{1}{25\times10^{-6}}\int_{0}^{t}\mathrm{e}^{-500\tau}\,d\tau \\
&= -\frac{1}{0.0125}\mathrm{e}^{-500\tau}\Big|_0^t u(t) \\
&= -80\left[\mathrm{e}^{-500(t)} - \mathrm{e}^{-500(0)}\right]u(t) \\
&= 80\left[1-\mathrm{e}^{-500t}\right]u(t)
\end{aligned}$$

Calculate the instantaneous power. The units will be mW, as the current is in terms of mA and the voltage is in terms of V.

$$\begin{aligned}
p(t) &= i(t)v(t) \\
&= \left(\mathrm{e}^{-500t}u(t)\right)\left(80\left[1-\mathrm{e}^{-500t}\right]u(t)\right) \\
&= 80\left[\mathrm{e}^{-500t}-\mathrm{e}^{-1000t}\right]u(t)\ \text{mW}
\end{aligned}$$

2. The voltage dropped over a 330 μF capacitor is $v(t) = 100t\ u(t)$ V. Derive equations for the current flowing through the capacitor and the instantaneous power consumed by the capacitor.

Calculate the current flow.

$$\begin{aligned}
i(t) &= C\frac{d}{dt}v(t) \\
&= \left(330\times10^{-6}\right)\frac{d}{dt}\left(100t\ u(t)\right) \\
&= \left(330\times10^{-6}\right)\left(100\ u(t)\right) \\
&= 0.033\ u(t)\ \text{A}
\end{aligned}$$

Calculate the instantaneous power.

$$\begin{aligned} p(t) &= i(t)v(t) \\ &= (0.033\ u(t)\ \text{A})\,(100t\ u(t)\ \text{V}) \\ &= 3.3\ u(t)\ \text{W} \end{aligned}$$

3. Calculate the equivalent capacitance of the circuit shown in figure 5.1.

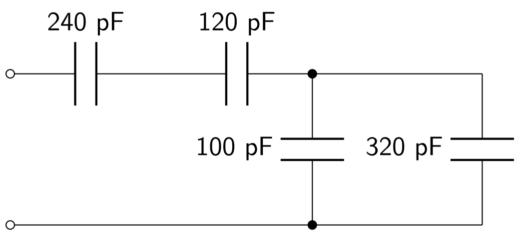


Figure 5.1: Circuit diagram for capacitance and equivalent capacitance question 3.

Calculate the equivalent capacitance.

$$\begin{aligned} C_{EQ} &= 240\ \text{pF}//120\ \text{pF}//(100\ \text{pF} + 320\ \text{pF}) \\ &= 240\ \text{pF}//120\ \text{pF}//420\ \text{pF} \\ &= 80\ \text{pF}//420\ \text{pF} \\ &= 67.2\ \text{pF} \end{aligned}$$

4. Calculate the equivalent capacitance of the circuit shown in figure 5.2.

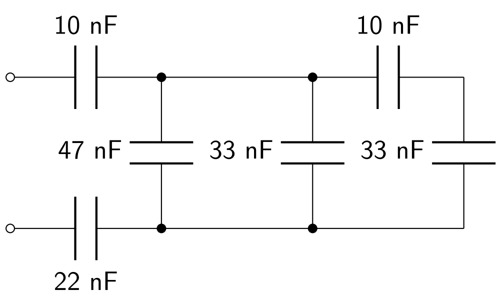


Figure 5.2: Circuit diagram for capacitance and equivalent capacitance question 4.

Calculate the equivalent capacitance.

$$\begin{aligned} C_{EQ} &= ((10 \text{ nF}//33 \text{ nF}) + 33 \text{ nF} + 47 \text{ nF})//10 \text{ nF}//22 \text{ nF} \\ &= (7.67 \text{ nF} + 33 \text{ nF} + 47 \text{ nF})//10 \text{ nF}//22 \text{ nF} \\ &= 87.67 \text{ nF}//10 \text{ nF}//22 \text{ nF} \\ &= 8.98 \text{ nF}//22 \text{ nF} \\ &= 6.38 \text{ nF} \end{aligned}$$

5. Determine the value of the capacitor C_X given that the circuit shown in figure 5.3 has an equivalent capacitance of 10 μF.

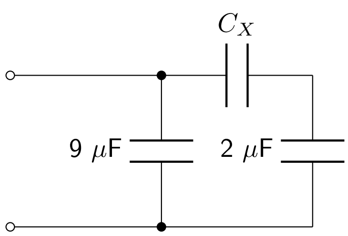


Figure 5.3: Circuit diagram for capacitance and equivalent capacitance question 5.

Use parallel and series combinations of capacitors. All capacitor units are in μF.

$$\begin{aligned} 10 &= 9 + (C_X//2) \\ &= 9 + \frac{2C_X}{C_X + 2} \\ 1 &= \frac{2C_X}{C_X + 2} \\ C_X + 2 &= 2C_X \\ 2 &= C_X \\ C_X &= 2 \ \mu\text{F} \end{aligned}$$

5.2 Resistor-Capacitor Circuits

6. Calculate an expression for $v(t)$ and $i(t)$ given the circuit shown in figure 5.4. The switch moves from position a to b at time of zero seconds.

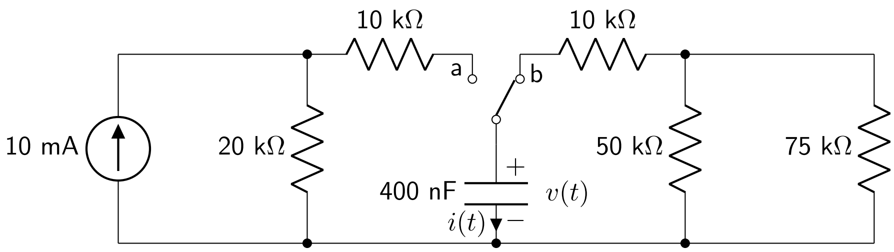
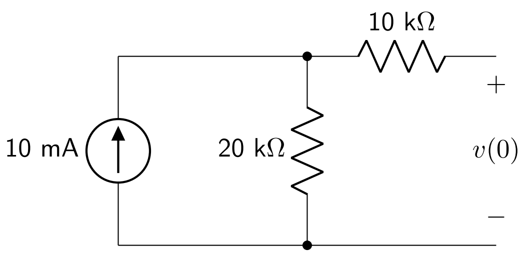


Figure 5.4: Circuit diagram for resistor-capacitor circuits question 6.

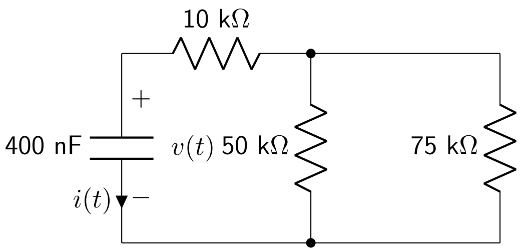
Draw the circuit in the initial steady-state condition. The capacitor can be replaced as an open.



Use Ohm's law to calculate $v(0)$.

$$\begin{aligned} v(0) &= (10 \text{ mA})(20 \text{ k}\Omega) \\ &= 200 \text{ V} \end{aligned}$$

Draw the circuit in the final conditions.



Calculate the equivalent resistance as seen by the capacitor.

$$\begin{aligned} R_{EQ} &= 75 \text{ k}\Omega // 50 \text{ k}\Omega + 10 \text{ k}\Omega \\ &= 30 \text{ k}\Omega + 10 \text{ k}\Omega \\ &= 40 \text{ k}\Omega \end{aligned}$$

Calculate the RC time constant.

$$\begin{aligned}\tau &= RC \\ &= (40000\ \Omega)(400 \times 10^{-9}\ \text{F}) \\ &= 0.016\ \text{s}\end{aligned}$$

Plug these values into the equation for the voltage of a discharging RC circuit.

$$v(t) = 200\ \mathrm{e}^{-62.5t}\ u(t)\ \text{V}$$

Calculate the current through a discharging RC circuit.

$$i(t) = -5\ \mathrm{e}^{-62.5t}\ u(t)\ \text{mA}$$

7. Calculate an expression for $v(t)$ and $i(t)$ given the circuit shown in figure 5.5. The switch opens at a time of zero seconds.

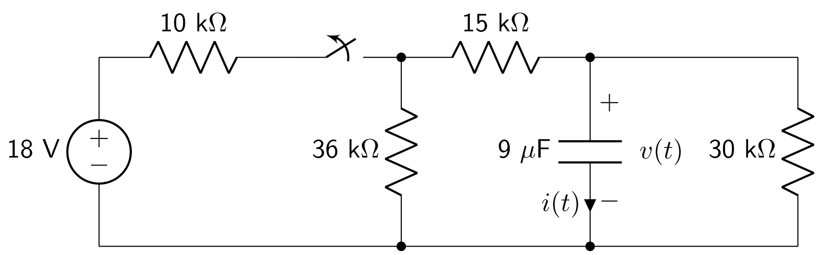
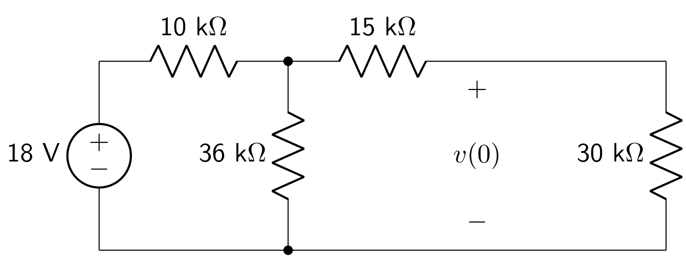


Figure 5.5: Circuit diagram for resistor-capacitor circuits question 7.

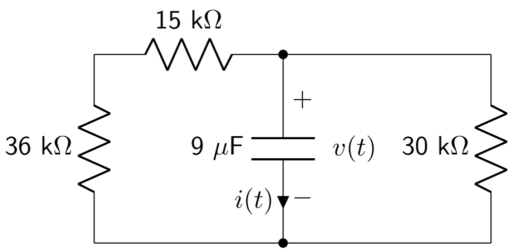
Draw the circuit in the initial steady-state condition. The capacitor can be replaced as an open.



Use the voltage divider rule to calculate $v(0)$.

$$\begin{aligned} v(0) &= 18\text{ V}\left(\frac{36\text{ k}\Omega//(15\text{ k}\Omega+30\text{ k}\Omega)}{10\text{ k}\Omega+36\text{ k}\Omega//(15\text{ k}\Omega+30\text{ k}\Omega)}\right)\left(\frac{30\text{ k}\Omega}{45\text{ k}\Omega}\right) \\ &= 18\text{ V}\left(\frac{36\text{ k}\Omega//45\text{ k}\Omega}{10\text{ k}\Omega+36\text{ k}\Omega//45\text{ k}\Omega}\right)\left(\frac{2}{3}\right) \\ &= 18\text{ V}\left(\frac{20\text{ k}\Omega}{10\text{ k}\Omega+20\text{ k}\Omega}\right)\left(\frac{2}{3}\right) \\ &= 8\text{ V} \end{aligned}$$

Draw the circuit in the final conditions.



Calculate the equivalent resistance as seen by the capacitor.

$$\begin{aligned} R_{EQ} &= 30\text{ k}\Omega//(15\text{ k}\Omega+36\text{ k}\Omega) \\ &= 30\text{ k}\Omega//51\text{ k}\Omega \\ &= 18.89\text{ k}\Omega \end{aligned}$$

Calculate the RC time constant.

$$\begin{aligned} \tau &= RC \\ &= (18888.89\ \Omega)(9\ \mu\text{F}) \\ &= 0.17\text{ s} \end{aligned}$$

Plus these values into the equation for the voltage of a discharging RC circuit.

$$v(t) = 8\ \text{e}^{-5.88t}\ u(t)\text{ V}$$

Calculate the current through a discharging RC circuit.

$$i(t) = -0.42\ \text{e}^{-5.88t}\ u(t)\text{ mA}$$

8. Calculate an expression for $v(t)$ and $i(t)$ given the circuit shown in figure 5.6. The switch closes at a time of zero seconds.

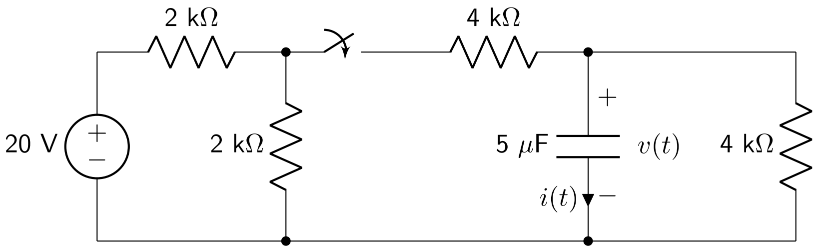
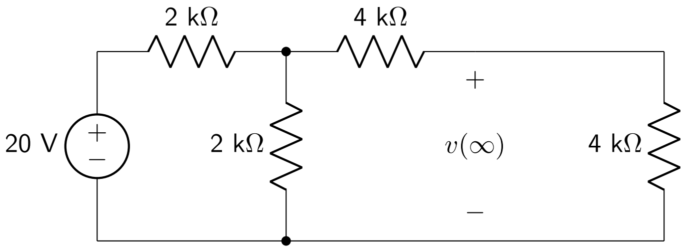


Figure 5.6: Circuit diagram for resistor-capacitor circuits question 8.

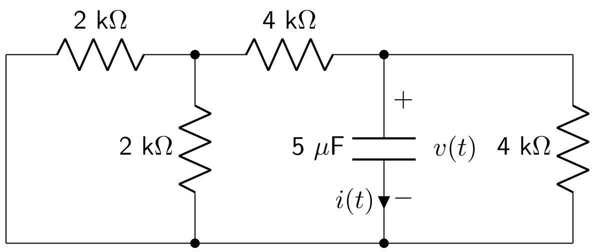
In the initial steady-state, the capacitor will be fully discharged and have an initial voltage of zero. Draw the circuit in the final steady-state. The capacitor can be replaced as an open.



Use the voltage divider rule to calculate $v(\infty)$.

$$\begin{aligned} v(\infty) &= 20 \text{ V} \left(\frac{2 \text{ k}\Omega // (4 \text{ k}\Omega + 4 \text{ k}\Omega)}{2 \text{ k}\Omega + 2 \text{ k}\Omega // (4 \text{ k}\Omega + 4 \text{ k}\Omega)} \right) \left(\frac{4 \text{ k}\Omega}{8 \text{ k}\Omega} \right) \\ &= 20 \text{ V} \left(\frac{2 \text{ k}\Omega // 8 \text{ k}\Omega}{2 \text{ k}\Omega + 2 \text{ k}\Omega // 8 \text{ k}\Omega} \right) \left(\frac{1}{2} \right) \\ &= 20 \text{ V} \left(\frac{1.6 \text{ k}\Omega}{3.6 \text{ k}\Omega} \right) \left(\frac{1}{2} \right) \\ &= 4.44 \text{ V} \end{aligned}$$

Draw the circuit at $t = 0^+$ and deactivate the source.



Calculate the equivalent resistance as seen by the capacitor.

$$\begin{aligned} R_{EQ} &= (2\ \text{k}\Omega // 2\ \text{k}\Omega + 4\ \text{k}\Omega) // 4\ \text{k}\Omega \\ &= (1\ \text{k}\Omega + 4\ \text{k}\Omega) // 4\ \text{k}\Omega \\ &= 5\ \text{k}\Omega // 4\ \text{k}\Omega \\ &= 2.22\ \text{k}\Omega \end{aligned}$$

Calculate the RC time constant.

$$\begin{aligned} \tau &= RC \\ &= (2222\ \Omega)\left(5 \times 10^{-6}\ \text{F}\right) \\ &= 0.011\ \text{s} \end{aligned}$$

Plug these values into the equation for the voltage of a charging RC circuit.

$$v(t) = \left[4.44 - 4.44\text{e}^{-90t}\right]\ u(t)\ \text{V}$$

Calculate the current through a charging RC circuit.

$$i(t) = 2\text{e}^{-90t}\ u(t)\ \text{mA}$$

9. Calculate an expression for $v(t)$ and $i(t)$ given the circuit shown in figure 5.7. The switch closes at a time of zero seconds.

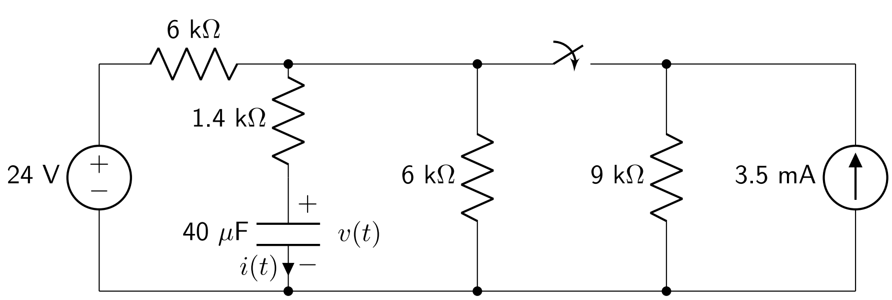
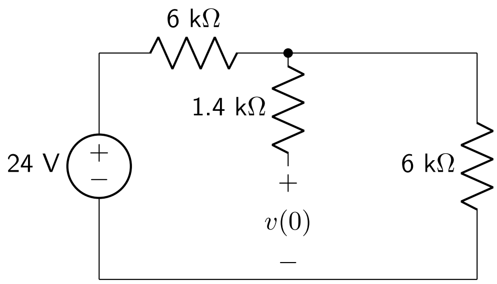


Figure 5.7: Circuit diagram for resistor-capacitor circuits question 9.

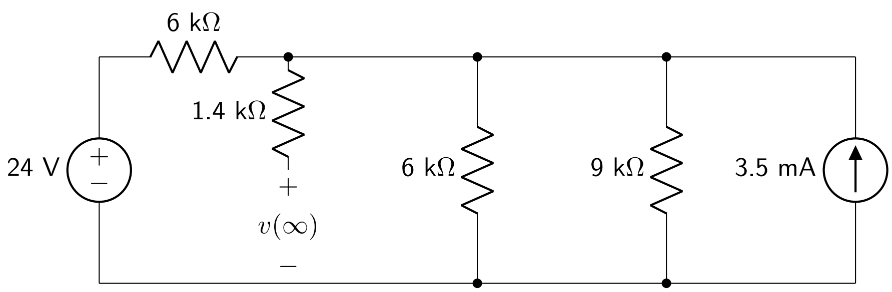
Draw the circuit in the initial steady-state condition. The capacitor can be replaced as an open.



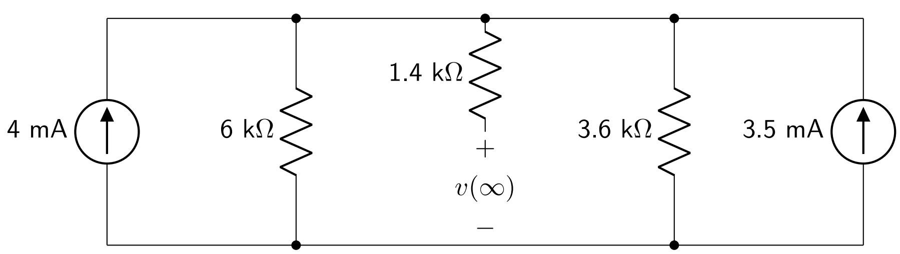
Use the voltage divider rule to calculate $v(0)$.

$$
\begin{aligned}
v(0) &= 24\ \text{V}\left(\frac{6\ \text{k}\Omega}{6\ \text{k}\Omega + 6\ \text{k}\Omega}\right) \\
&= 12\ \text{V}
\end{aligned}
$$

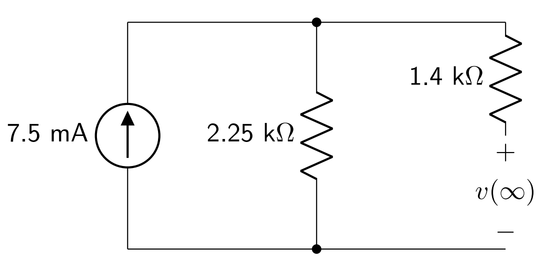
Draw the circuit in the final steady-state condition. The capacitor can be replaced as an open.



Combine parallel resistors, and source transform the 24 V source and re-draw the circuit.



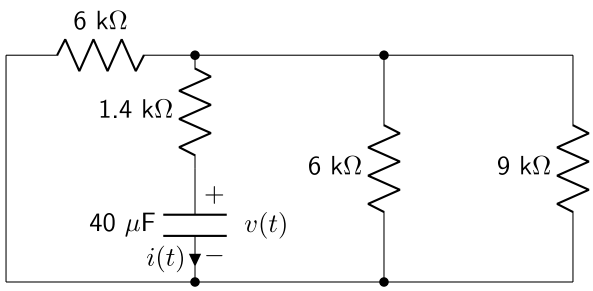
Combine remaining resistors and both current sources.



Use Ohm's law to calculate $v(\infty)$.

$$v(\infty) = (7.5 \text{ mA})(2.25 \text{ k}\Omega)$$
$$= 16.875 \text{ V}$$

Draw the circuit at $t = 0^+$ and deactivate the sources.



Calculate the equivalent resistance as seen by the capacitor.

$$\begin{aligned} R_{EQ} &= 9 \text{ k}\Omega // 6 \text{ k}\Omega // 6 \text{ k}\Omega + 1.4 \text{ k}\Omega \\ &= 9 \text{ k}\Omega // 3 \text{ k}\Omega + 1.4 \text{ k}\Omega \\ &= 2.25 \text{ k}\Omega + 1.4 \text{ k}\Omega \\ &= 3.65 \text{ k}\Omega \end{aligned}$$

Calculate the RC time constant.

$$\begin{aligned} \tau &= RC \\ &= (3650 \ \Omega)(40 \times 10^{-6} \text{ F}) \\ &= 0.146 \text{ s} \end{aligned}$$

Plug these values into the equation for a general RC circuit.

$$v(t) = \left[16.875 - 4.875\text{e}^{-6.85t}\right] \ u(t) \text{ V}$$

Calculate the current.

$$i(t) = 1.34\text{e}^{-6.85t} \ u(t) \text{ mA}$$

10. Calculate an expression for $v(t)$ and $i(t)$ given the circuit shown in figure 5.8. The switch opens at a time of zero seconds.

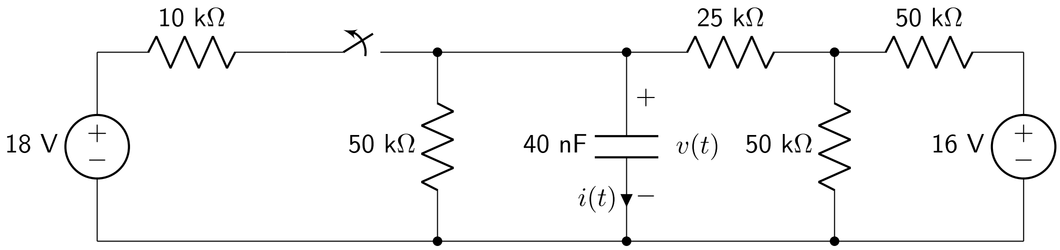
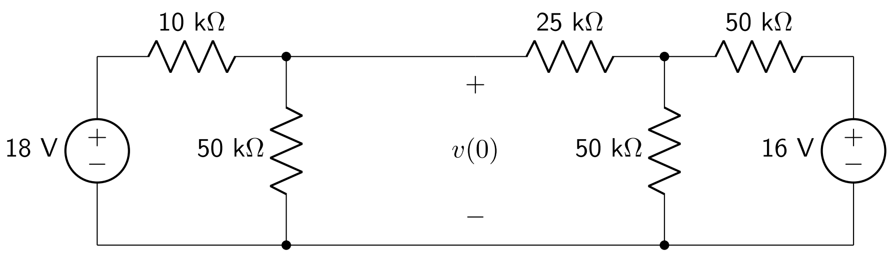
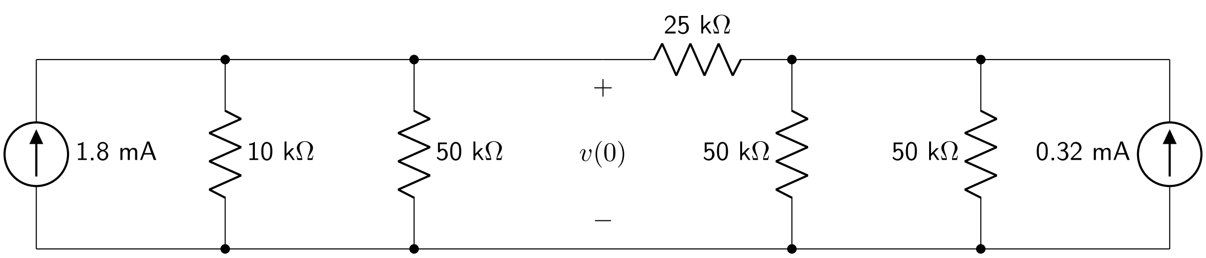


Figure 5.8: Circuit diagram for resistor-capacitor circuits question 10.

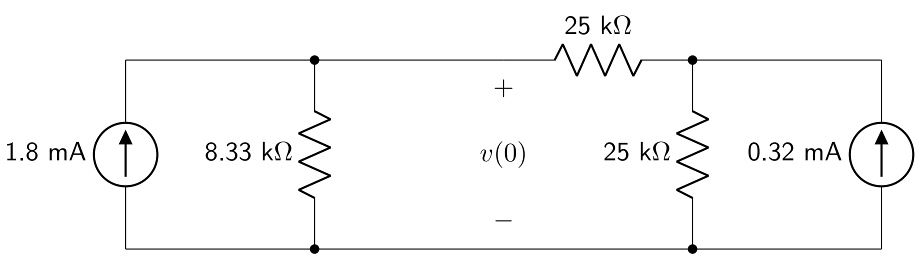
Draw the circuit in the initial steady-state condition. The capacitor can be replaced as an open.



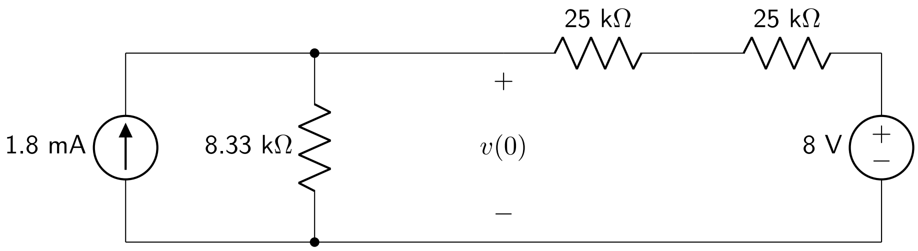
Convert both sources to current sources.



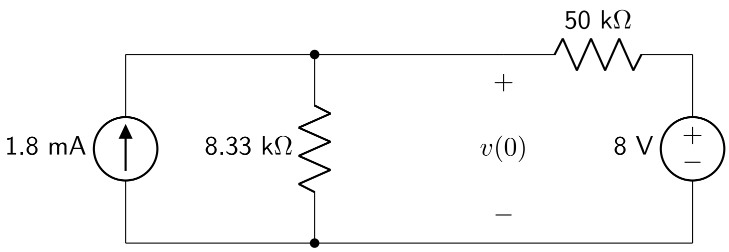
Combine parallel resistors.



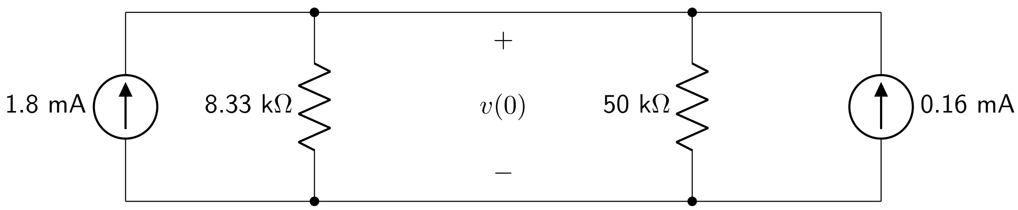
Convert the 0.32 mA source to a voltage source.



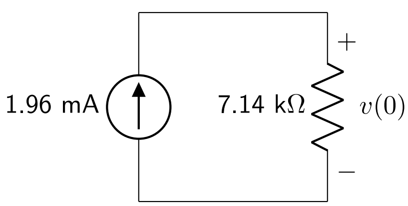
Combine series resistors.



Convert the voltage source to a current source.



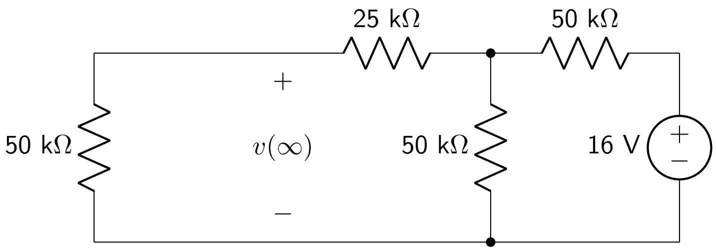
Combine sources and resistors.



Use Ohm's law to calculate $v(0)$.

$$\begin{aligned} v(0) &= (1.96 \text{ mA})(7.14 \text{ k}\Omega) \\ &= 14 \text{ V} \end{aligned}$$

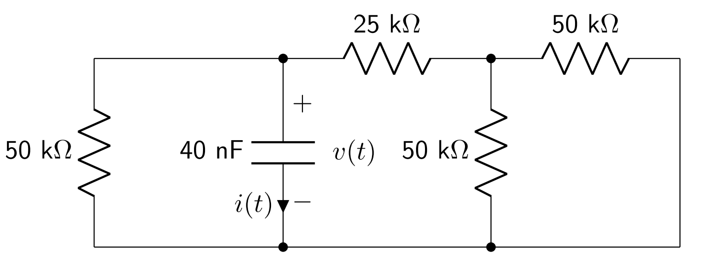
Draw the circuit in the final steady-state condition. The capacitor can be replaced as an open.



Use the voltage divider law to calculate $v(\infty)$.

$$\begin{aligned} v(\infty) &= 16\text{ V}\left(\frac{50\text{ k}\Omega//75\text{ k}\Omega}{50\text{ k}\Omega + 50\text{ k}\Omega//75\text{ k}\Omega}\right)\left(\frac{50\text{ k}\Omega}{75\text{ k}\Omega}\right) \\ &= 16\text{ V}\left(\frac{30\text{ k}\Omega}{50\text{ k}\Omega + 30\text{ k}\Omega}\right)\left(\frac{2}{3}\right) \\ &= 4\text{ V} \end{aligned}$$

Draw the circuit at $t = 0^+$ and deactivate the source.



Calculate the equivalent resistance as seen by the capacitor.

$$\begin{aligned} R_{EQ} &= (50\text{ k}\Omega//50\text{ k}\Omega + 25\text{ k}\Omega)//50\text{ k}\Omega \\ &= (25\text{ k}\Omega + 25\text{ k}\Omega)//50\text{ k}\Omega \\ &= 50\text{ k}\Omega//50\text{ k}\Omega \\ &= 25\text{ k}\Omega \end{aligned}$$

Calculate the RC time constant.

$$\begin{aligned} \tau &= RC \\ &= (25000\ \Omega)(40 \times 10^{-9}\text{F}) \\ &= 0.001\text{ s} \end{aligned}$$

Plug these values into the equation for a general RC circuit.

$$v(t) = \left[4 + 10\text{e}^{-1000t}\right]\ u(t)\text{ V}$$

Calculate the current.

$$i(t) = -0.4\mathrm{e}^{-1000t}\ u(t)\ \mathrm{mA}$$

6 Chapter 6 Solutions

6.1 Inductance and Equivalent Inductance

1. The current flowing through a 20 mH inductor is $i(t) = 50t\ u(t)$ mA. Derive equations for the voltage dropped over the inductor and the instantaneous power consumed by the inductor.

Calculate the voltage drop. Use units of H and A.

$$\begin{aligned} v(t) &= L\frac{d}{dt}i(t) \\ &= 0.02\frac{d}{dt}(0.05t\ u(t)) \\ &= 0.001\ u(t)\ \text{V} \\ &= u(t)\ \text{mV} \end{aligned}$$

Calculate the instantaneous power.

$$\begin{aligned} p(t) &= i(t)v(t) \\ &= (0.05t\ u(t)\ \text{A})(0.001\ u(t)\ \text{V}) \\ &= 5 \times 10^{-5}\ u(t)\ \text{W} \\ &= 0.05\ u(t)\ \text{mW} \end{aligned}$$

2. The voltage dropped over a 15 μH inductor is $v(t) = \cos(5000t)u(t)$ V. Derive equations for the current flowing through the inductor and the instantaneous power consumed by the inductor.

Calculate the current flow. Use units of H and V.

$$\begin{aligned} i(t) &= \frac{1}{L}\int_{-\infty}^{t} v(\tau)\,d\tau \\ &= \frac{1}{15 \times 10^{-6}}\int_{-\infty}^{t} \cos(5000\tau)\ u(\tau)\,d\tau \\ &= \frac{1}{15 \times 10^{-6}}\int_{0}^{t} \cos(5000\tau)\,d\tau \\ &= \frac{1}{0.075}\sin(5000\tau)\Big|_0^t \\ &= 13.33\sin(5000t)\ u(t)\ \text{A} \end{aligned}$$

Calculate the instantaneous power. Use the double angle formula.

$$\begin{aligned} p(t) &= i(t)v(t) \\ &= (13.33\sin(5000t)\ u(t)\ \text{A})(\cos(5000t)\ u(t)\ \text{V}) \\ &= 6.67\sin(10000t)\ u(t)\ \text{W} \end{aligned}$$

i(t) = 13.333 sin(5000t) u(t) A, p(t) = 6.667 sin(10000t) u(t) W – To calculate the current, use equation. Use equation to calculate the instantaneous power.

3. Calculate the equivalent inductance of the circuit shown in figure 6.1.

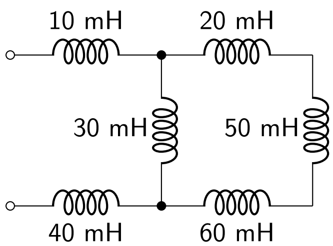


Figure 6.1: Circuit diagram for inductance and equivalent inductance question 3.

Calculate the equivalent inductance.

$$\begin{aligned} L_{EQ} &= (50\text{ mH} + 20\text{ mH} + 60\text{ mH})//30\text{ mH} + 10\text{ mH} + 40\text{ mH} \\ &= 130\text{ mH}//30\text{ mH} + 50\text{ mH} \\ &= 24.375\text{ mH} + 50\text{ mH} \\ &= 74.375\text{ mH} \end{aligned}$$

4. Calculate the equivalent inductance of the circuit shown in figure 6.2.

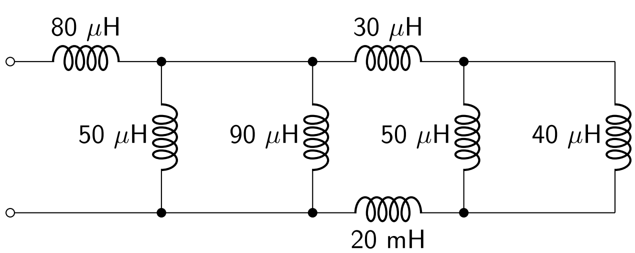


Figure 6.2: Circuit diagram for inductance and equivalent inductance question 4.

Calculate the equivalent inductance.

$$
\begin{aligned}
L_{EQ} &= (40\ \mu\text{F}//50\ \mu\text{F} + 30\ \mu\text{F} + 20\ \mu\text{F})//90\ \mu\text{F}//50\ \mu\text{F} + 80\ \mu\text{F} \\
&= (22.22\ \mu\text{F} + 50\ \mu\text{F})//32.14\ \mu\text{F} + 80\ \mu\text{F} \\
&= 72.22\ \mu\text{F}//32.14\ \mu\text{F} + 80\ \mu\text{F} \\
&= 22.24\ \mu\text{F} + 80\ \mu\text{F} \\
&= 102.24\ \mu\text{F}
\end{aligned}
$$

5. Determine the value of the inductor L_X given that the circuit shown in figure 6.3 has an equivalent inductance of 250 μH.

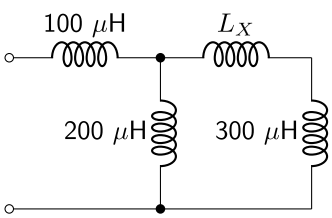


Figure 6.3: Circuit diagram for inductance and equivalent inductance question 5.

Use the equivalent inductance relationships to calculate L_X. All units are in μH below.

$$
\begin{aligned}
250 &= (300 + L_X)//200 + 100 \\
150 &= \frac{200(300 + L_X)}{200 + 300 + L_X} \\
&= \frac{60000 + 200L_X}{500 + L_X} \\
75000 + 150L_X &= 60000 + 200L_X \\
15000 &= 50L_X \\
300 &= L_X \\
L_X &= 300\ \mu\text{H}
\end{aligned}
$$

6.2 Resistor-Inductor Circuits

6. Calculate an expression for $i(t)$ and $v(t)$ given the circuit shown in figure 6.4. The switch moves from position a to b at time of zero seconds.

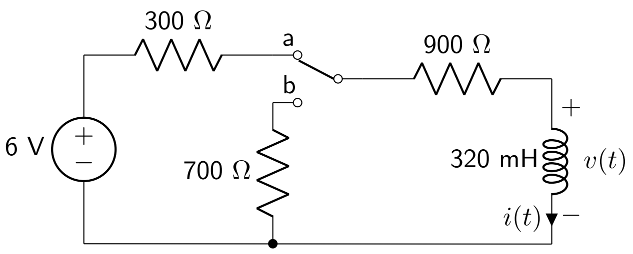
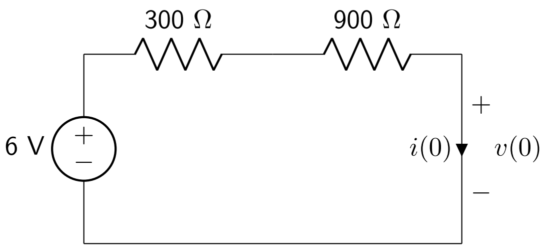


Figure 6.4: Circuit diagram for resistor-inductor circuits question 6.

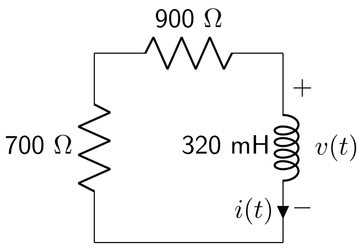
Draw the circuit in the initial steady-state condition. The inductor can be replaced by a short.



Use Ohm's law to calculate $i(0)$.

$$i(0) = \frac{6 \text{ V}}{300 \ \Omega + 900 \ \Omega}$$
$$= 5 \text{ mA}$$

Draw the circuit at $t = 0^+$.



Calculate the equivalent resistance as seen by the inductor.

$$R_{EQ} = 700 \ \Omega + 900 \ \Omega$$
$$= 1600 \ \Omega$$

Calculate the RL time constant.

$$\begin{aligned}\tau &= \frac{L}{R} \\ &= \frac{0.32 \text{ H}}{1600 \ \Omega} \\ &= 2 \times 10^{-4} \text{ s}\end{aligned}$$

Plug these values into the equation for a discharging RL circuit.

$$i(t) = 5\text{e}^{-5000t} \ u(t) \text{ mA}$$

Calculate the voltage drop.

$$v(t) = -8\text{e}^{-5000t} \ u(t) \text{ V}$$

7. Calculate an expression for $i(t)$ and $v(t)$ given the circuit shown in figure 6.5. The switch opens at a time of zero seconds.

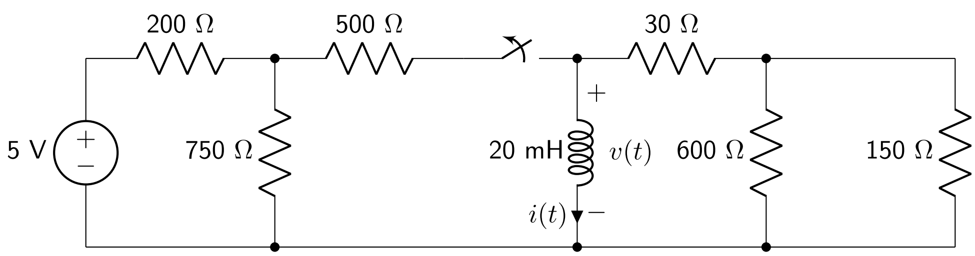
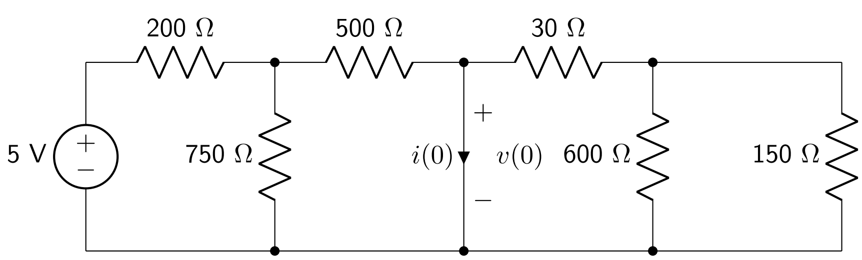
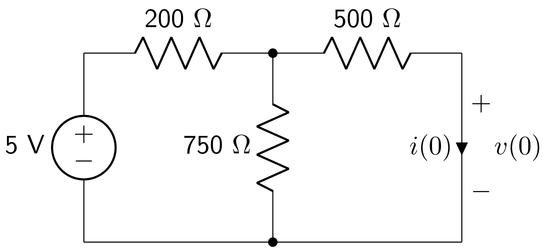


Figure 6.5: Circuit diagram for resistor-inductor circuits question 7.

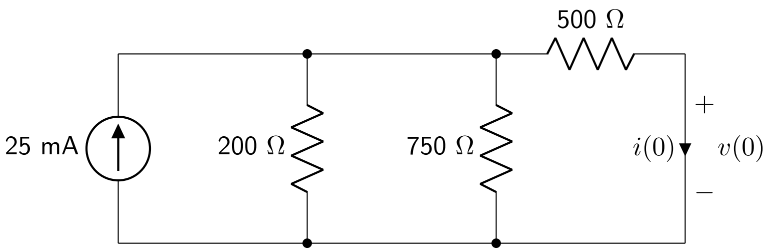
Draw the circuit in the initial steady-state condition. The inductor can be replaced by a short.



The 30 Ω, 600 Ω, and 150 Ω resistors are shorted by the inductor in the steady-state.



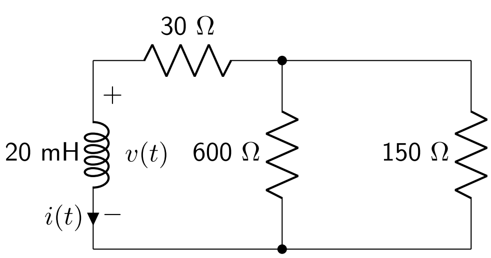
Convert the voltage source to a current source.



Use the current divider rule to calculate $i(0)$.

$$
\begin{aligned}
i(0) &= 25\text{ mA}\left(\frac{200\ \Omega//750\ \Omega//500\ \Omega}{500\ \Omega}\right) \\
&= 25\text{ mA}\left(\frac{120\ \Omega}{500\ \Omega}\right) \\
&= 6\text{ mA}
\end{aligned}
$$

Draw the circuit at $t = 0^+$.



Calculate the equivalent resistance as seen by the inductor.

$$
\begin{aligned}
R_{EQ} &= 600\ \Omega//150\ \Omega + 30\ \Omega \\
&= 120\ \Omega + 30\ \Omega \\
&= 150\ \Omega
\end{aligned}
$$

Calculate the RL time constant.

$$\begin{aligned}\tau &= \frac{L}{R} \\ &= \frac{0.02\ \text{H}}{150\ \Omega} \\ &= 1.33 \times 10^{-4}\ \text{s}\end{aligned}$$

Plug these values into the equation for a discharging RL circuit.

$$i(t) = 6\text{e}^{-7500t}\ u(t)\ \text{mA}$$

Calculate the voltage drop.

$$v(t) = -0.9\text{e}^{-7500t}\ u(t)\ \text{V}$$

8. Calculate an expression for $i(t)$ and $v(t)$ given the circuit shown in figure 6.6. The switch closes at a time of zero seconds.

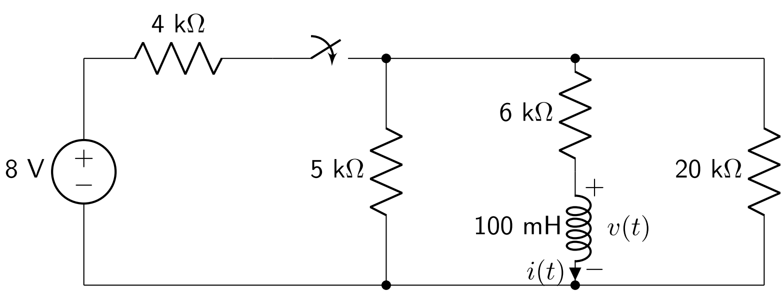
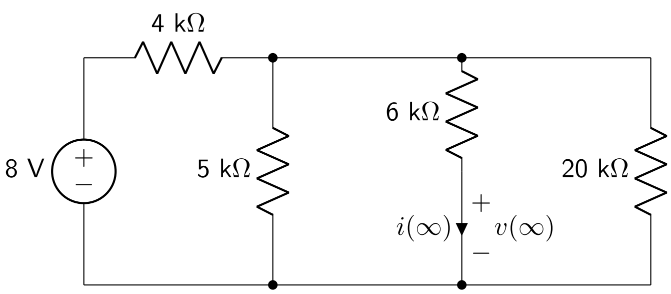
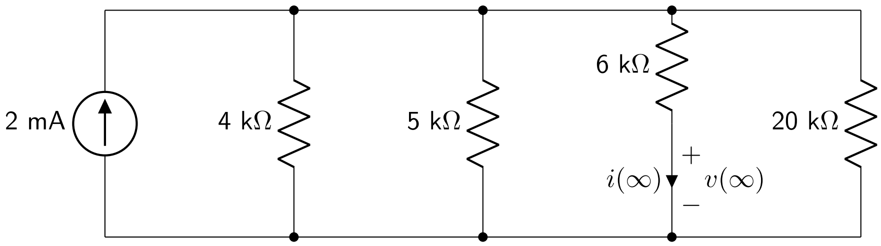


Figure 6.6: Circuit diagram for resistor-inductor circuits question 8.

Draw the circuit in the final steady-state condition. The inductor can be replaced by a short.



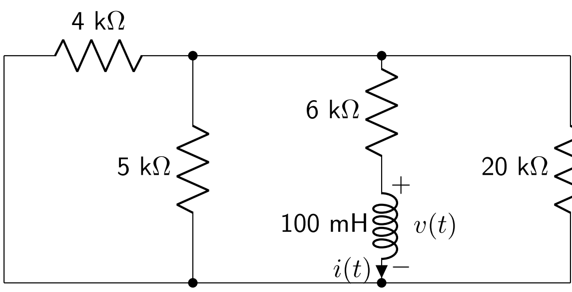
Convert the voltage source to a current source.



Use the current divider rule to calculate $i(\infty)$.

$$\begin{aligned} i(\infty) &= 2 \text{ mA} \left(\frac{4 \text{ k}\Omega // 5 \text{ k}\Omega // 6 \text{ k}\Omega // 20 \text{ k}\Omega}{6 \text{ k}\Omega} \right) \\ &= 2 \text{ mA} \left(\frac{1.5 \text{ k}\Omega}{6 \text{ k}\Omega} \right) \\ &= 0.5 \text{ mA} \end{aligned}$$

Draw the circuit at $t = 0^+$ and deactivate the source.



Calculate the equivalent resistance as seen by the inductor.

$$\begin{aligned} R_{EQ} &= 4 \text{ k}\Omega // 5 \text{ k}\Omega // 20 \text{ k}\Omega + 6 \text{ k}\Omega \\ &= 2 \text{ k}\Omega + 6 \text{ k}\Omega \\ &= 8 \text{ k}\Omega \end{aligned}$$

Calculate the RL time constant.

$$\begin{aligned} \tau &= \frac{L}{R} \\ &= \frac{0.1 \text{ H}}{8000 \ \Omega} \\ &= 1.25 \times 10^{-5} \text{ s} \end{aligned}$$

Plug these values into the equation for a charging RL circuit.

$$i(t) = 0.5 \left[1 - \text{e}^{-80000t} \right] \ u(t) \text{ mA}$$

Calculate the voltage drop over the inductor.

$$v(t) = 4\mathrm{e}^{-80000t}\ u(t)\ \mathrm{V}$$

9. Calculate an expression for $i(t)$ and $v(t)$ given the circuit shown in figure 6.7. The switch closes at a time of zero seconds.

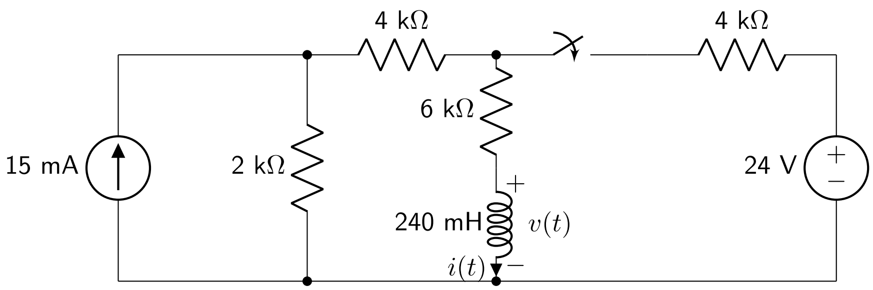
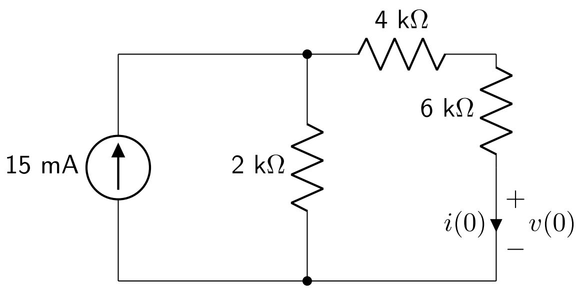


Figure 6.7: Circuit diagram for resistor-inductor circuits question 9.

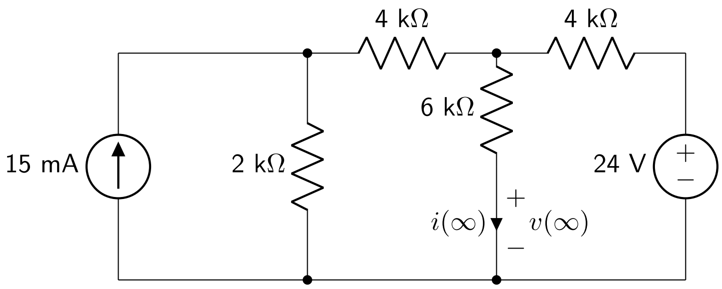
Draw the circuit in the initial steady-state condition. The inductor can be replaced by a short.



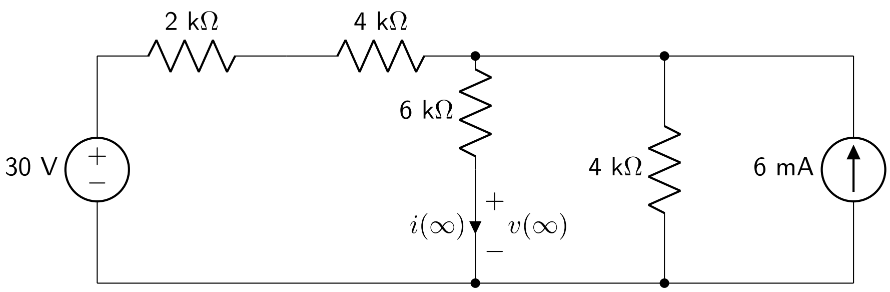
Use the current divider rule to calculate $i(0)$.

$$\begin{aligned} i(0) &= 15\ \mathrm{mA}\left(\frac{2\ \mathrm{k\Omega}//10\ \mathrm{k\Omega}}{10\ \mathrm{k\Omega}}\right) \\ &= 15\ \mathrm{mA}\left(\frac{1.67\ \mathrm{k\Omega}}{10\ \mathrm{k\Omega}}\right) \\ &= 2.5\ \mathrm{mA} \end{aligned}$$

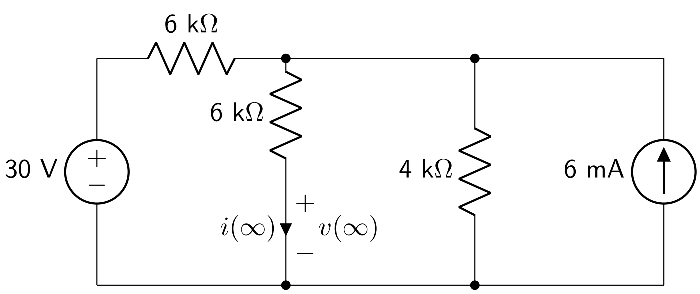
Draw the circuit in the final steady-state condition. The inductor can be replaced by a short.



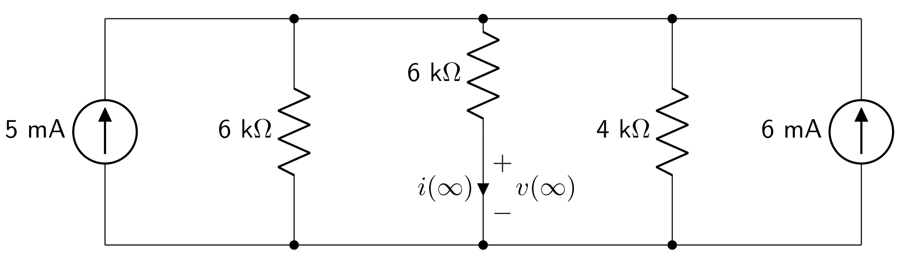
Perform source transformation on both sources.



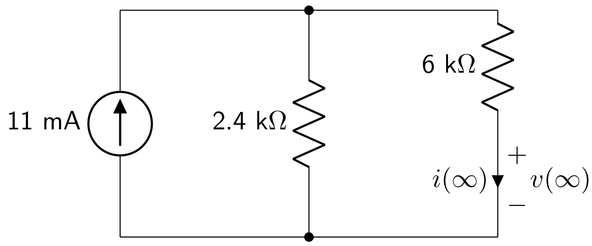
Combine the series resistors.



Convert the voltage source to a current source.



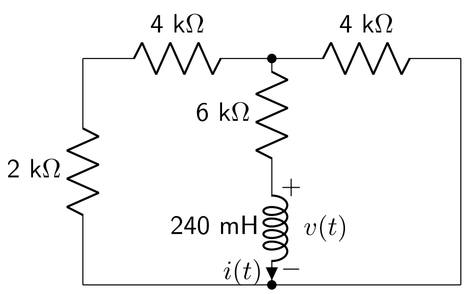
Combine resistors and sources.



Use the current divider rule to calculate $i(\infty)$.

$$\begin{aligned} i(\infty) &= 11 \text{ mA} \left(\frac{2.4 \text{ k}\Omega // 6 \text{ k}\Omega}{6 \text{ k}\Omega} \right) \\ &= 11 \text{ mA} \left(\frac{1.71 \text{ k}\Omega}{6 \text{ k}\Omega} \right) \\ &= 3.14 \text{ mA} \end{aligned}$$

Draw the circuit at $t = 0^+$ and deactivate the sources.



Calculate the equivalent resistance as seen by the inductor.

$$\begin{aligned} R_{EQ} &= (2 \text{ k}\Omega + 4 \text{ k}\Omega) // 4 \text{ k}\Omega + 6 \text{ k}\Omega \\ &= 6 \text{ k}\Omega // 4 \text{ k}\Omega + 6 \text{ k}\Omega \\ &= 2.4 \text{ k}\Omega + 6 \text{ k}\Omega \\ &= 8.4 \text{ k}\Omega \end{aligned}$$

Calculate the RL time constant.

$$\begin{aligned} \tau &= \frac{L}{R} \\ &= \frac{.24 \text{ H}}{8400 \ \Omega} \\ &= 2.86 \times 10^{-5} \text{ s} \end{aligned}$$

Plug these values into the equation for a general RL circuit.

$$i(t) = \left[3.14 - 0.64\mathrm{e}^{-35000t}\right]\ u(t)\ \mathrm{mA}$$

Calculate the voltage drop.

$$v(t) = 5.4\mathrm{e}^{-35000t}\ u(t)\ \mathrm{V}$$

10. Calculate an expression for $i(t)$ and $v(t)$ given the circuit shown in figure 6.7. Switch S1 opens at a time of zero seconds, and switch S2 closes at a time of zero seconds.

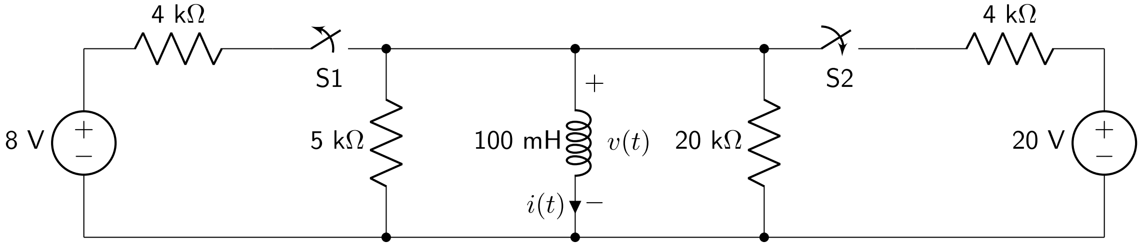
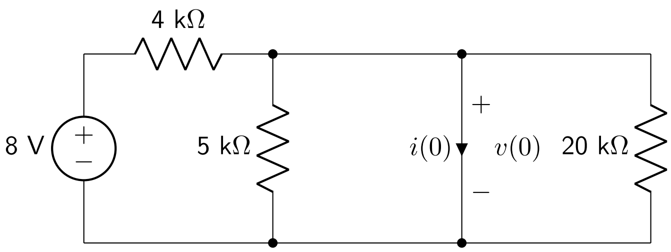
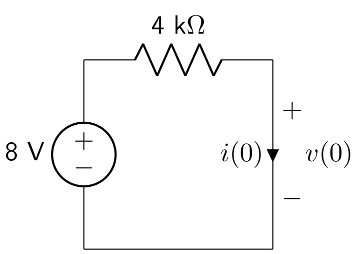


Figure 6.8: Circuit diagram for resistor-inductor circuits question 10.

Draw the circuit in the initial steady-state condition. The inductor can be replaced by a short.



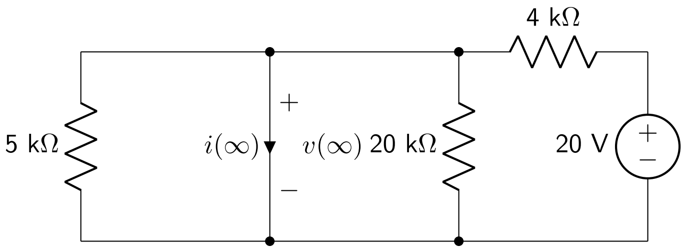
The 5 kΩ and 20 kΩ resistors are shorted by the inductor in the steady-state.



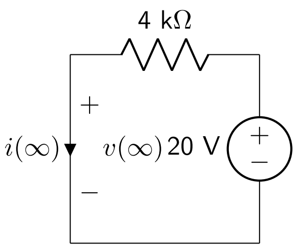
Use Ohm's law to calculate $i(0)$.

$$\begin{aligned} i(0) &= \frac{8\ \mathrm{V}}{4\ \mathrm{k}\Omega} \\ &= 2\ \mathrm{mA} \end{aligned}$$

Draw the circuit in the final steady-state condition. The inductor can be replaced by a short.



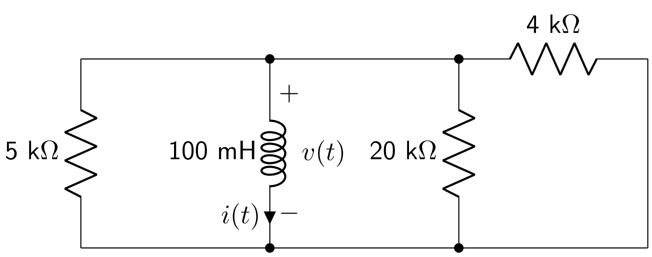
The 5 kΩ and 20 kΩ resistors are shorted by the inductor in the steady-state.



Use Ohm's law to calculate $i(\infty)$.

$$\begin{aligned} i(\infty) &= \frac{20 \text{ V}}{4 \text{ k}\Omega} \\ &= 5 \text{ mA} \end{aligned}$$

Draw the circuit at $t = 0^+$ and deactivate the source.



Calculate the equivalent resistance as seen by the inductor.

$$\begin{aligned} R_{EQ} &= 5 \text{ k}\Omega // 20 \text{ k}\Omega // 4 \text{ k}\Omega \\ &= 2 \text{ k}\Omega \end{aligned}$$

Calculate the RL time constant.

$$\begin{aligned}\tau &= \frac{L}{R} \\ &= \frac{.1 \text{ H}}{2000 \ \Omega} \\ &= 5 \times 10^{-5} \text{ s}\end{aligned}$$

Plug these values into the equation for a general RL circuit.

$$i(t) = \left[5 - 3\mathrm{e}^{-20000t}\right] \ u(t) \text{ mA}$$

Calculate the voltage drop.

$$v(t) = 6\mathrm{e}^{-20000t} \ u(t) \text{ V}$$

7 Chapter 7 Solutions

7.1 Homogeneous Second Order Circuits

1. Calculate an expression for $v(t)$ given the circuit shown in figure 7.1. The switch opens at a time of zero seconds.

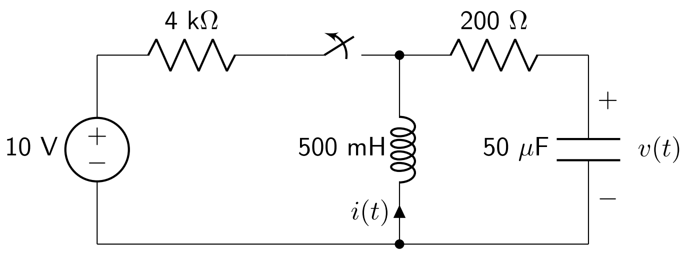
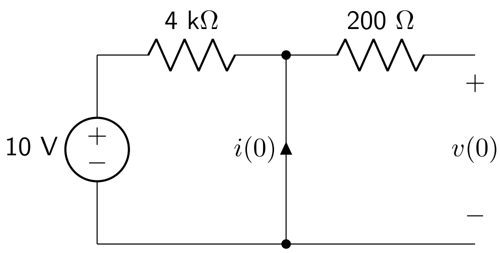


Figure 7.1: Circuit diagram for homogeneous second order circuits question 1.

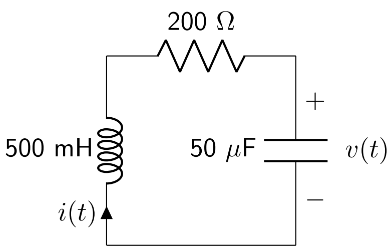
Draw the circuit in the initial steady-state condition. The capacitor can be replaced by an open and the inductor can be replaced by a short.



Because the initial voltage drop is measured over the inductor (short), it will be zero. Use Ohm's law to calculate the initial current flow.

$$\begin{aligned} i(0) &= \frac{-10 \text{ V}}{4 \text{ k}\Omega} \\ &= -2.5 \text{ mA} \end{aligned}$$

Draw the circuit for $t = 0^+$.



This is a series RLC circuit. Calculate the damping parameter.

$$\begin{aligned}\alpha &= \frac{R}{2L} \\ &= \frac{200\ \Omega}{2(0.5\ \text{H})} \\ &= 200\ \text{Np/s}\end{aligned}$$

Calculate the resonant frequency.

$$\begin{aligned}\omega_0 &= \frac{1}{\sqrt{LC}} \\ &= \frac{1}{\sqrt{(0.5\ \text{H})(50 \times 10^{-6}\ \text{F})}} \\ &= \frac{1}{0.005} \\ &= 200\ \text{rad/s}\end{aligned}$$

Calculate the initial first derivative of the voltage drop.

$$\begin{aligned}v'(0) &= \frac{i(0)}{C} \\ &= \frac{-0.0025\ \text{A}}{50\ \times 10^{-6}\ \text{F}} \\ &= -50\ \text{V/s}\end{aligned}$$

The circuit is critically damped. Calculate the coefficients.

$$\begin{aligned}A_1 &= v(0) \\ &= 0 \\ A_2 &= v'(0) + \alpha v(0) \\ &= -50\ \text{V/s}\end{aligned}$$

Plug these values into the equation for a critically damped homogeneous circuit.

$$v(t) = -50t\text{e}^{-200t}\ u(t)\ \text{V}$$

2. Calculate an expression for $i(t)$ given the circuit shown in figure 7.2. The switch opens at a time of zero seconds.

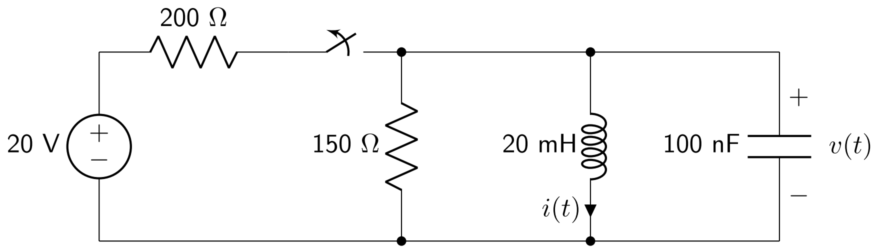
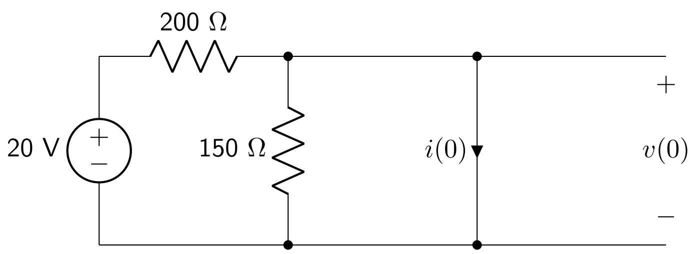


Figure 7.2: Circuit diagram for homogeneous second order circuits question 2.

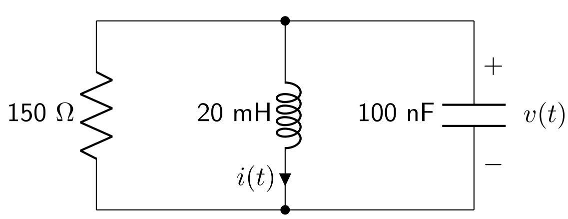
Draw the circuit in the initial steady-state condition. The capacitor can be replaced by an open and the inductor can be replaced by a short.



Because the initial voltage drop is measured over the inductor (short), it will be zero. Use Ohm's law to calculate the initial current flow.

$$\begin{aligned} i(0) &= \frac{20\text{ V}}{0.2\text{ k}\Omega} \\ &= 100\text{ mA} \end{aligned}$$

Draw the circuit at $t = 0^+$.



This is a parallel RLC circuit. Calculate the damping parameter.

$$\begin{aligned}\alpha &= \frac{1}{2RC} \\ &= \frac{1}{2(150\ \Omega)(100 \times 10^{-9}\ \text{F})} \\ &= \frac{1}{3 \times 10^{-5}} \\ &= 33333.33\ \text{Np/s}\end{aligned}$$

Calculate the resonant frequency.

$$\begin{aligned}\omega_0 &= \frac{1}{\sqrt{LC}} \\ &= \frac{1}{\sqrt{(0.02\ \text{H})(100 \times 10^{-9}\ \text{F})}} \\ &= \frac{1}{4.47 \times 10^{-5}} \\ &= 22360.68\ \text{rad/s}\end{aligned}$$

This circuit is overdamped. Calculate the roots.

$$\begin{aligned}s_1 &= -\alpha + \sqrt{\alpha^2 - \omega_0^2} \\ &= -33333.33 + \sqrt{33333.33^2 - 22360.68^2} \\ &= -33333.33 + 24720.66 \\ &= -8612.67 \\ s_2 &= -\alpha - \sqrt{\alpha^2 - \omega_0^2} \\ &= -33333.33 - \sqrt{33333.33^2 - 22360.68^2} \\ &= -33333.33 - 24720.66 \\ &= -58056.99\end{aligned}$$

The first derivative of the initial current flow through the inductor will be zero due to the initial voltage drop of zero. Calculate the coefficients of the equation.

$$\begin{aligned} A_1 &= \frac{i(0)s_2 - i'(0)}{s_2 - s_1} \\ &= \frac{(100)(-58056.99)}{-58056.99 - (-8612.67)} \\ &= 117.42 \text{ mA} \\ A_2 &= \frac{i'(0) - i(0)s_1}{s_2 - s_1} \\ &= \frac{-(100)(-8612.67)}{-58056.99 - (-8612.67)} \\ &= -17.42 \text{ mA} \end{aligned}$$

Plug into the equation for an overdamped homogeneous circuit.

$$i(t) = \left[117.42\mathrm{e}^{-8612.67t} - 17.4\mathrm{e}^{-58056.99t}\right]\ u(t) \text{ mA}$$

3. Calculate an expression for $v(t)$ given the circuit shown in figure 7.3. The switch opens at a time of zero seconds.

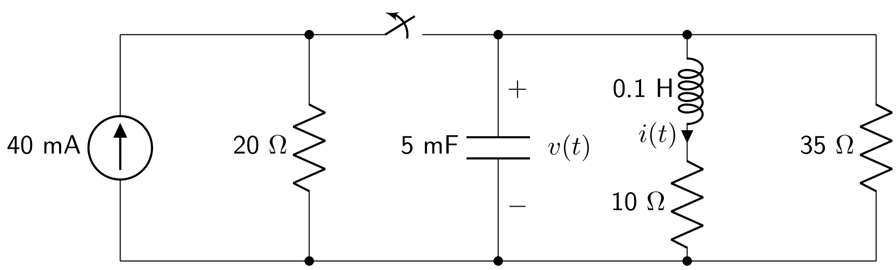
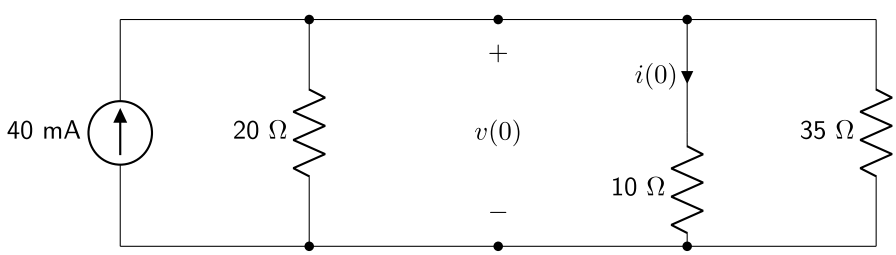


Figure 7.3: Circuit diagram for homogeneous second order circuits question 3.

Draw the circuit in the initial steady-state condition. The capacitor can be replaced by an open and the inductor can be replaced by a short.



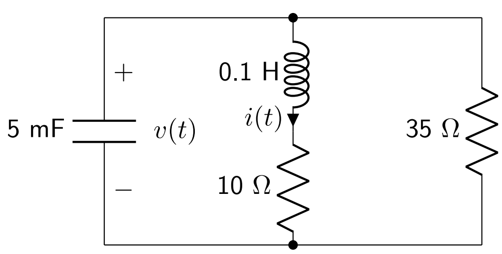
Use Ohm's law to calculate $v(0)$.

$$\begin{aligned} v(0) &= (0.04\text{ A})(20\ \Omega//10\ \Omega//35\ \Omega) \\ &= (0.04\text{ A})(5.6\ \Omega) \\ &= 0.224\text{ V} \end{aligned}$$

Use the current divider rule to calculate $i(0)$.

$$\begin{aligned} i(0) &= 40\text{ mA}\left(\frac{20\ \Omega//10\ \Omega//35\ \Omega}{10\ \Omega}\right) \\ &= 40\text{ mA}\left(\frac{5.6\ \Omega}{10\ \Omega}\right) \\ &= 22.4\text{ mA} \end{aligned}$$

Draw the circuit at $t = 0^+$.



This is a parallel RLC circuit with parasitic resistance. Calculate the damping parameter. (All units are in Ω, H, and F.)

$$\begin{aligned} \alpha &= \frac{1}{2RC} + \frac{R_P}{2L} \\ &= \frac{1}{2(35)(0.005)} + \frac{10}{2(0.1)} \\ &= 2.86 + 50 \\ &= 52.86\text{ Np/s} \end{aligned}$$

Calculate the resonant frequency.

$$\begin{aligned} \omega_0 &= \sqrt{\frac{1}{LC} + \frac{R_P}{RLC}} \\ &= \sqrt{\frac{1}{(0.1)(0.005)} + \frac{10}{(35)(0.1)(0.005)}} \\ &= \sqrt{2000 + 571.43} \\ &= 50.71\text{ rad/s} \end{aligned}$$

This circuit is overdamped. Calculate the roots.

$$\begin{aligned}
s_1 &= -\alpha + \sqrt{\alpha^2 - \omega_0^2} \\
&= -52.86 + \sqrt{52.86^2 - 50.71^2} \\
&= -52.86 + 14.91 \\
&= -37.94 \\
s_2 &= -\alpha - \sqrt{\alpha^2 - \omega_0^2} \\
&= -52.86 - \sqrt{52.86^2 - 50.71^2} \\
&= -52.86 - 14.91 \\
&= -67.77
\end{aligned}$$

Calculate $v'(0)$.

$$\begin{aligned}
v'(0) &= \frac{-v(0) - Ri(0)}{RC} \\
&= \frac{-0.224 \text{ V} - (35\ \Omega)(0.0224 \text{ A})}{(35\ \Omega)(0.005 \text{ F})} \\
&= \frac{-0.224 \text{ V} - 0.784 \text{ V}}{0.175 \text{ s}} \\
&= \frac{-1.008 \text{ V}}{0.175 \text{ s}} \\
&= -5.76 \text{ V/s}
\end{aligned}$$

Calculate the coefficients of the equation.

$$\begin{aligned}
A_1 &= \frac{v(0)s_2 - v'(0)}{s_2 - s_1} \\
&= \frac{(0.224)(-67.77) + 5.76}{-67.77 - (-37.94)} \\
&= 0.32 \text{ V} \\
A_2 &= \frac{v'(0) - v(0)s_1}{s_2 - s_1} \\
&= \frac{-5.76 - (0.224)(-37.94)}{-67.77 - (-37.94)} \\
&= -0.09 \text{ V}
\end{aligned}$$

Plug into the equation for an overdamped homogeneous circuit.

$$v(t) = \left[315.83\mathrm{e}^{-37.94} - 91.83\mathrm{e}^{-67.77}\right]\ u(t) \text{ mV}$$

4. Calculate an expression for $v(t)$ given the circuit shown in figure 7.4. The switch moves from position a to position b at a time of zero seconds.

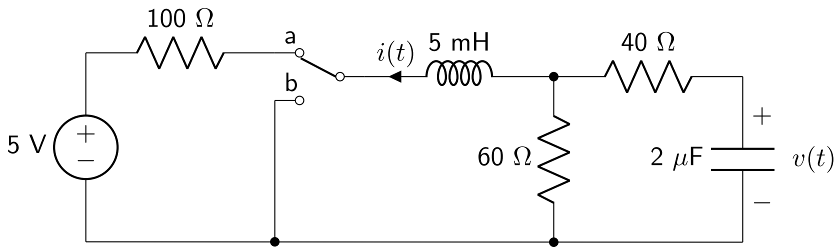
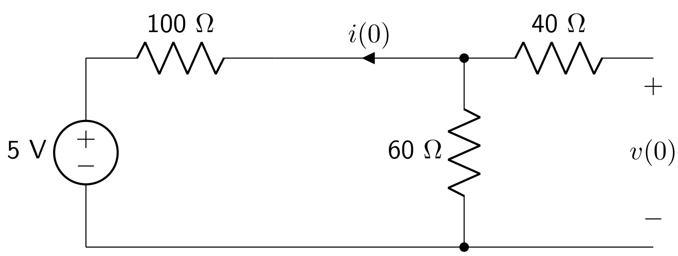


Figure 7.4: Circuit diagram for homogeneous second order circuits question 4.

Draw the circuit in the initial steady-state condition. The capacitor can be replaced by an open and the inductor can be replaced by a short.



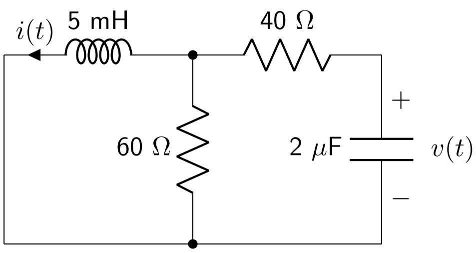
Use Ohm's law to calculate $i(0)$.

$$\begin{aligned} i(0) &= \frac{-5 \text{ V}}{0.16 \text{ k}\Omega} \\ &= 31.25 \text{ mA} \end{aligned}$$

Use the voltage divider rule to calculate $v(0)$.

$$\begin{aligned} v(0) &= 5 \text{ V} \left(\frac{60 \ \Omega}{100 \ \Omega + 60 \ \Omega} \right) \\ &= 1.875 \text{ V} \end{aligned}$$

Draw the circuit at $t = 0^+$.



Solve for the second order differential equation using symbolic form. $L = 5$ mH, $R_1 = 60\ \Omega$, $R_2 = 40\ \Omega$,

$C = 2\ \mu$F.

Perform KCL.

$$0 = i(t) + \frac{L}{R_1}\frac{d}{dt}i(t) + C\frac{d}{dt}v(t)$$

Perform KVL around the right loop.

$$L\frac{d}{dt}i(t) = R_2C\frac{d}{dt}v(t) + v(t)$$

Solve the KVL equation for i(t).

$$\begin{aligned}
L\frac{d}{dt}i(t) &= R_2C\frac{d}{dt}v(t) + v(t) \\
\frac{d}{dt}i(t) &= \frac{R_2C}{L}\frac{d}{dt}v(t) + \frac{1}{L}v(t) \\
i(t) &= \frac{R_2C}{L}v(t) + \frac{1}{L}\int v(t)
\end{aligned}$$

Plug the equation for $i(t)$ into the KCL equation, and put into standard form.

$$\begin{aligned}
0 &= \frac{R_2C}{L}v(t) + \frac{1}{L}\int v(t) + \frac{L}{R_1}\frac{d}{dt}\left[\frac{R_2C}{L}v(t) + \frac{1}{L}\int v(t)\right] + C\frac{d}{dt}v(t) \\
&= \frac{R_2C}{L}v(t) + \frac{1}{L}\int v(t) + \frac{R_2C}{R_1}\frac{d}{dt}v(t) + \frac{1}{R_1}v(t) + C\frac{d}{dt}v(t) \\
&= \left[\frac{R_2C}{R_1} + C\right]\frac{d}{dt}v(t) + \left[\frac{R_2C}{L} + \frac{1}{R_1}\right]v(t) + \frac{1}{L}\int v(t) \\
&= \left[\frac{R_2C}{R_1} + C\right]\frac{d^2}{dt^2}v(t) + \left[\frac{R_2C}{L} + \frac{1}{R_1}\right]\frac{d}{dt}v(t) + \frac{1}{L}v(t) \\
&= \frac{d^2}{dt^2}v(t) + \left[\frac{R_1R_2C + L}{LC(R_1 + R_2)}\right]\frac{d}{dt}v(t) + \left[\frac{R_1}{LC(R_1 + R_2)}\right]v(t)
\end{aligned}$$

Plug in component values.

$$0 = \frac{d^2}{dt^2}v(t) + 9800\frac{d}{dt}v(t) + 60000000v(t)$$

Calculate the damping parameter and resonant frequency.

$$\alpha = 4900 \text{ Np/s}$$

$$\omega_0 = 7745.97 \text{ rad/s}$$

The circuit is underdamped. Calculate the oscillation frequency.

$$\begin{aligned}\beta &= \sqrt{\omega_0^2 - \alpha^2} \\ &= \sqrt{7745.97^2 - 4900^2} \\ &= 5999.17 \text{ rad/s}\end{aligned}$$

The KCL equation can be restated to calculate the initial first derivative of the voltage, where $i_{R1}(0)$ is the initial current flow through the 60 Ω resistor.

$$\begin{aligned}0 &= i(0) + i_{R1}(0) + Cv'(0) \\ &= -31.25 \text{ mA} + 31.25 \text{ mA} + (2 \times 10^{-6} \text{ F})v'(0) \\ v'(0) &= 0 \text{ V/s}\end{aligned}$$

Calculate the coefficients of the underdamped equation.

$$\begin{aligned}B_1 &= v(0) \\ &= 1.875 \text{ V} \\ B_2 &= \frac{v'(0) + \alpha v(0)}{\beta} \\ &= \frac{4900(1.875)}{5999.17} \\ &= 1.531 \text{ V}\end{aligned}$$

Plug into the equation for an underdamped homogeneous circuit.

$$v(t) = \mathrm{e}^{-4900t} \left[1.875 \cos(5999.17t) + 1.531 \sin(5999.17t)\right] \ u(t) \text{ V}$$

5. Derive a second order differential equation in terms of $i(t)$ given the circuit shown in figure 7.5. The switch opens at a time of zero seconds.

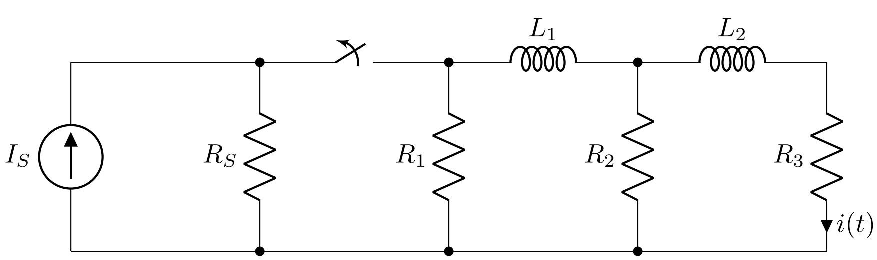
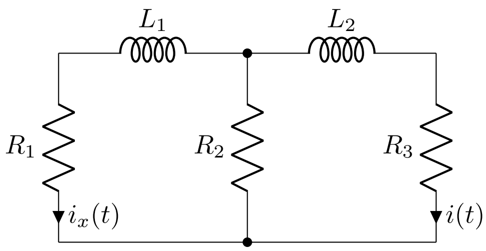


Figure 7.5: Circuit diagram for homogeneous second order circuits question 5.

Draw the circuit at $t = 0^+$. It is useful at this time to define $i_x(t)$ and $v_x(t)$.



Perform KVL around the outer loop.

$$\begin{aligned} 0 &= -R_1 i_x(t) - L_1 \frac{d}{dt} i_x(t) + L_2 \frac{d}{dt} i(t) + R_3 i(t) \\ &= \left[-R_1 - L_1 \frac{d}{dt}\right] i_x(t) + L_2 \frac{d}{dt} i(t) + R_3 i(t) \end{aligned}$$

Perform KCL and solve for $i_x(t)$.

$$\begin{aligned} 0 &= i_x(t) + i(t) + \frac{L_2 \frac{d}{dt} i(t) + R_3 i(t)}{R_2} \\ i_x(t) &= -i(t) - \frac{L_2}{R_2}\frac{d}{dt} i(t) - \frac{R_3}{R_2} i(t) \\ &= -\left[1 + \frac{R_3}{R_2}\right] i(t) - \frac{L_2}{R_2}\frac{d}{dt} i(t) \end{aligned}$$

Plug the expression for $i_x(t)$ into the KVL equation. Then normalize the second order equation.

$$\begin{aligned} 0 &= \left[-R_1 - L_1 \frac{d}{dt}\right] \left(-\left[1 + \frac{R_3}{R_2}\right] i(t) - \frac{L_2}{R_2}\frac{d}{dt} i(t)\right) + L_2 \frac{d}{dt} i(t) + R_3 i(t) \\ &= \left[R_1 + L_1 \frac{d}{dt}\right] \left(\left[1 + \frac{R_3}{R_2}\right] i(t) + \frac{L_2}{R_2}\frac{d}{dt} i(t)\right) + L_2 \frac{d}{dt} i(t) + R_3 i(t) \\ &= \left[R_1 + \frac{R_1 R_3}{R_2}\right] i(t) + \left[L_1 + \frac{L_1 R_3}{R_2}\right] \frac{d}{dt} i(t) + \frac{R_1 L_2}{R_2}\frac{d}{dt} i(t) + \frac{L_1 L_2}{R_2}\frac{d^2}{dt^2} i(t) + L_2 \frac{d}{dt} i(t) + R_3 i(t) \\ &= \frac{L_1 L_2}{R_2}\frac{d^2}{dt^2} i(t) + \left[L_1 + L_2 + \frac{L_1 R_3}{R_2} + \frac{R_1 L_2}{R_2}\right] \frac{d}{dt} i(t) + \left[R_1 + \frac{R_1 R_3}{R_2} + R_3\right] i(t) \\ &= \frac{d^2}{dt^2} i(t) + \left[\frac{R_2}{L_1} + \frac{R_2}{L_2} + \frac{R_3}{L_2} + \frac{R_1}{L_1}\right] \frac{d}{dt} i(t) + \left[\frac{R_1 R_2}{L_1 L_2} + \frac{R_1 R_3}{L_1 L_2} + \frac{R_2 R_3}{L_1 L_2}\right] i(t) \end{aligned}$$

7.2 Non-Homogeneous Second Order Circuits

6. Calculate an expression for $v(t)$ given the circuit shown in figure 7.6. The switch moves from position a to b at a time of zero seconds.

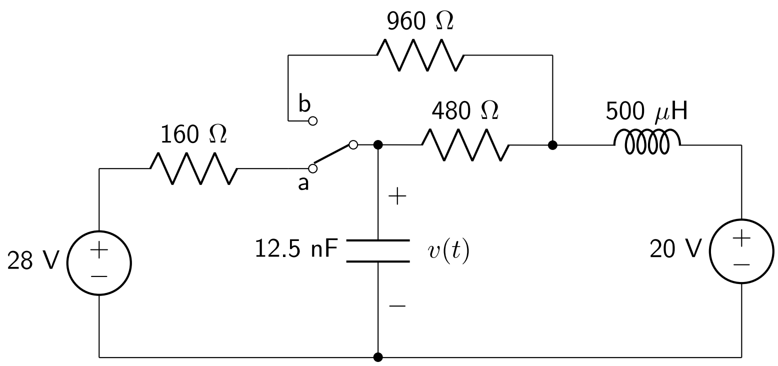
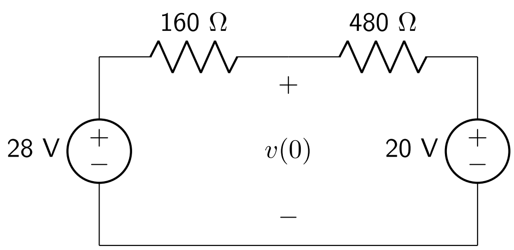


Figure 7.6: Circuit diagram for non-homogeneous second order circuits question 6.

Draw the circuit in the initial steady-state condition. The capacitor can be replaced by an open and the inductor can be replaced by a short.



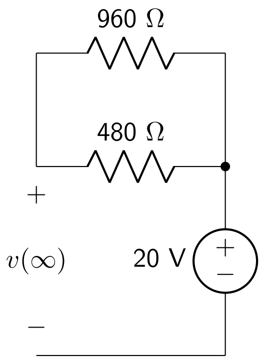
Use superposition and the voltage divider rule to calculate $v(0)$.

$$\begin{aligned} v(0) &= 28 \text{ V} \left(\frac{480 \ \Omega}{480 \ \Omega + 160 \ \Omega} \right) + 20 \text{ V} \left(\frac{160 \ \Omega}{480 \ \Omega + 160 \ \Omega} \right) \\ &= 21 \text{ V} + 5 \text{ V} \\ &= 26 \text{ V} \end{aligned}$$

Use Ohm's law to calculate the initial current flow through the inductor (from right to left to be consistent with the direction of the defined capacitor voltage).

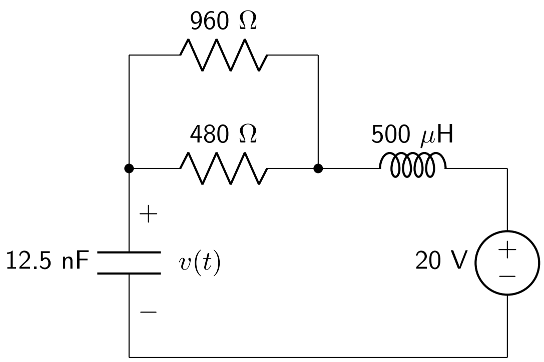
$$\begin{aligned} i(0) &= \frac{-8 \text{ V}}{480 \ \Omega + 160 \ \Omega} \\ &= -12.5 \text{ mA} \end{aligned}$$

Draw the circuit in the final steady-state condition. The capacitor can be replaced by an open and the inductor can be replaced by a short.

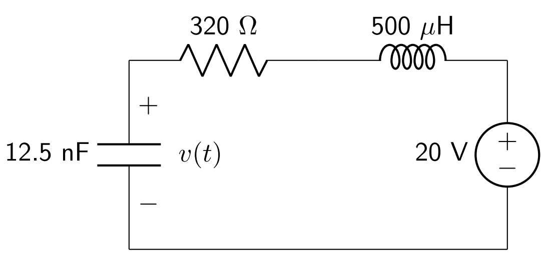


The value of $v(\infty)$ is 20 V.

Draw the circuit at $t = 0^+$.



Combine resistors in parallel.



This is a series RLC circuit. Calculate the damping parameter.

$$\begin{aligned}\alpha &= \frac{R}{2L} \\ &= \frac{320\ \Omega}{2(500 \times 10^{-6}\ \text{H})} \\ &= 320000\ \text{Np/s}\end{aligned}$$

Calculate the resonant frequency.

$$\begin{aligned}\omega_0 &= \frac{1}{\sqrt{LC}} \\ &= \frac{1}{\sqrt{(500 \times 10^{-6}\text{ H})(12.5 \times 10^{-9}\text{ F})}} \\ &= 400000\text{ rad/s}\end{aligned}$$

The circuit is underdamped. Calculate the oscillation frequency.

$$\begin{aligned}\beta &= \sqrt{\omega_0^2 - \alpha^2} \\ &= \sqrt{400000^2 - 320000^2} \\ &= 240000\text{ rad/s}\end{aligned}$$

Calculate the initial first derivative of the voltage drop.

$$\begin{aligned}v'(0) &= \frac{i(0)}{C} \\ &= \frac{-0.0125\text{ A}}{12.5 \times 10^{-9}\text{ F}} \\ &= -1000000\text{ V/s}\end{aligned}$$

Calculate the coefficients of the underdamped equation.

$$\begin{aligned}B_1 &= v(0) - v(\infty) \\ &= 26\text{ V} - 20\text{ V} \\ &= 6\text{ V} \\ B_2 &= \frac{v'(0) + \alpha[v(0) - v(\infty)]}{\beta} \\ &= \frac{-1000000 + 320000[26 - 20]}{240000} \\ &= 3.83\text{ V}\end{aligned}$$

Plug these values into the equation for an underdamped non-homogeneous circuit.

$$v(t) = \left[20 + \mathrm{e}^{-320000t}\left(6\cos(240000) + 3.83\sin(240000t)\right)\right]\ u(t)\text{ V}$$

7. Calculate an expression for $i(t)$ given the circuit shown in figure 7.7. The switch closes at a time of zero seconds.

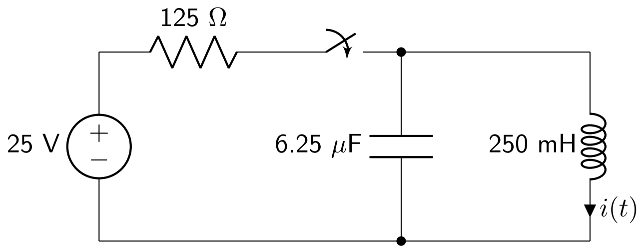
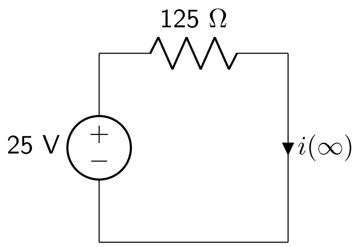


Figure 7.7: Circuit diagram for non-homogeneous second order circuits question 7.

The initial voltage drops and current flows are zero.

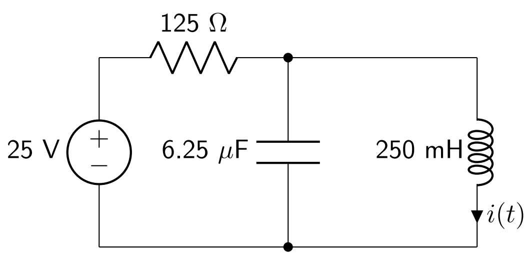
Draw the circuit in the final steady-state condition. The capacitor can be replaced by an open and the inductor can be replaced by a short.



Use Ohm's law to calculate $i(\infty)$.

$$\begin{aligned} i(\infty) &= \frac{25 \text{ V}}{125 \text{ } \Omega} \\ &= 0.2 \text{ A} \end{aligned}$$

Draw the circuit at $t = 0^+$.



This is a non-homogeneous parallel RLC circuit. Calculate the damping parameter.

$$\begin{aligned}\alpha &= \frac{1}{2RC} \\ &= \frac{1}{2(125\ \Omega)(6.25 \times 10^{-6}\ \text{F})} \\ &= 640\ \text{Np/s}\end{aligned}$$

Calculate the resonant frequency.

$$\begin{aligned}\omega_0 &= \frac{1}{\sqrt{LC}} \\ &= \frac{1}{\sqrt{(0.25\ \text{H})(6.25 \times 10^{-6}\ \text{F})}} \\ &= 800\ \text{rad/s}\end{aligned}$$

The circuit is underdamped. Calculate the oscillation frequency.

$$\begin{aligned}\beta &= \sqrt{\omega_0^2 - \alpha^2} \\ &= \sqrt{800^2 - 640^2} \\ &= 480\ \text{rad/s}\end{aligned}$$

The initial first derivative of the current will be zero. Calculate the coefficients of the underdamped equation.

$$\begin{aligned}B_1 &= i(0) - i(\infty) \\ &= 0\ \text{A} - 0.2\ \text{A} \\ &= -0.2\ \text{A} \\ &= -200\ \text{mA} \\ B_2 &= \frac{i'(0) + \alpha[i(0) - i(\infty)]}{\beta} \\ &= \frac{640[0 - 0.2]}{480} \\ &= -0.267\ \text{A} \\ &= -266.67\ \text{mA}\end{aligned}$$

Plug these values into the equation for an underdamped non-homogeneous circuit.

$$i(t) = \left[200 - \text{e}^{-640t}\left(200\cos(480t) + 266.67\sin(480t)\right)\right]\ u(t)\ \text{mA}$$

8. Calculate an expression for $v(t)$ given the circuit shown in figure 7.8. The switch moves from position a to b at a time of zero seconds.

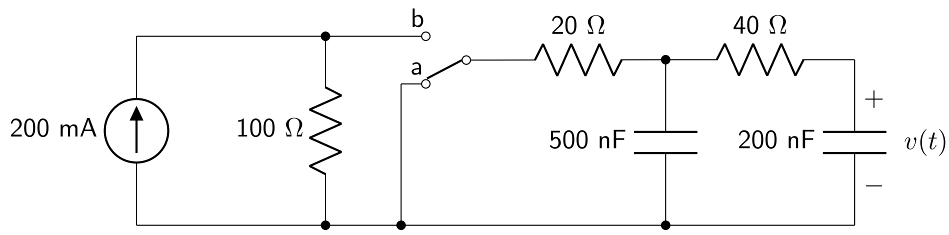
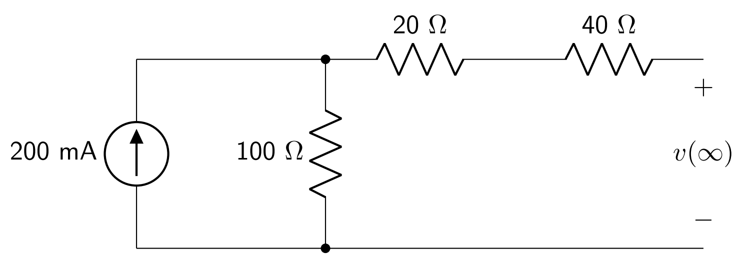


Figure 7.8: Circuit diagram for non-homogeneous second order circuits question 8.

The initial voltage drops and current flows are zero.

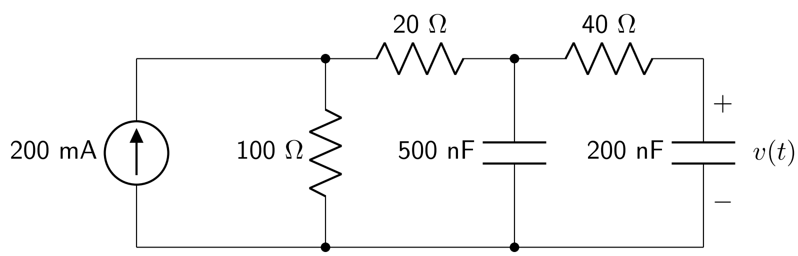
Draw the circuit in the final steady-state condition. The capacitor can be replaced by an open and the inductor can be replaced by a short.



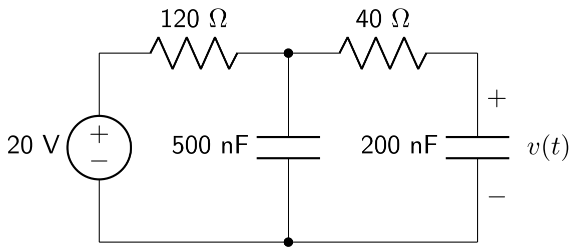
Use Ohm's law to calculate $v(\infty)$.

$$\begin{aligned} v(\infty) &= (0.2\text{ A})(100\ \Omega) \\ &= 20\text{ V} \end{aligned}$$

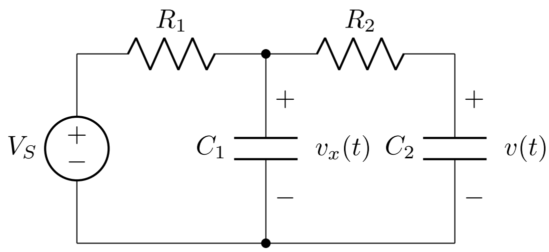
Draw the circuit at $t = 0^+$.



Convert the current source to a voltage source. Then combine the series resistors.



Define symbolic values for each component, and define the voltage drop over C_1.



Perform KCL and collect like terms.

$$\begin{aligned}\frac{V_S - v_x(t)}{R_1} &= C_1\frac{d}{dt}v_x(t) + C_2\frac{d}{dt}v(t)\\ \frac{V_S}{R_1} &= \left[\frac{1}{R_1} + C_1\frac{d}{dt}\right]v_x(t) + C_2\frac{d}{dt}v(t)\end{aligned}$$

Perform KVL around the right loop.

$$v_x(t) = R_2C_2\frac{d}{dt}v(t) + v(t)$$

Plug the KVL equation into the KCL equation and find the normalized second order differential equation.

$$\begin{aligned}\frac{V_S}{R_1} &= \left[\frac{1}{R_1} + C_1\frac{d}{dt}\right]\left[R_2C_2\frac{d}{dt}v(t) + v(t)\right] + C_2\frac{d}{dt}v(t)\\ &= \frac{R_2C_2}{R_1}\frac{d}{dt}v(t) + \frac{1}{R_1}v(t) + C_1C_2R_2\frac{d^2}{dt^2}v(t) + C_1\frac{d}{dt}v(t) + C_2\frac{d}{dt}v(t)\\ &= C_1C_2R_2\frac{d^2}{dt^2}v(t) + \left[\frac{R_2C_2}{R_1} + C_1 + C_2\right]\frac{d}{dt}v(t) + \frac{1}{R_1}v(t)\\ \frac{V_S}{R_1R_2C_1C_2} &= \frac{d^2}{dt^2}v(t) + \left[\frac{1}{R_1C_1} + \frac{1}{R_2C_2} + \frac{1}{R_2C_1}\right]\frac{d}{dt}v(t) + \frac{1}{R_1R_2C_1C_2}v(t)\end{aligned}$$

Plug in component values.

$$4.167\times 10^{10} = \frac{d^2}{dt^2}v(t) + 191666.667\frac{d}{dt}v(t) + 2.083\times 10^{9}v(t)$$

Calculate the damping parameter and resonant frequency.

$$\alpha = 95833.33 \text{ Np/s}$$
$$\omega_0 = 45643.55 \text{ rad/s}$$

The circuit is overdamped. Calculate the roots

$$\begin{aligned} s_1 &= -\alpha + \sqrt{\alpha^2 - \omega_0^2} \\ &= -95833.33 + \sqrt{95833.33^2 - 45643.55^2} \\ &= -95833.33 + 84265.61 \\ &= -11567.72 \\ s_2 &= -\alpha - \sqrt{\alpha^2 - \omega_0^2} \\ &= -95833.33 - \sqrt{95833.33^2 - 45643.55^2} \\ &= -95833.33 - 84265.61 \\ &= -180098.94 \end{aligned}$$

The initial first derivative of the voltage will be zero. Calculate the coefficients of the equation.

$$\begin{aligned} A_1 &= \frac{[v(0) - v(\infty)]s_2 - v'(0)}{s_2 - s_1} \\ &= \frac{(0 - 20)(-180098.94)}{-180098.94 - (-11567.72)} \\ &= -21.373 \text{ V} \\ A_2 &= \frac{v'(0) - [v(0) - v(\infty)]s_1}{s_2 - s_1} \\ &= \frac{-(0 - 20)(-11567.72)}{-180098.94 - (-11567.72)} \\ &= 1.373 \text{ V} \end{aligned}$$

Plug these values into the equation for an overdamped non-homogeneous circuit.

$$v(t) = \left[20 - 21.373\mathrm{e}^{-11567.72t} + 1.373\mathrm{e}^{-180098.94t}\right]\ u(t) \text{ V}$$

9. Calculate an expression for $i(t)$ given the circuit shown in figure 7.9. Switch S1 opens at a time of zero seconds and switch S2 closes at a time of zero seconds.

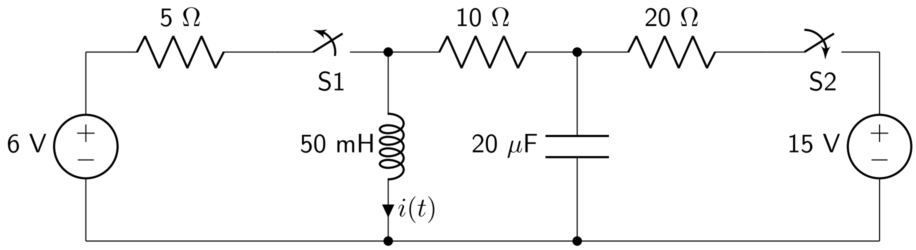
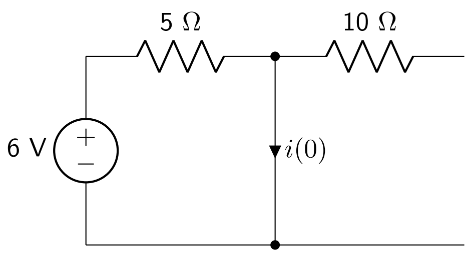


Figure 7.9: Circuit diagram for non-homogeneous second order circuits question 9.

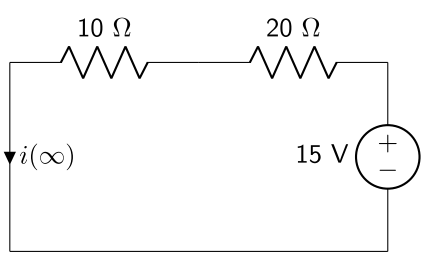
Draw the circuit in the initial steady-state condition. The capacitor can be replaced by an open and the inductor can be replaced by a short.



Use Ohm's law to calculate $i(0)$.

$$\begin{aligned} i(0) &= \frac{6 \text{ V}}{5 \ \Omega} \\ &= 1.2 \text{ A} \end{aligned}$$

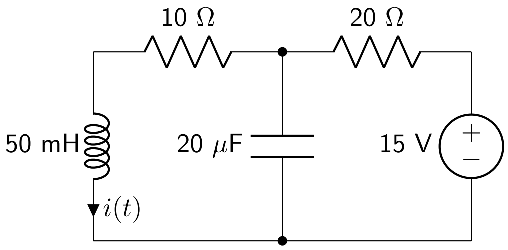
Draw the circuit in the final steady-state condition. The capacitor can be replaced by an open and the inductor can be replaced by a short.



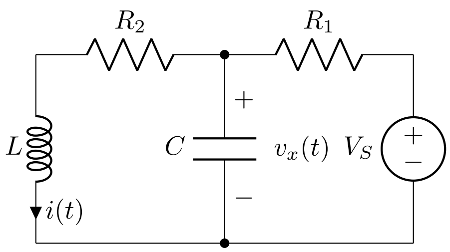
Use Ohm's law to calculate $i(\infty)$.

$$\begin{aligned} i(\infty) &= \frac{15 \text{ V}}{30 \ \Omega} \\ &= 0.5 \text{ A} \end{aligned}$$

Draw the circuit at $t = 0^+$.



Define symbolic values for each component, and define the voltage drop over the capacitor.



Perform KCL and collect like terms.

$$\frac{V_S - v_x(t)}{R_1} = C\frac{d}{dt}v_x(t) + i(t)$$
$$\frac{V_S}{R_1} = \left[\frac{1}{R_1} + C\frac{d}{dt}\right] v_x(t) + i(t)$$

Perform KVL around the left loop.

$$v_x(t) = R_2 i(t) + L\frac{d}{dt}i(t)$$

Plug the KVL equation into the KCL equation, and normalize the second order differential equation.

$$\begin{aligned}
\frac{V_S}{R_1} &= \left[\frac{1}{R_1} + C\frac{d}{dt}\right]\left[R_2 i(t) + L\frac{d}{dt}i(t)\right] v_x(t) + i(t) \\
&= \frac{R_2}{R_1}i(t) + \frac{L}{R_1}\frac{d}{dt}i(t) + R_2 C\frac{d}{dt}i(t) + LC\frac{d^2}{dt^2}i(t) + i(t) \\
&= LC\frac{d^2}{dt^2}i(t) + \left[R_2 C + \frac{L}{R_1}\right]\frac{d}{dt}i(t) + \left[\frac{R_2}{R_1} + 1\right] i(t) \\
\frac{V_S}{R_1 LC} &= \frac{d^2}{dt^2}i(t) + \left[\frac{R_2}{L} + \frac{1}{R_1 C}\right]\frac{d}{dt}i(t) + \left[\frac{R_2}{R_1 LC} + \frac{1}{LC}\right] i(t)
\end{aligned}$$

Plug in component values.

$$750000 = \frac{d^2}{dt^2}i(t) + 2700\frac{d}{dt}i(t) + 1500000 i(t)$$

Calculate the damping parameter and resonant frequency.

$$\alpha = 1350 \text{ Np/s}$$
$$\omega_0 = 1224.74 \text{ rad/s}$$

The circuit is overdamped. Calculate the roots.

$$\begin{aligned}
s_1 &= -\alpha + \sqrt{\alpha^2 - \omega_0^2} \\
&= -1350 + \sqrt{1350^2 - 1224.74^2} \\
&= -1350 + 567.89 \\
&= -782.10 \\
s_2 &= -\alpha - \sqrt{\alpha^2 - \omega_0^2} \\
&= -1350 - \sqrt{1350^2 - 1224.74^2} \\
&= -1350 - 567.89 \\
&= -1917.90
\end{aligned}$$

Use the KVL equation to calculate $i'(0)$. The initial voltage drop over the capacitor ($v_x(0)$) is zero.

$$\begin{aligned}
v_x(0) &= R_2 i(0) + L i'(0) \\
0 &= (10\ \Omega)(1.2\ \text{A}) + (0.05\ \text{H}) i'(0) \\
i'(0) &= \frac{-12}{0.05}\ \text{A/s} \\
&= -240\ \text{A/s}
\end{aligned}$$

Calculate the coefficients of the equation.

$$\begin{aligned}
A_1 &= \frac{[i(0) - i(\infty)]s_2 - i'(0)}{s_2 - s_1} \\
&= \frac{(1.2 - 0.5)(-1917.90) - (-240)}{-1917.90 - (-782.10)} \\
&= 0.97\ \text{A} \\
A_2 &= \frac{i'(0) - [i(0) - i(\infty)]s_1}{s_2 - s_1} \\
&= \frac{-240 - (1.5 - 0.5)(-782.10)}{-1917.90 - (-782.10)} \\
&= -0.27\ \text{A}
\end{aligned}$$

Plug these values into the equation for an overdamped non-homogeneous circuit.

$$i(t) = \left[0.5 + 0.97\mathrm{e}^{-782.10} - 0.27\mathrm{e}^{-1917.90}\right]\ u(t)\ \mathrm{A}$$

i(t) = [0.5 + 0.971 e$^{-782.099t}$ – 0.271 e$^{-1917.901t}$] u(t) A – This is an overdamped general second order circuit.

10. Calculate an expression for $v(t)$ given the circuit shown in figure 7.10. The switch closes at a time of zero seconds.

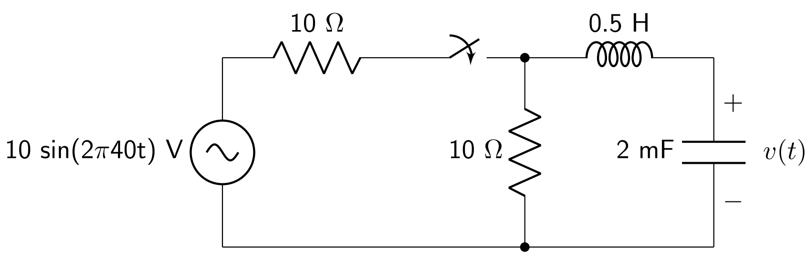
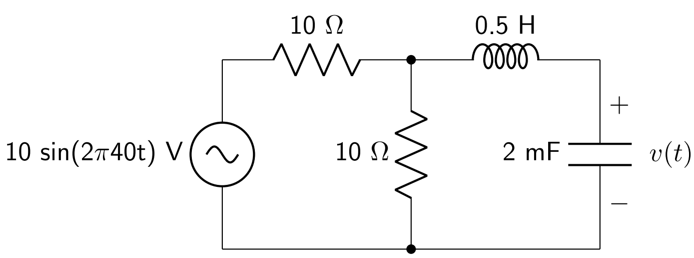


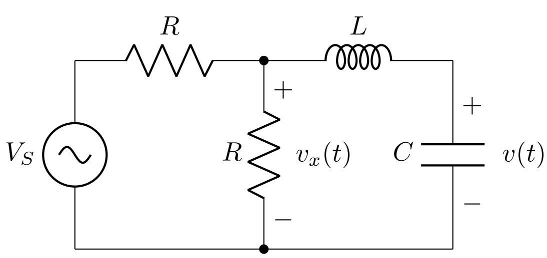
Figure 7.10: Circuit diagram for non-homogeneous second order circuits question 10.

The initial values of the RLC circuit will be zero.

Draw the circuit at $t = 0^+$.



Define symbolic values for each component, and define the voltage drop over the resistor.



Perform KCL and combine like terms.

$$\frac{V_S - v_x(t)}{R} = \frac{1}{R}v_x(t) + C\frac{d}{dt}v(t)$$
$$\frac{V_S}{R} = \frac{2}{R}v_x(t) + C\frac{d}{dt}v(t)$$

Perform KVL around the right loop.

$$v_x(t) = LC\frac{d^2}{dt^2}v(t) + v(t)$$

Plug the KVL equation into the KVL equation, and normalize the second order differential equation.

$$\begin{aligned}\frac{V_S}{R} &= \frac{2}{R}\left[LC\frac{d^2}{dt^2}v(t) + v(t)\right] + C\frac{d}{dt}v(t)\\ &= \frac{2LC}{R}\frac{d^2}{dt^2}v(t) + \frac{2}{R}v(t) + C\frac{d}{dt}v(t)\\ \frac{V_S}{2LC} &= \frac{d^2}{dt^2}v(t) + \frac{R}{2L}\frac{d}{dt}v(t) + \frac{1}{LC}v(t)\end{aligned}$$

Plug in component values.

$$5000\sin(2\pi 40t) = \frac{d^2}{dt^2}v(t) + 10\frac{d}{dt}v(t) + 1000v(t)$$

Find the form of the particular solution.

$$v_p(t) = K_1\sin(2\pi 40t) + K_2\cos(2\pi 40t)$$

Plug the particular solution into the differential equation.

$$\begin{aligned}5000\sin(2\pi 40t) &= \frac{d^2}{dt^2}\left[K_1\sin(2\pi 40t) + K_2\cos(2\pi 40t)\right]\\ &+ 10\frac{d}{dt}\left[K_1\sin(2\pi 40t) + K_2\cos(2\pi 40t)\right]\\ &+ 1000\left[K_1\sin(2\pi 40t) + K_2\cos(2\pi 40t)\right]\\ &= -6400\pi^2\left[K_1\sin(2\pi 40t) + K_2\cos(2\pi 40t)\right]\\ &+ 800\pi\left[K_1\cos(2\pi 40t) - K_2\sin(2\pi 40t)\right]\\ &+ 1000\left[K_1\sin(2\pi 40t) + K_2\cos(2\pi 40t)\right]\end{aligned}$$

One equation will relate all of the sine terms. Divide the $\sin(2\pi 40t)$ term out of each term.

$$5000 = -6400\pi^2 K_1 - 800\pi K_2 + 1000K_1$$

The other equation will relate all of the cosine terms. Divide the $\cos(2\pi 40t)$ term out of each term.

$$0 = -6400\pi^2 K_2 + 800\pi K_1 + 1000K_2$$

Place both equations into form $\alpha K_1 + \beta K_2 = c$, and then place each coefficient into a matrix.

$$\begin{bmatrix} -6400\pi^2 + 1000 & -800\pi & 5000 \\ 800\pi & -6400\pi^2 + 1000 & 0 \end{bmatrix}$$

Solve the matrix for K_1 and K_2.

$$K_1 = -0.08 \text{ V}$$
$$K_2 = 0.00 \text{ V}$$

The particular form of the equation is now known.

$$v_p(t) = -0.080 \sin(2\pi 40t) \text{ V}$$

To find the solution to the homogeneous equation, first calculate the damping parameter and resonant frequency.

$$\alpha = 5 \text{ Np/s}$$
$$\omega_0 = 31.62 \text{ rad/s}$$

The circuit is underdamped. Calculate the oscillation frequency.

$$\begin{aligned} \beta &= \sqrt{\omega_0^2 - \alpha^2} \\ &= \sqrt{31.62^2 - 5^2} \\ &= 31.22 \text{ rad/s} \end{aligned}$$

Find the form of the homogeneous equation.

$$v_c(t) = \mathrm{e}^{-5t} \left[B_1 \cos(31.22t) + B_2 \sin(31.22t)\right]$$

Find the form of $v(t)$.

$$\begin{aligned} v(t) &= v_p(t) + v_c(t) \\ &= -0.080 \sin(2\pi 40t) + \mathrm{e}^{-5t} \left[B_1 \cos(31.22t) + B_2 \sin(31.22t)\right] \end{aligned}$$

Use the initial conditions to calculate B_1 and B_2. Start with $v(0)$.

$$v(0) = B_1$$
$$B_1 = 0$$

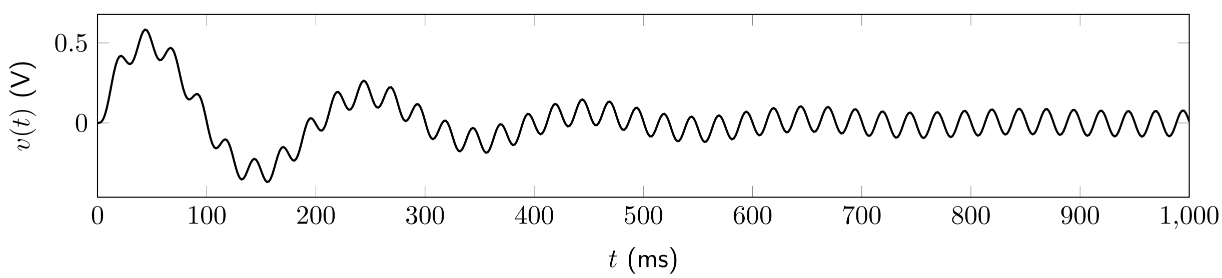
Use the initial first derivative of the voltage to calculate B_2.

$$\begin{aligned} v'(t) &= -20.11\cos(2\pi 40t) + \mathrm{e}^{-5t}\left[31.22B_2\cos(31.22t) - 5B_2\sin(31.22t)\right] \\ v'(0) &= -20.11 + 31.22B_2 \\ 0 &= -20.11 + 31.22B_2 \\ B_2 &= \frac{20.11}{31.22} \\ &= 0.64 \text{ V} \end{aligned}$$

Now $v(t)$ is known.

$$v(t) = \left[-0.080\sin(2\pi 40t) + 0.64\mathrm{e}^{-5t}\sin(31.22t)\right]\ u(t) \text{ V}$$

A graph of $v(t)$ is shown below.



8 Chapter 8 Solutions

8.1 Phasor Arithmetic

1. Convert $-200 - j100$ to polar form.

Use the Pythagorean theorem to calculate the magnitude.

$$\begin{aligned} r &= \sqrt{200^2 + 100^2} \\ &= 223.61 \end{aligned}$$

Use the atan2 function to calculate the angle.

$$\begin{aligned} \theta &= \text{atan2}(-100, -200) \\ &= -153.43^\circ \end{aligned}$$

2. Convert $5\angle -120^\circ$ to Cartesian form.

Use sine and cosine to calculate the real and imaginary parts.

$$\begin{aligned} \mathbf{Z} &= 5\cos(-120^\circ) + j5\sin(-120^\circ) \\ &= -2.5 - j4.33 \end{aligned}$$

3. Calculate the value of $(10 + j20)(-30 + j50) + (-15 - j40)$ and express the answer in both Cartesian and polar forms.

Complete the addition in Cartesian form.

$$\begin{aligned} \mathbf{Z} &= (10 + j20)(-30 + j50) + (-15 - j40) \\ &= -1300 - j100 - 15 - j40 \\ &= -1315 - j140 \end{aligned}$$

Convert to polar form.

$$\begin{aligned} \mathbf{Z} &= \sqrt{1315^2 + 140^2}\angle\, \text{atan2}(-140, -1315) \\ &= 1322.43\angle -173.92^\circ \end{aligned}$$

4. Calculate the value of $(10\angle 60° - 4\angle -140°)(20\angle 20°)$ and express the answer in both Cartesian and polar forms.

Convert the phasors to Cartesian form.

$$10\angle 60° = 5 + j8.66$$
$$4\angle -140° = -3.06 - j2.57$$
$$20\angle 20° = 18.79 + j6.84$$

Perform the arithmetic.

$$\begin{aligned}\mathbf{Z} &= \left[(5 + j8.66) - (-3.06 - j2.57)\right](18.79 + j6.84)\\ &= (8.06 + j11.23)(18.79 + j6.84)\\ &= 74.73 + j266.24\end{aligned}$$

Convert to polar form.

$$\begin{aligned}\mathbf{Z} &= \sqrt{74.73^2 + 266.24^2}\angle \operatorname{atan2}(266.24, 74.73)\\ &= 276.53\angle 74.32°\end{aligned}$$

5. Calculate the value of $(10 + j14)(60\angle -140°) + (38\angle 20°)/(-5 - j18)$ and express the answer in both Cartesian and polar forms.

Convert polar phasors to Cartesian form.

$$60\angle -140° = -45.96 - j38.57$$
$$38\angle 20° = 35.71 + j13.00$$

Perform the arithmetic.

$$\begin{aligned}\mathbf{Z} &= (10 + j14)(-45.96 - j38.57) + \frac{35.71 + j13.00}{-5 - j18}\\ &= (80.31 - j1029.15) + (-1.18 + j1.66)\\ &= 79.13 - j1027.49\end{aligned}$$

Convert to polar form.

$$\begin{aligned}\mathbf{Z} &= \sqrt{79.13^2 + 1027.49^2}\angle\,\text{atan2}(-1027.49, 79.13)\\ &= 1030.54\angle - 85.60^\circ\end{aligned}$$

8.2 Impedance and Equivalent Impedance

6. Calculate the impedance of a 25 mH inductor at a frequency of 60 Hz.

$$\begin{aligned}\mathbf{Z_L} &= j\omega L\\ &= j(2\pi 60)(0.025)\\ &= j9.42\ \Omega\end{aligned}$$

7. Calculate the impedance of a 470 nF capacitor at a frequency of 2 kHz.

$$\begin{aligned}\mathbf{Z_C} &= \frac{-j}{\omega C}\\ &= \frac{-j}{(2\pi 2000)(470 \times 10^{-9}}\\ &= -j169.34\ \Omega\end{aligned}$$

8. Calculate the equivalent impedance of the circuit shown in figure 8.1. The frequency of operation is 200 Hz.

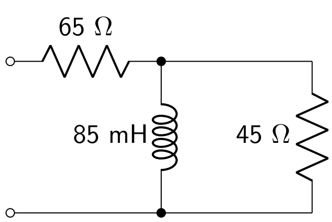


Figure 8.1: Circuit diagram for impedance and equivalent impedance question 8.

Calculate the impedance of the inductor

$$\begin{aligned}\mathbf{Z_L} &= j\omega L\\ &= j(2\pi 200)(0.085)\\ &= j106.81\ \Omega\end{aligned}$$

Calculate the equivalent impedance.

$$\begin{aligned}\mathbf{Z_{EQ}} &= 45\ \Omega // j106.81\ \Omega + 65\ \Omega \\ &= 38.22 + j16.10\ \Omega + 65\ \Omega \\ &= 103.22 + j16.10\ \Omega\end{aligned}$$

9. Calculate the equivalent impedance of the circuit shown in figure 8.2. The frequency of operation is 450 Hz.

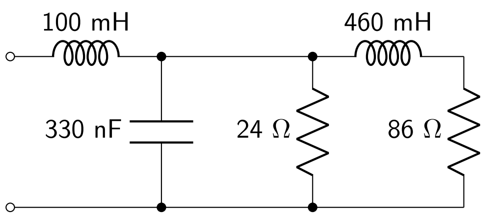


Figure 8.2: Circuit diagram for impedance and equivalent impedance question 9.

Calculate the impedance of the capacitor and inductors.

$$\begin{aligned}\mathbf{Z_{L100}} &= j(2\pi 450)(0.1) \\ &= j282.74\ \Omega \\ \mathbf{Z_{L460}} &= j(2\pi 450)(0.46) \\ &= j1300.62\ \Omega \\ \mathbf{Z_{C}} &= \frac{-j}{(2\pi 450)(330 \times 10^{-9})} \\ &= -j1071.75\ \Omega\end{aligned}$$

Calculate the equivalent impedance.

$$\begin{aligned}\mathbf{Z_{EQ}} &= (86\ \Omega + j1300.62\ \Omega)//24\ \Omega// - j1071.75\ \Omega + j282.74\ \Omega \\ &= 86 + j1300.62\ \Omega//24\ \Omega// - j1071.75\ \Omega + j282.74\ \Omega \\ &= 23.97 - j0.10\ \Omega + j282.74\ \Omega \\ &= 23.97 + j282.65\ \Omega\end{aligned}$$

10. Calculate the equivalent impedance of the circuit shown in figure 8.3. The frequency of operation is 6 kHz.

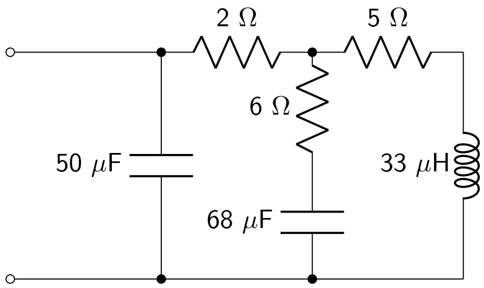


Figure 8.3: Circuit diagram for impedance and equivalent impedance question 10.

Calculate the impedances of the inductor and both capacitors.

$$\begin{aligned}
\mathbf{Z_{C50}} &= \frac{-j}{(2\pi 6000)(50 \times 10^{-6})} \\
&= -j0.53\ \Omega \\
\mathbf{Z_{C68}} &= \frac{-j}{(2\pi 6000)(68 \times 10^{-6})} \\
&= -j0.39\ \Omega \\
\mathbf{Z_L} &= j(2\pi 6000)(33 \times 10^{-6}) \\
&= j1.24\ \Omega
\end{aligned}$$

Calculate the equivalent impedance.

$$\begin{aligned}
Z_{EQ} &= ((5\ \Omega + j1.24\ \Omega)//(6\ \Omega - j0.39\ \Omega) + 2\ \Omega)// - j0.53\ \Omega \\
&= (2.79 + j0.28\ \Omega + 2\ \Omega)// - j0.53\ \Omega \\
&= 58.56 - j527.51\ \text{m}\Omega
\end{aligned}$$

8.3 Delta-Wye and Wye-Delta Transforms

11. Convert the circuit shown in figure 8.4 to a delta circuit.

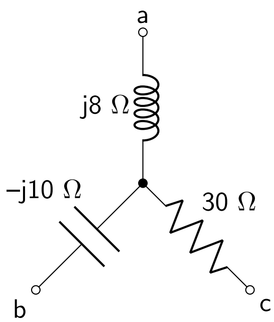
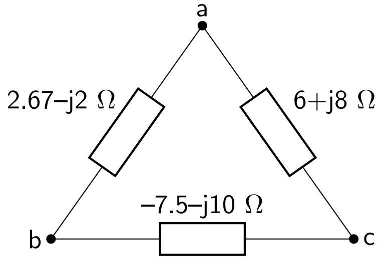


Figure 8.4: Circuit diagram for delta-wye and wye-delta transforms question 11.

Calculate each of the delta impedances.

$$\begin{aligned}
\mathbf{Z_1} &= \frac{\mathbf{Z_a Z_b} + \mathbf{Z_b Z_c} + \mathbf{Z_a Z_c}}{Z_c} \\
&= \frac{(j8\ \Omega)(-j10\ \Omega) + (-j10\ \Omega)(30\ \Omega) + (j8\ \Omega)(30\ \Omega)}{30\ \Omega} \\
&= \frac{80 - j60\ \Omega^2}{30\ \Omega} \\
&= 2.67 - j2\ \Omega \\
\mathbf{Z_2} &= \frac{\mathbf{Z_a Z_b} + \mathbf{Z_b Z_c} + \mathbf{Z_a Z_c}}{Z_a} \\
&= \frac{80 - j60\ \Omega^2}{j8\ \Omega} \\
&= -7.5 - j10\ \Omega \\
\mathbf{Z_3} &= \frac{\mathbf{Z_a Z_b} + \mathbf{Z_b Z_c} + \mathbf{Z_a Z_c}}{Z_b} \\
&= \frac{80 - j60\ \Omega^2}{-j10\ \Omega} \\
&= 6 + j8\ \Omega
\end{aligned}$$

Draw the delta circuit diagram.



12. Convert the circuit shown in figure 8.5 to a wye circuit.

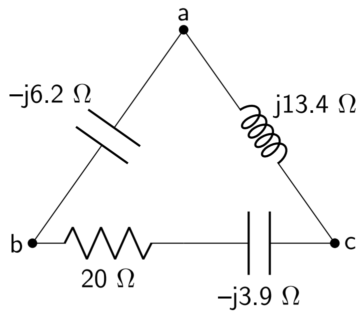
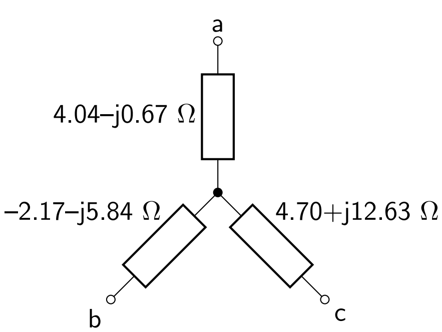


Figure 8.5: Circuit diagram for delta-wye and wye-delta transforms question 12.

Calculate each of the wye impedances.

$$\begin{aligned}
\mathbf{Z_a} &= \frac{\mathbf{Z_1 Z_3}}{\mathbf{Z_1} + \mathbf{Z_2} + \mathbf{Z_3}} \\
&= \frac{(-j6.2\ \Omega)(j13.4\ \Omega)}{(-j6.2\ \Omega) + (20 - j3.9\ \Omega) + (j13.4\ \Omega)} \\
&= \frac{83.08\ \Omega^2}{20 + j3.3\ \Omega} \\
&= 4.04 - j0.67\ \Omega \\
\mathbf{Z_b} &= \frac{\mathbf{Z_1 Z_2}}{\mathbf{Z_1} + \mathbf{Z_2} + \mathbf{Z_3}} \\
&= \frac{(-j6.2\ \Omega)(20 - j3.9\ \Omega)}{20 + j3.3\ \Omega} \\
&= -2.17 - j5.84\ \Omega \\
\mathbf{Z_c} &= \frac{\mathbf{Z_2 Z_3}}{\mathbf{Z_1} + \mathbf{Z_2} + \mathbf{Z_3}} \\
&= \frac{(20 - j3.9\ \Omega)(j13.4\ \Omega)}{20 + j3.3\ \Omega} \\
&= 4.70 + j12.63\ \Omega
\end{aligned}$$

Draw the wye circuit diagram.



13. Calculate the equivalent impedance of the circuit shown in figure 8.6.

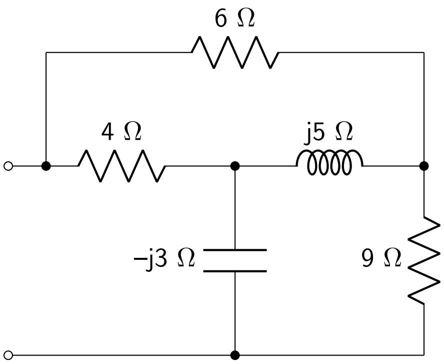
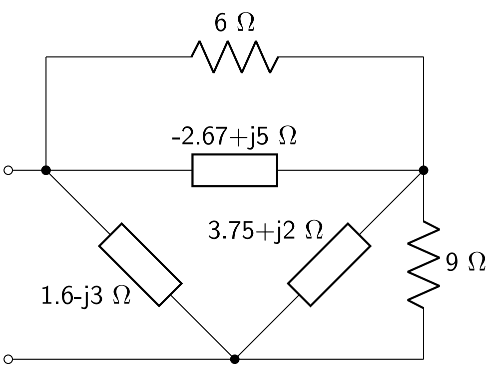


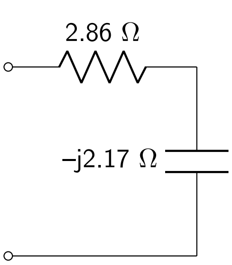
Figure 8.6: Circuit diagram for delta-wye and wye-delta transforms question 13.



Calculate the equivalent impedance.

$$\begin{aligned}
\mathbf{Z_{EQ}} &= (9\ \Omega//(3.75 + j2\ \Omega) + 6\ \Omega//(-2.67 + j5\ \Omega))//(1.6 - j3\ \Omega) \\
&= ((2.80 + 0.97\ \Omega) + (2.68 + j4.98\ \Omega))//(1.6 - j3\ \Omega) \\
&= (5.48 + j5.96\ \Omega)//(1.6 - j3\ \Omega) \\
&= 2.86 - j2.17\ \Omega
\end{aligned}$$

Draw the equivalent circuit.



14. Calculate the equivalent impedance of the circuit shown in figure 8.7. The frequency of operation is 10 kHz.

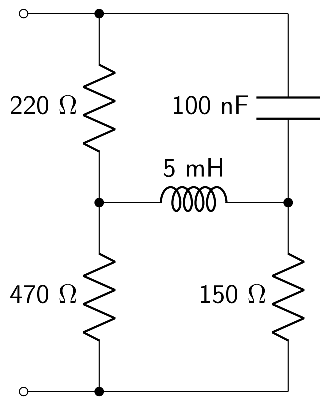
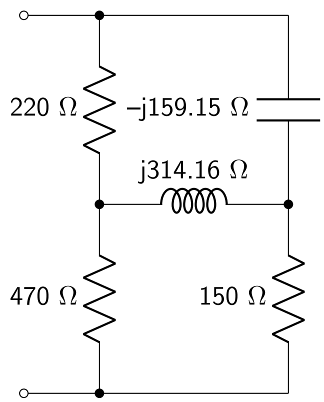
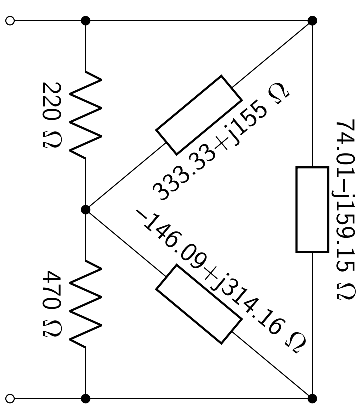


Figure 8.7: Circuit diagram for delta-wye and wye-delta transforms question 14.

Convert to impedances.



Perform a wye-delta transform.



Calculate the equivalent impedance.

$$\begin{aligned}\mathbf{Z_{EQ}} &= [(220\ \Omega)//(333.33 + j155\ \Omega) + (470\ \Omega)//(-j146.09 + j314.16\ \Omega)]\,//(74.01 - j159.15\ \Omega)\\ &= [(138.89 + j22.72\ \Omega) + (118.59 + j340.83\ \Omega)]\,//(74.01 - j159.15\ \Omega)\\ &= (257.49 + j363.55\ \Omega)//(74.01 - j159.15\ \Omega)\\ &= 149.15 - j134.42\ \Omega\end{aligned}$$

15. Calculate the equivalent impedance of the circuit shown in figure 8.8. The frequency of operation is 50 Hz.

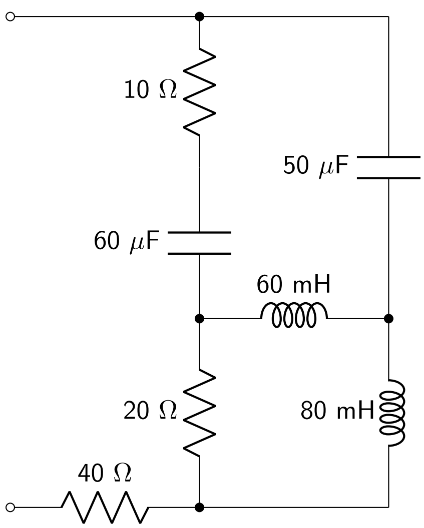
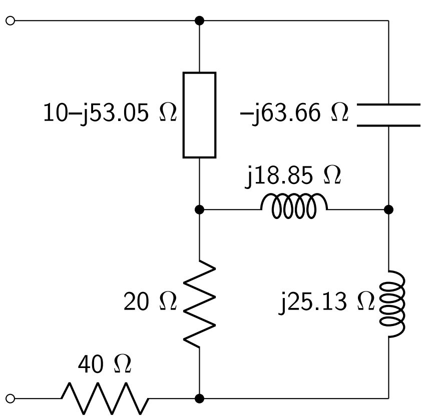
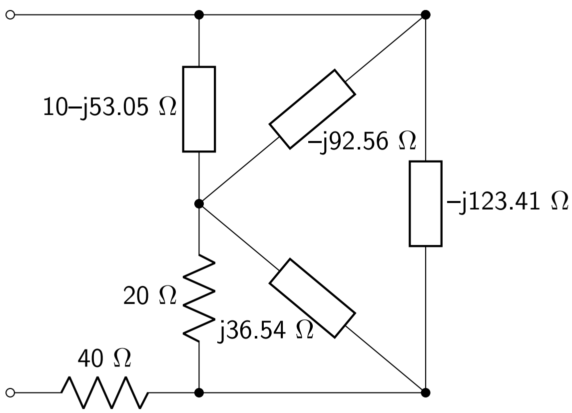


Figure 8.8: Circuit diagram for delta-wye and wye-delta transforms question 15.

Convert to impedances.



Do a wye-delta transform.



Calculate the equivalent impedance.

$$\begin{aligned}\mathbf{Z_{EQ}} &= [(10 - j53.05\ \Omega)//(-j92.56\ \Omega) + (20\ \Omega)//(j36.54\ \Omega)]\,//(-j123.41\ \Omega) + 40\ \Omega \\ &= [(4.02 - j34.00\ \Omega) + (15.39 + j8.42\ \Omega)]\,//(-j123.41\ \Omega) + 40\ \Omega \\ &= (19.41 - j25.58\ \Omega)//(-j123.41\ \Omega) + 40\ \Omega \\ &= 13.10 - j22.89\ \Omega + 40\ \Omega \\ &= 53.10 - j22.89\ \Omega\end{aligned}$$

9 Chapter 9 Solutions

9.1 Complex Voltage and Current Divider

1. Calculate $v(t)$ given the circuit diagram shown in figure 9.1.

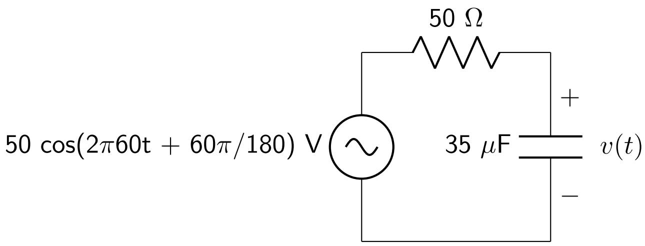


Figure 9.1: Circuit diagram for complex voltage and current divider circuits question 1.

Convert the source to phasor form.

$$\begin{aligned}\mathbf{V_S} &= 50\cos(60^\circ) + j50\sin(60^\circ)\ \text{V} \\ &= 25 + j43.30\ \text{V}\end{aligned}$$

Convert the capacitor to an impedance.

$$\begin{aligned}\mathbf{Z_C} &= \frac{-j}{(2\pi 60)(35 \times 10^{-6}} \\ &= -j75.79\ \Omega\end{aligned}$$

Apply the complex voltage divider rule.

$$\begin{aligned}\mathbf{V_{OUT}} &= (25 + j43.30\ \text{V})\left(\frac{-j75.79\ \Omega}{50\ \Omega - j75.79\ \Omega}\right) \\ &= 37.32 + j18.68\ \text{V} \\ &= 41.74\ \text{V}\angle 26.59^\circ\end{aligned}$$

Convert to time-varying form.

$$v_{out}(t) = 41.74\cos(2\pi 60t + 26.59\pi/180)\ \text{V}$$

2. Calculate $v(t)$ given the circuit diagram shown in figure 9.2. The frequency of operation is 80 Hz.

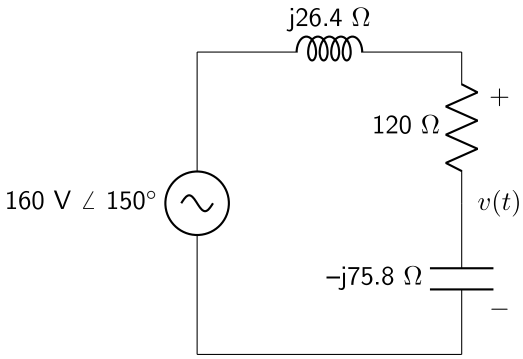


Figure 9.2: Circuit diagram for complex voltage and current divider circuits question 2.

Apply the complex voltage divider rule.

$$\begin{aligned}\mathbf{V_{OUT}} &= (-138.56 + j80\ \text{V})\left(\frac{120 - j75.8\ \Omega}{120 - j75.8\ \Omega + j26.4\ \Omega}\right)\\ &= -134.25 + j112.26\ \text{V}\\ &= 175.00\ \text{V}\angle 140.10^\circ\end{aligned}$$

Convert to time-varying form.

$$v_{out}(t) = 175.00\cos(2\pi 80t + 140.10\pi/180)\ \text{V}$$

3. Calculate $i(t)$ given the circuit diagram shown in figure 9.3. The frequency of operation is 150 Hz.

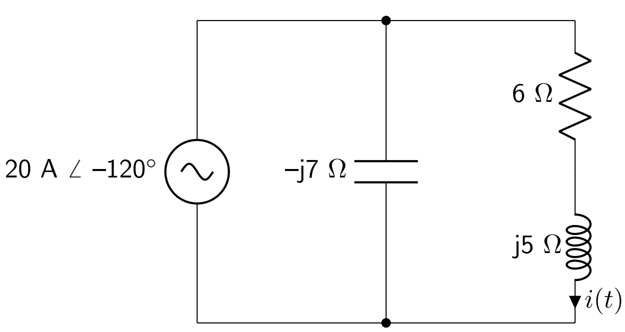


Figure 9.3: Circuit diagram for complex voltage and current divider circuits question 3.

Apply the complex current divider rule.

$$\mathbf{I_{OUT}} = (-10 - j17.32 \text{ A}) \left(\frac{(-j7 \ \Omega)//(6 + j5 \ \Omega)}{6 + j5 \ \Omega} \right)$$
$$= -21.69 + j4.44 \text{ A}$$
$$= 22.14 \text{ A}\angle 168.43°$$

Convert to time-varying form.

$$i_{out}(t) = 22.14 \cos(2\pi 150t + 168.43\pi/180) \text{ A}$$

4. Calculate $v(t)$ given the circuit diagram shown in figure 9.4.

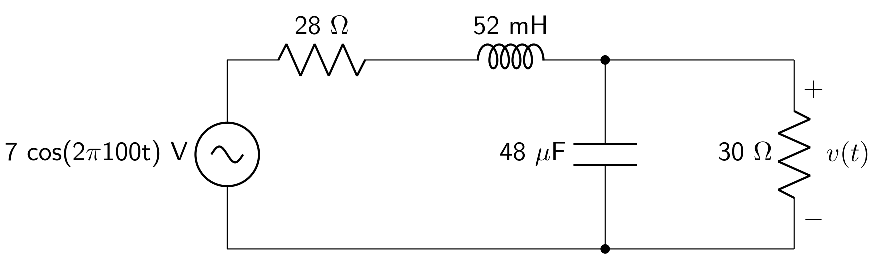
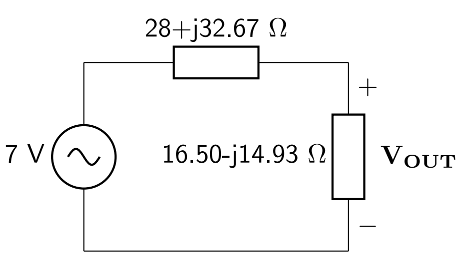


Figure 9.4: Circuit diagram for complex voltage and current divider circuits question 4.

Calculate equivalent impedances.



Apply the complex voltage divider rule.

$$\mathbf{V_{OUT}} = 7 \text{ V} \left(\frac{16.50 - j14.93 \ \Omega}{16.50 - j14.93 \ \Omega + 28 + j32.67 \ \Omega} \right)$$
$$= 1.43 - j2.92 \text{ V}$$
$$= 3.23 \text{ V}\angle -63.88°$$

Convert to time-varying form.

$$v_{out}(t) = 3.23 \cos(2\pi 100t - 63.88\pi/180) \text{ V}$$

5. Calculate $i(t)$ given the circuit diagram shown in figure 9.5.

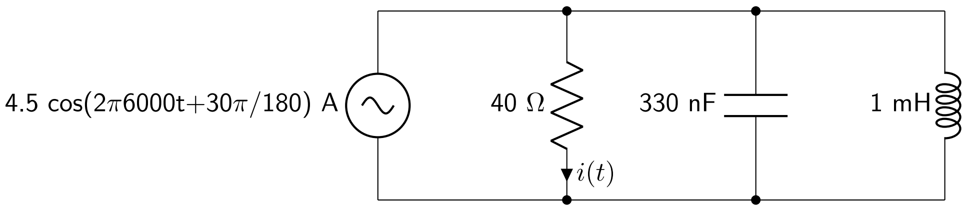
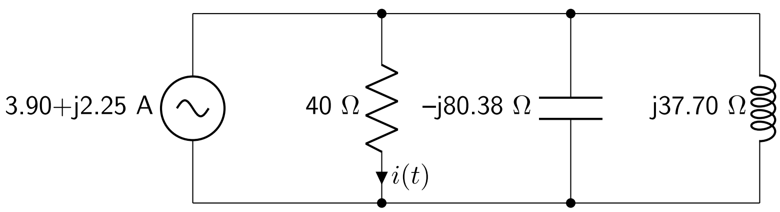


Figure 9.5: Circuit diagram for complex voltage and current divider circuits question 5.

Convert to phasors and impedances.



Apply the complex current divider rule.

$$\begin{aligned}\mathbf{I_{OUT}} &= (3.90 + j2.25\text{ A})\left(\frac{(40\ \Omega)//(-j80.38\ \Omega)//(j37.70\ \Omega)}{40\ \Omega}\right)\\ &= (3.90 + j2.25\text{ A})\left(\frac{30.36 + j17.11\ \Omega}{40\ \Omega}\right)\\ &= (3.90 + j2.25\text{ A})(0.76 + j0.43)\\ &= 2.00 + j3.37\text{ A}\\ &= 3.92\text{ A}\angle 59.40^\circ\end{aligned}$$

Convert to time-varying form.

$$i_{out}(t) = 3.92\cos(2\pi 6000t + 59.40\pi/180)\text{ A}$$

9.2 Complex Kirchhoff's Laws

6. Calculate $i(t)$ given the circuit diagram shown in figure 9.6.

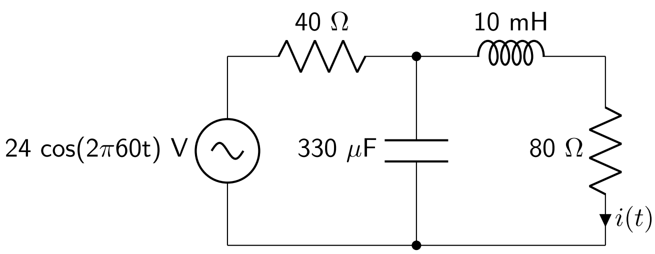
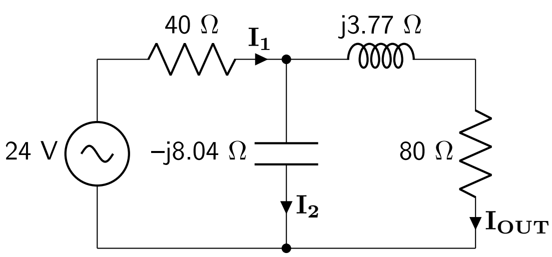


Figure 9.6: Circuit diagram for complex Kirchhoff's laws question 6.

Convert to impedances and define branch currents.



Perform KCL.

$$0 = \mathbf{I_1} - \mathbf{I_2} - \mathbf{I_{OUT}}$$

Perform KVL.

$$24 = 40\mathbf{I_1} - j8.04\mathbf{I_2}$$
$$0 = j8.04\mathbf{I_2} + (80 + j3.77)\mathbf{I_{OUT}}$$

Place the equations into form $\alpha\mathbf{I_1} + \beta\mathbf{I_2} + \gamma\mathbf{I_{OUT}} = \mathbf{c}$, and then place each coefficient into a matrix.

$$\begin{bmatrix} 1 & -1 & -1 & 0 \\ 40 & -j8.04 & 0 & 24 \\ 0 & j8.04 & 80 + j3.77 & 0 \end{bmatrix}$$

Solve the matrix for $\mathbf{I_{OUT}}$.

$$\mathbf{I_{OUT}} = 14.15 - j56.16 \text{ mA}$$

Convert to time-varying form.

$$i_{out}(t) = 57.91\cos(2\pi 60t - 75.86\pi/180) \text{ mA}$$

7. Calculate $v(t)$ given the circuit diagram shown in figure 9.7. The frequency of operation is 20 Hz.

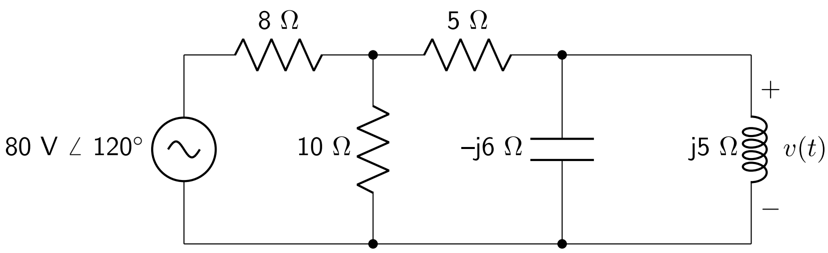
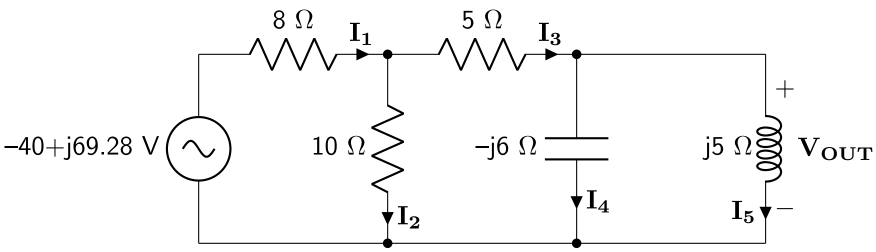


Figure 9.7: Circuit diagram for complex Kirchhoff's laws question 7.

Define branch currents.



Perform KCL.

$$0 = \mathbf{I_1} - \mathbf{I_2} - \mathbf{I_3}$$
$$0 = \mathbf{I_3} - \mathbf{I_4} - \mathbf{I_5}$$

Perform KVL.

$$-40 + j69.28 = 8\mathbf{I_1} + 10\mathbf{I_2}$$
$$0 = -10\mathbf{I_2} + 5\mathbf{I_3} - j6\mathbf{I_4}$$
$$0 = -10\mathbf{I_2} + 5\mathbf{I_3} + j5\mathbf{I_5}$$

Place the equations into form $\alpha\mathbf{I_1} + \beta\mathbf{I_2} + \gamma\mathbf{I_3} + \delta\mathbf{I_4} + \epsilon\mathbf{I_5} = \mathbf{c}$, and then place each coefficient into a

matrix.

$$\begin{bmatrix} 1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & -1 & 0 \\ 8 & 10 & 0 & 0 & 0 & -40+j69.28 \\ 0 & -10 & 5 & -j6 & 0 & 0 \\ 0 & -10 & 5 & 0 & j5 & 0 \end{bmatrix}$$

Solve the matrix for $\mathbf{I_5}$.

$$\mathbf{I_5} = 5.73 + j6.25 \text{ A}$$

Use Ohm's law to calculate $\mathbf{V_{OUT}}$.

$$\begin{aligned} \mathbf{V_{OUT}} &= (5.73 + j6.25 \text{ A})(j5\ \Omega) \\ &= -31.24 + j28.65 \text{ V} \\ &= 42.39 \text{ V}\angle 137.47^\circ \end{aligned}$$

Convert to time-varying form.

$$v_{out}(t) = 42.39 \cos(2\pi 20t + 137.47\pi/180) \text{ V}$$

8. Calculate $v(t)$ given the circuit diagram shown in figure 9.8. The frequency of operation is 20 kHz.

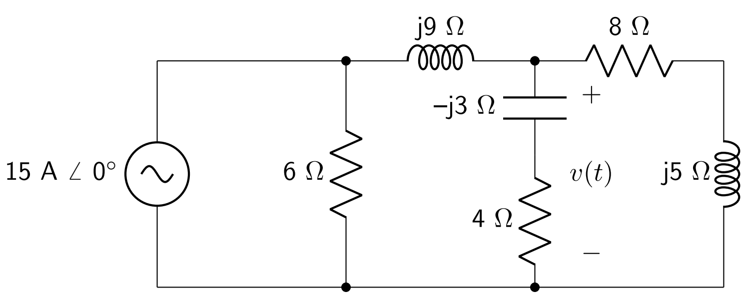
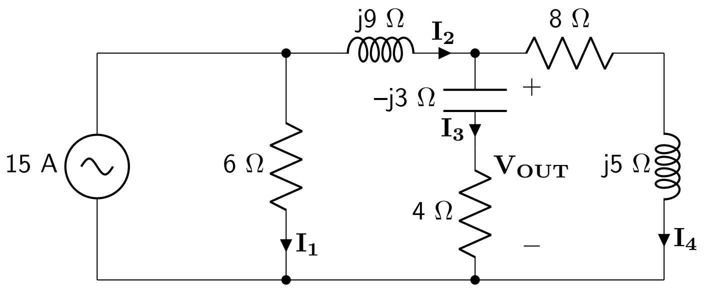


Figure 9.8: Circuit diagram for complex Kirchhoff's laws question 8.

Define branch currents.



Perform KCL.

$$15 = \mathbf{I_1} + \mathbf{I_2}$$
$$0 = \mathbf{I_2} - \mathbf{I_3} - \mathbf{I_4}$$

Perform KVL.

$$0 = -6\mathbf{I_1} + j9\mathbf{I_2} + (4 - j3)\mathbf{I_3}$$
$$0 = -6\mathbf{I_1} + j9\mathbf{I_2} + (8 + j5)\mathbf{I_4}$$

Place the equations into form $\alpha\mathbf{I_1} + \beta\mathbf{I_2} + \gamma\mathbf{I_3} + \delta\mathbf{I_4} = \mathbf{c}$, and then place each coefficient into a matrix.

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 15 \\ 0 & 1 & -1 & -1 & 0 \\ -6 & j9 & 4-j3 & 0 & 0 \\ -6 & j9 & 0 & 8+j5 & 0 \end{bmatrix}$$

Solve the matrix for $\mathbf{I_3}$.

$$\mathbf{I_3} = 5.28 - j1.61 \text{ A}$$

Use Ohm's law to calculate $\mathbf{V_{OUT}}$.

$$\begin{aligned} \mathbf{V_{OUT}} &= (5.28 - j1.61 \text{ A})(4 - j3\ \Omega) \\ &= 16.29 - j22.28 \text{ V} \\ &= 27.60 \text{ V}\angle - 53.82^\circ \end{aligned}$$

Convert to time-varying form.

$$v_{out}(t) = 27.60\cos(2\pi 20000t - 53.82\pi/180) \text{ V}$$

9. Calculate $v(t)$ given the circuit diagram shown in figure 9.9. The frequency of operation is 30 Hz.

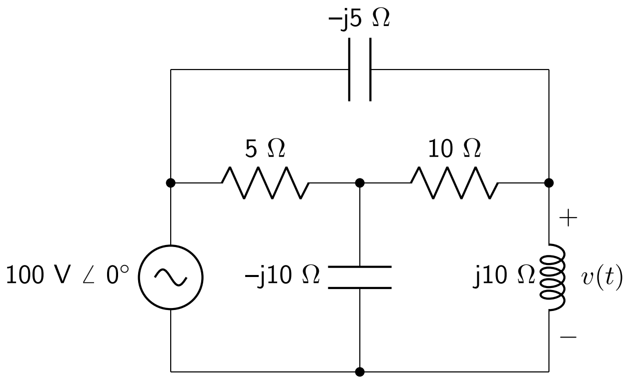
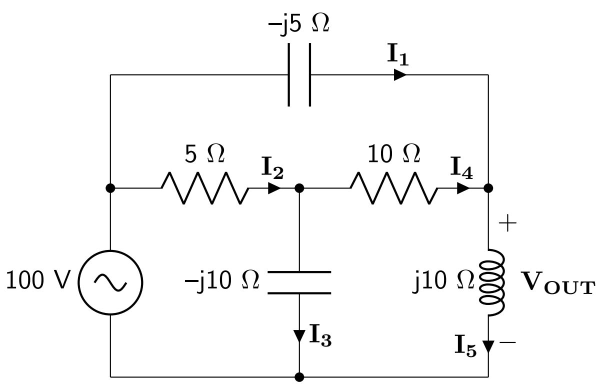


Figure 9.9: Circuit diagram for complex Kirchhoff's laws question 9.

Define branch currents.



Perform KCL.

$$0 = \mathbf{I_2} - \mathbf{I_3} - \mathbf{I_4}$$

$$0 = \mathbf{I_1} + \mathbf{I_4} - \mathbf{I_5}$$

Perform KVL.

$$100 = 5\mathbf{I_2} - j10\mathbf{I_3}$$

$$0 = j10\mathbf{I_3} + 10\mathbf{I_4} + j10\mathbf{I_5}$$

$$0 = -j5\mathbf{I_1} - 10\mathbf{I_4} - 5\mathbf{I_2}$$

Place the equations into form $\alpha\mathbf{I_1} + \beta\mathbf{I_2} + \gamma\mathbf{I_3} + \delta\mathbf{I_4} + \epsilon\mathbf{I_5} = \mathbf{c}$, and then place each coefficient into a

matrix.

$$\begin{bmatrix} 0 & 1 & -1 & -1 & 0 & 0 \\ 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 5 & -j10 & 0 & 0 & 100 \\ 0 & 0 & j10 & 10 & j10 & 0 \\ -j5 & -5 & 0 & -10 & 0 & 0 \end{bmatrix}$$

Solve the matrix for $\mathbf{I_5}$.

$$\mathbf{I_5} = 3.53 - j14.12 \text{ A}$$

Use Ohm's law to calculate $\mathbf{V_{OUT}}$.

$$\begin{aligned} \mathbf{V_{OUT}} &= (3.53 - j14.12 \text{ A})(j10\ \Omega) \\ &= 141.18 + j35.29 \text{ V} \\ &= 145.52 \text{ V}\angle 14.04^\circ \end{aligned}$$

Convert to time-varying form.

$$v_{out}(t) = 145.52\cos(2\pi 30t + 14.04\pi/180) \text{ V}$$

10. Calculate $v(t)$ given the circuit diagram shown in figure 9.10.

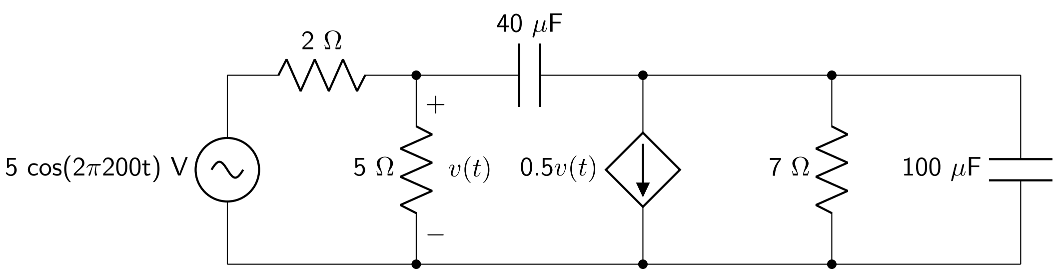
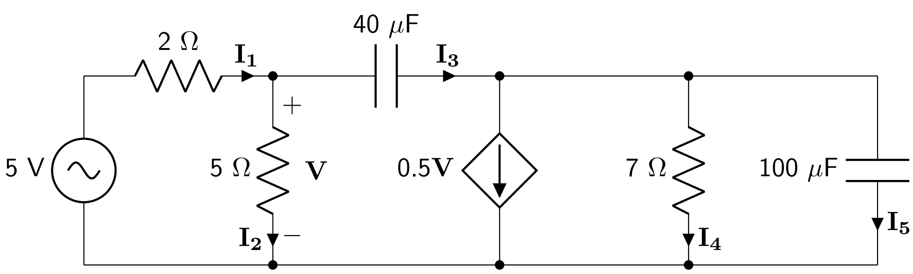


Figure 9.10: Circuit diagram for complex Kirchhoff's laws question 10.

Define branch currents.



Perform KCL.

$$0 = \mathbf{I_1} - \mathbf{I_2} - \mathbf{I_3}$$
$$0 = \mathbf{I_3} - 0.5\mathbf{V} - \mathbf{I_4} - \mathbf{I_5}$$

Perform KVL.

$$5 = 2\mathbf{I_1} + 5\mathbf{I_2}$$
$$0 = -5\mathbf{I_2} - j19.89\mathbf{I_3} + 7\mathbf{I_4}$$
$$0 = -7\mathbf{I_4} - j7.96\mathbf{I_5}$$

Derive a dependent source equation.

$$\mathbf{V} = 5\mathbf{I_2}$$

Place the equations into form $\alpha\mathbf{I_1} + \beta\mathbf{I_2} + \gamma\mathbf{I_3} + \delta\mathbf{I_4} + \epsilon\mathbf{I_5} + \zeta\mathbf{V} = \mathbf{c}$, and then place each coefficient into a matrix.

$$\begin{bmatrix} 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & -1 & -0.5 & 0 \\ 2 & 5 & 0 & 0 & 0 & 0 & 5 \\ 0 & -5 & -j19.89 & 7 & 0 & 0 & 0 \\ 0 & 0 & 0 & -7 & -j7.96 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}$$

Solve the matrix for $\mathbf{V}$.

$$\mathbf{V} = 3.09 - j0.43 \text{ V}$$
$$= 3.12 \text{ V}\angle - 8.00^\circ$$

Convert to time-varying form.

$$v(t) = 3.12\cos(2\pi 200t - 8.00\pi/180) \text{ V}$$

9.3 Complex Mesh Analysis

11. Calculate $v(t)$ given the circuit diagram shown in figure 9.11. The frequency of operation is 50 Hz.

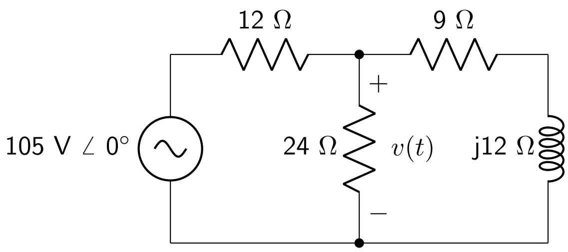
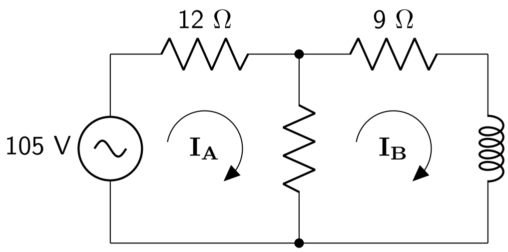


Figure 9.11: Circuit diagram for complex mesh analysis question 11.

Define mesh currents. Component values may be hidden so that mesh current labels can be read.



Derive the mesh equations.

$$105 = 12\mathbf{I_A} + 24(\mathbf{I_A} - \mathbf{I_B})$$
$$0 = 24(\mathbf{I_B} - \mathbf{I_A}) + (9 + j12)\mathbf{I_B}$$

Place the equations into form $\alpha\mathbf{I_A} + \beta\mathbf{I_B} = \mathbf{c}$, and then place each coefficient into a matrix.

$$\begin{bmatrix} 36 & -24 & 105 \\ -24 & 33 + j12 & 0 \end{bmatrix}$$

Solve the matrix for $\mathbf{I_A}$ and $\mathbf{I_B}$.

$$\mathbf{I_A} = 4.75 - j1.29 \text{ A}$$
$$\mathbf{I_B} = 2.75 - j1.94 \text{ A}$$

Calculate the voltage.

$$\begin{aligned}\mathbf{V} &= (24\ \Omega)(\mathbf{I_A} - \mathbf{I_B})\\ &= (24\ \Omega)((4.75 - j1.29\ \text{A}) - (2.75 - j1.94\ \text{A}))\\ &= 48.01 + j15.52\ \text{V}\\ &= 50.46\ \text{V}\angle 17.91^\circ\end{aligned}$$

Convert to time-varying form.

$$v(t) = 50.46\cos(2\pi 50t + 17.91\pi/180)\ \text{V}$$

12. Calculate $i(t)$ given the circuit diagram shown in figure 9.12.

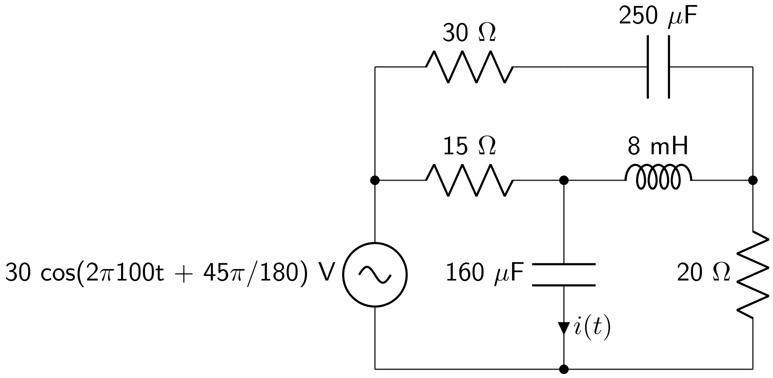
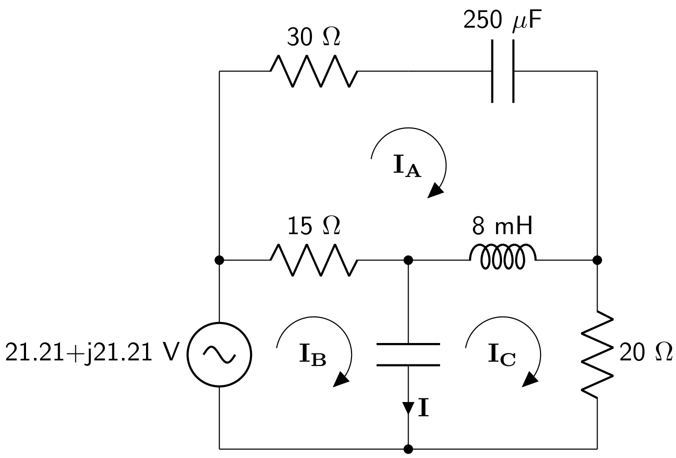


Figure 9.12: Circuit diagram for complex mesh analysis question 12.

Define mesh currents. Component values may be hidden so that mesh current labels can be read.



Derive the mesh equations.

$$\begin{aligned} 0 &= (30 - j6.37)\mathbf{I_A} + j5.03(\mathbf{I_A} - \mathbf{I_C}) + 15(\mathbf{I_A} - \mathbf{I_B}) \\ 21.21 + j21.21 &= 15(\mathbf{I_B} - \mathbf{I_A}) - j9.95(\mathbf{I_B} - \mathbf{I_C}) \\ 0 &= -j9.95(\mathbf{I_C} - \mathbf{I_B}) + j5.03(\mathbf{I_C} - \mathbf{I_A}) + 20\mathbf{I_C} \end{aligned}$$

Place the equations into form $\alpha\mathbf{I_A} + \beta\mathbf{I_B} + \gamma\mathbf{I_C} = \mathbf{c}$, and then place each coefficient into a matrix.

$$\begin{bmatrix} 45 - j1.34 & -15 & -j5.03 & 0 \\ -15 & 15 - j9.95 & j9.95 & 21.21 + j21.21 \\ -j5.03 & j9.95 & 20 - j4.92 & 0 \end{bmatrix}$$

Solve the matrix for $\mathbf{I_B}$ and $\mathbf{I_C}$.

$$\mathbf{I_B} = 0.31 + j1.83 \text{ A}$$

$$\mathbf{I_C} = 0.73 + j0.05 \text{ A}$$

Calculate the branch current.

$$\begin{aligned} \mathbf{I} &= (\mathbf{I_B} - \mathbf{I_C}) \\ &= (0.31 + j1.83 \text{ A}) - (0.73 + j0.05 \text{ A}) \\ &= -0.42 + j1.79 \text{ A} \\ &= 1.83 \text{ A}\angle 103.18^\circ \end{aligned}$$

Convert to time-varying form.

$$i(t) = 1.83\cos(2\pi 100t + 103.18\pi/180) \text{ A}$$

13. Calculate $v(t)$ given the circuit diagram shown in figure 9.13.

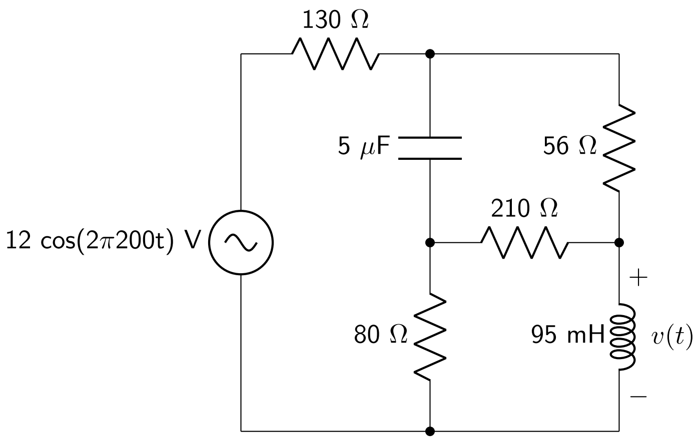
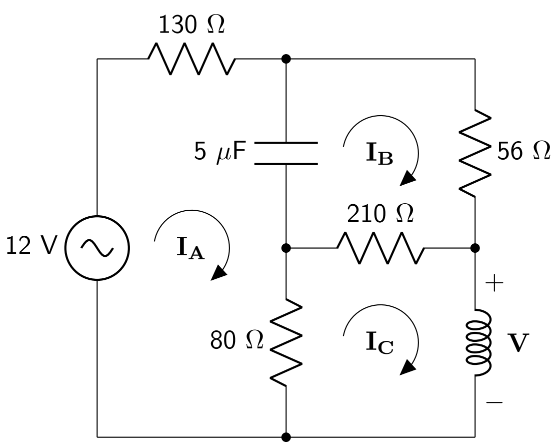


Figure 9.13: Circuit diagram for complex mesh analysis question 13.

Define mesh currents. Component values may be hidden so that mesh current labels can be read.



Derive the mesh equations.

$$12 = 130\mathbf{I_A} - j159.15(\mathbf{I_A} - \mathbf{I_B}) + 80(\mathbf{I_A} - \mathbf{I_C})$$

$$0 = -j159.15(\mathbf{I_B} - \mathbf{I_A}) + 56\mathbf{I_B} + 210(\mathbf{I_B} - \mathbf{I_C})$$

$$0 = 80(\mathbf{I_C} - \mathbf{I_A}) + 210(\mathbf{I_C} - \mathbf{I_B}) + j119.38\mathbf{I_C}$$

Place the equations into form $\alpha\mathbf{I_A} + \beta\mathbf{I_B} + \gamma\mathbf{I_C} = \mathbf{c}$, and then place each coefficient into a matrix.

$$\begin{bmatrix} 210 - j159.15 & j159.15 & -80 & 12 \\ j159.15 & 266 - j159.15 & -210 & 0 \\ -80 & -210 & 290 + j119.38 & 0 \end{bmatrix}$$

Solve the matrix for $\mathbf{I_C}$.

$$\mathbf{I_C} = 22.68 - j35.74 \text{ mA}$$

Use Ohm's law to calculate $\mathbf{V}$.

$$\begin{aligned}\mathbf{V} &= (j119.38\ \Omega)(\mathbf{I_C}) \\ &= (j0.11938\ \text{k}\Omega)(22.68 - j35.74 \text{ mA}) \\ &= 4.27 + j2.71 \text{ V} \\ &= 5.05 \text{ V}\angle 32.40^\circ\end{aligned}$$

Convert to time-varying form.

$$v(t) = 5.05 \cos(2\pi 200t + 32.40\pi/180) \text{ V}$$

14. Calculate $v(t)$ given the circuit diagram shown in figure 9.14. The frequency of operation is 5 kHz.

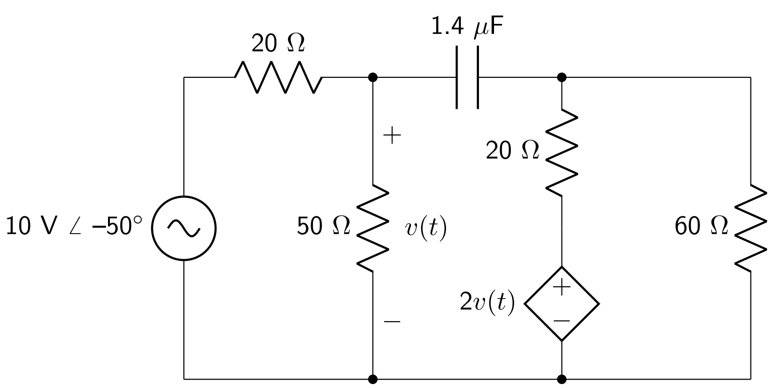
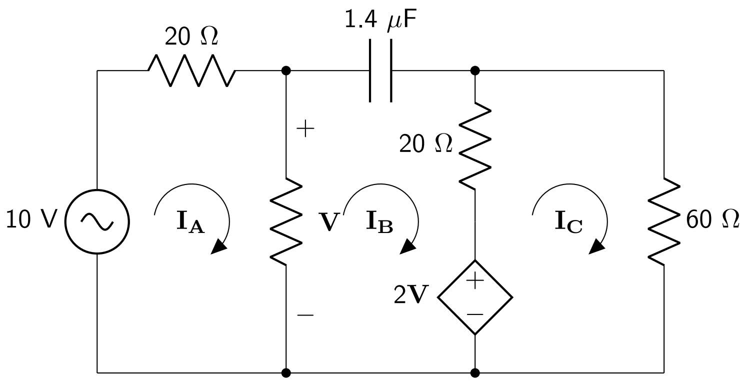


Figure 9.14: Circuit diagram for complex mesh analysis question 14.

Define mesh currents. Component values may be hidden so that mesh current labels can be read.



Derive the mesh equations.

$$10 = 20\mathbf{I_A} + 50(\mathbf{I_A} - \mathbf{I_B})$$
$$0 = 50(\mathbf{I_B} - \mathbf{I_A}) - j22.75\mathbf{I_B} + 20(\mathbf{I_B} - \mathbf{I_C}) + 2\mathbf{V}$$
$$0 = -2\mathbf{V} + 20(\mathbf{I_C} - \mathbf{I_B}) + 60\mathbf{I_C}$$

Derive an equation for the dependent source.

$$\mathbf{V} = 50(\mathbf{I_A} - \mathbf{I_B})$$

Place the equations into form $\alpha\mathbf{I_A} + \beta\mathbf{I_B} + \gamma\mathbf{I_C} + \delta\mathbf{V} = \mathbf{c}$, and then place each coefficient into a matrix.

$$\begin{bmatrix} 70 & -50 & 0 & 0 & 10 \\ -50 & 70 - j22.74 & -20 & 2 & 0 \\ 0 & -20 & 80 & -2 & 0 \\ -50 & 50 & 0 & 1 & 0 \end{bmatrix}$$

Solve the matrix for $\mathbf{V}$.

$$\mathbf{V} = 7.84 + j2.00 \text{ V}$$
$$= 8.09 \text{ V}\angle 14.35°$$

Convert to time-varying form.

$$v(t) = 8.09\cos(2\pi 5000t + 14.35\pi/180) \text{ V}$$

15. Calculate $v(t)$ given the circuit diagram shown in figure 9.15.

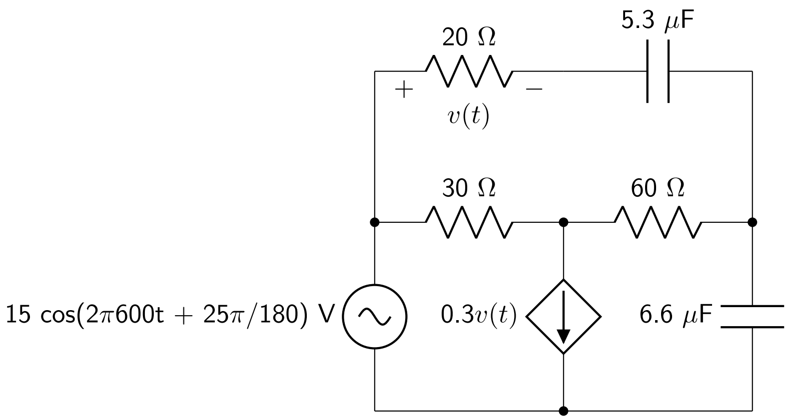
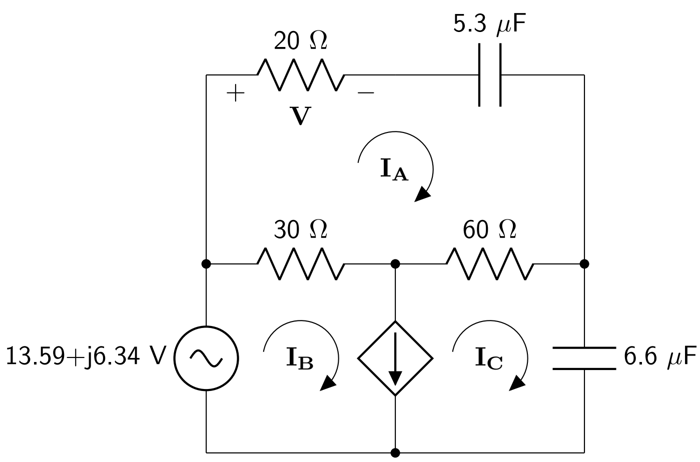


Figure 9.15: Circuit diagram for complex mesh analysis question 15.

Define mesh currents. Component values may be hidden so that mesh current labels can be read.



Derive the mesh equations.

$$0 = (20 - j50.05)\mathbf{I_A} + 60(\mathbf{I_A} - \mathbf{I_C}) + 30(\mathbf{I_A} - \mathbf{I_B})$$

$$13.59 + j6.34 = 30(\mathbf{I_B} - \mathbf{I_A}) + 60(\mathbf{I_C} - \mathbf{I_A}) - j40.19\mathbf{I_C}$$

Perform KCL at the supermesh.

$$0 = \mathbf{I_B} - 0.3\mathbf{V} - \mathbf{I_C}$$

Derive an equation for the dependent source.

$$\mathbf{V} = 20\mathbf{I_A}$$

Place the equations into form $\alpha\mathbf{I_A} + \beta\mathbf{I_B} + \gamma\mathbf{I_C} + \delta\mathbf{V} = \mathbf{c}$, and then place each coefficient into a matrix.

$$\begin{bmatrix} 110 - j50.05 & -30 & -60 & 0 & 0 \\ -90 & 30 & 60 - j40.19 & 0 & 13.59 + j6.34 \\ 0 & 1 & -1 & -0.3 & 0 \\ -20 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Solve the matrix for $\mathbf{V}$.

$$\begin{aligned} \mathbf{V} &= -8.43 + j13.42 \text{ V} \\ &= 15.84 \text{ V}\angle 122.13^\circ \end{aligned}$$

Convert to time-varying form.

$$v(t) = 15.84\cos(2\pi 600t + 122.13\pi/180) \text{ V}$$

9.4 Complex Superposition

16. Calculate $v(t)$ given the circuit diagram shown in figure 9.16. The frequency of operation is 10 kHz.

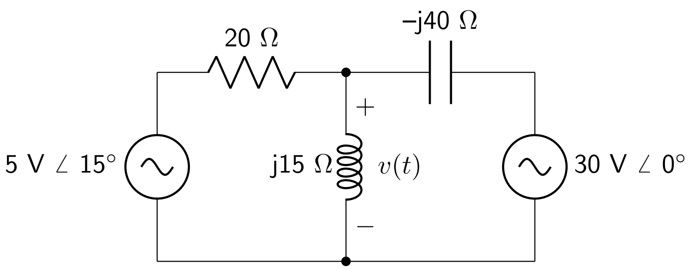
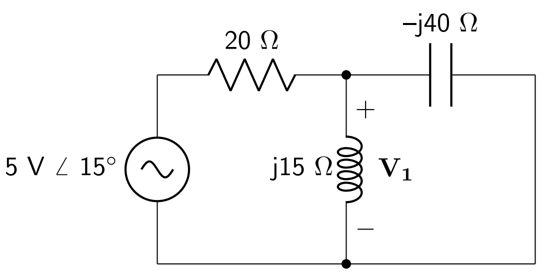


Figure 9.16: Circuit diagram for complex superposition question 16.

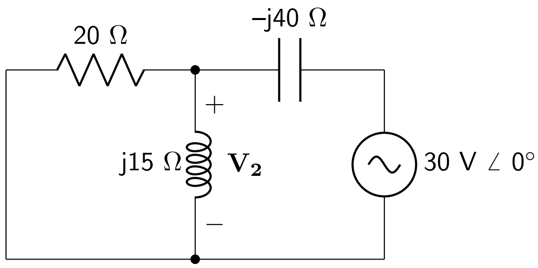
Calculate the voltage drop contributed by the 5 V source.



Use the complex voltage divider rule to calculate $\mathbf{V_1}$.

$$\begin{aligned}\mathbf{V_1} &= (4.83 + j1.29 \text{ V})\left(\frac{j15\ \Omega // -j40\ \Omega}{j15\ \Omega // -j40\ \Omega + 20\ \Omega}\right)\\ &= (4.83 + j1.29 \text{ V})\left(\frac{j24\ \Omega}{j24\ \Omega + 20\ \Omega}\right)\\ &= 2.21 + j3.14 \text{ V}\end{aligned}$$

Calculate the voltage drop contributed by the 30 V source.



Use the complex voltage divider rule to calculate $\mathbf{V_2}$.

$$\begin{aligned}\mathbf{V_2} &= (30 \text{ V})\left(\frac{j15\ \Omega//20\ \Omega}{j15\ \Omega//20\ \Omega - j40\ \Omega}\right)\\ &= (30 \text{ V})\left(\frac{7.2 + j9.6\ \Omega}{-32.8 + j9.6\ \Omega}\right)\\ &= -7.38 + j8.85 \text{ V}\end{aligned}$$

Calculate $\mathbf{V}$.

$$\begin{aligned}\mathbf{V} &= \mathbf{V_1} + \mathbf{V_2}\\ &= (2.21 + j3.14 \text{ V}) + (-7.38 + j8.85 \text{ V})\\ &= -5.16 + j11.99 \text{ V}\\ &= 13.06 \text{ V}\angle 113.30^\circ\end{aligned}$$

Convert to time-varying form.

$$v(t) = 13.06\cos(2\pi 10000t + 113.30\pi/180) \text{ V}$$

17. Calculate $v(t)$ given the circuit diagram shown in figure 9.17. The frequency of operation is 40 Hz.

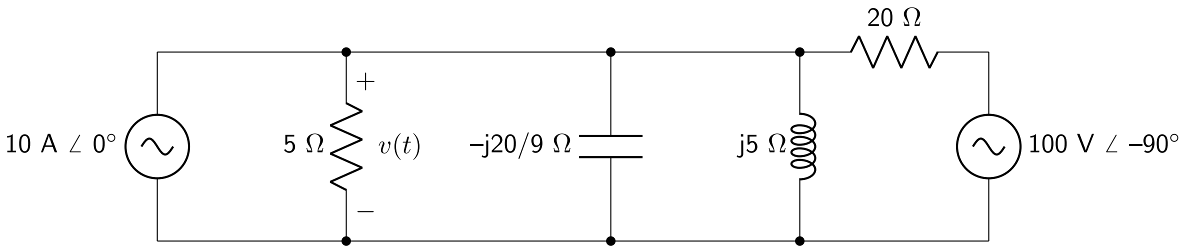
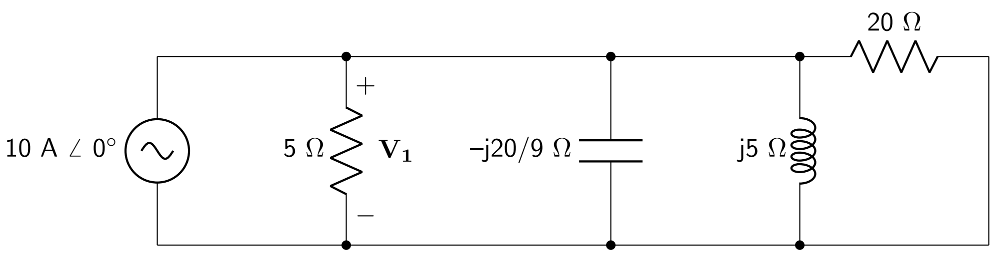


Figure 9.17: Circuit diagram for complex superposition question 17.

Calculate the voltage drop contributed by the 10 A source.



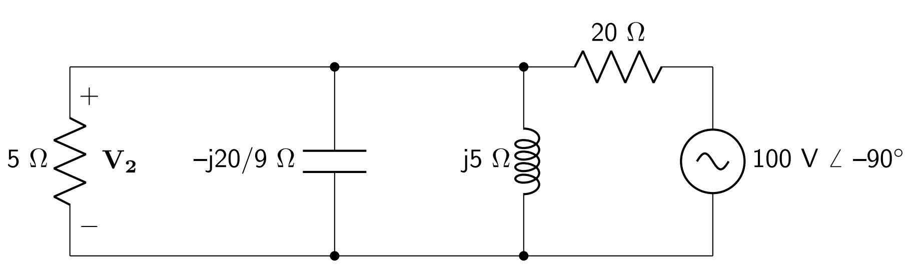
Calculate the equivalent impedance.

$$\begin{aligned} Z_{EQ} &= 5\ \Omega // -j20/9\ \Omega // j5\ \Omega // 20\ \Omega \\ &= 2 - j2\ \Omega \end{aligned}$$

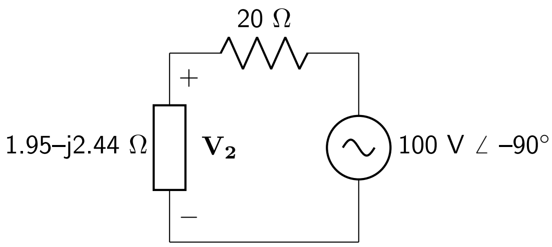
Use Ohm's law to calculate $\mathbf{V_1}$.

$$\begin{aligned} \mathbf{V_1} &= (10\text{ A})(2 - j2\ \Omega) \\ &= 20 - j20\text{ V} \end{aligned}$$

Calculate the voltage drop contributed by the 100 V source.



Calculate the equivalent impedance of the parallel impedances.



Use the complex voltage divider rule to calculate $\mathbf{V_2}$.

$$\begin{aligned}\mathbf{V_2} &= (-j100\text{ V})\left(\frac{1.95 - j2.44\ \Omega}{21.95 - j2.44\ \Omega}\right)\\ &= -10 - j10\text{ V}\end{aligned}$$

Calculate $\mathbf{V}$.

$$\begin{aligned}\mathbf{V} &= \mathbf{V_1} + \mathbf{V_2}\\ &= (20 - j20\text{ V}) + (-10 - j10\text{ V})\\ &= 10 - j30\text{ V}\\ &= 31.62\text{ V}\angle - 71.57^\circ\end{aligned}$$

Convert to time-varying form.

$$v(t) = 31.62\cos(2\pi 100t - 71.57\pi/180)\text{ V}$$

18. Calculate $v(t)$ given the circuit diagram shown in figure 9.18. The frequency of operation is 100 Hz.

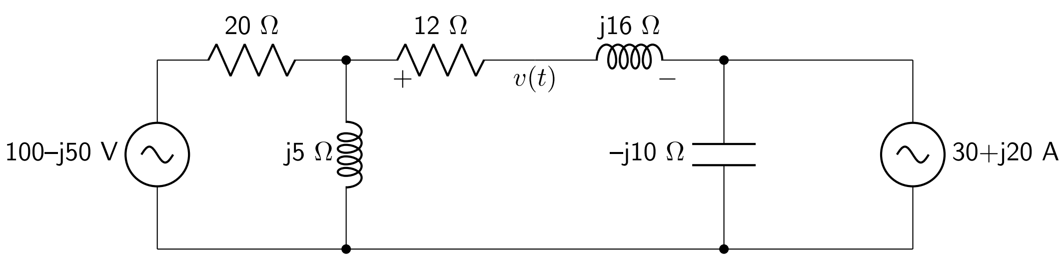
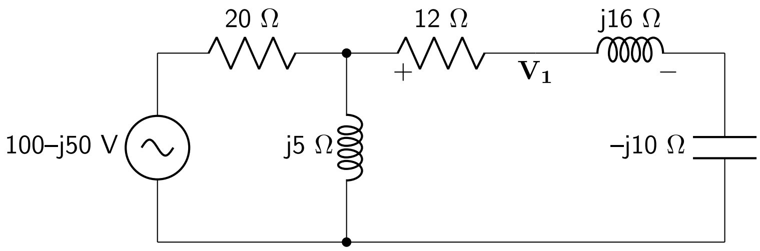


Figure 9.18: Circuit diagram for complex superposition question 18.

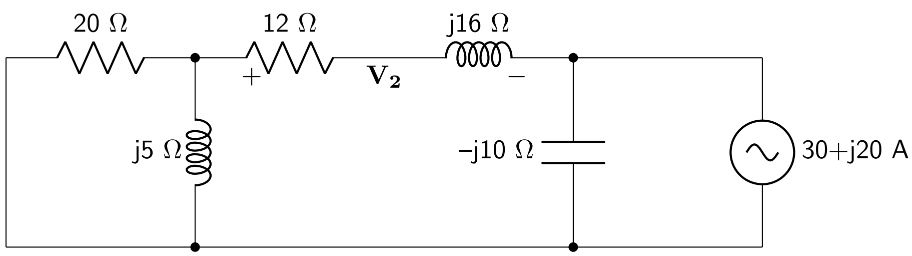
Calculate the voltage drop contributed by the voltage source.



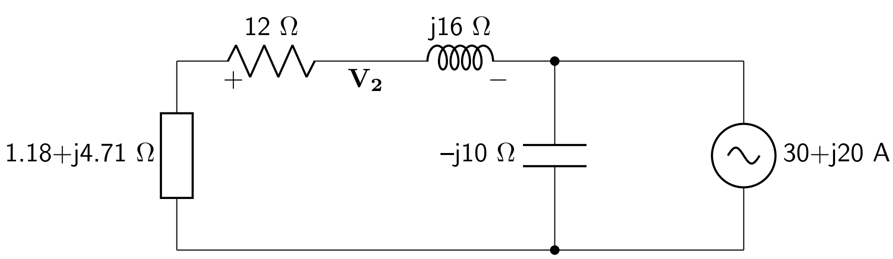
Use the complex voltage divider rule to calculate $\mathbf{V_1}$.

$$\begin{aligned}\mathbf{V_1} &= (100 - j50\text{ V})\left(\frac{j5\ \Omega//(12 + j6\ \Omega)}{j5\ \Omega//(12 + j6\ \Omega) + 20\ \Omega}\right)\left(\frac{12 + j16\ \Omega}{12 + j6\ \Omega}\right)\\ &= (100 - j50\text{ V})\left(\frac{1.13 + j3.96\ \Omega}{21.13 + j3.96\ \Omega}\right)(1.33 + j0.67)\\ &= (100 - j50\text{ V})(0.09 + j0.17)\,(1.33 + j0.67)\\ &= 14.29 + j28.57\text{ V}\end{aligned}$$

Calculate the voltage drop contributed by the current source.



Combine parallel impedances.



Use the complex current divider rule to calculate the current through the branch with many elements.

$$\begin{aligned}\mathbf{I} &= (30 + j20\text{ A})\left(\frac{-j10\ \Omega//13.18 + j20.71\ \Omega}{13.18 + j20.71\ \Omega}\right)\\ &= -2 - j21.14\text{ A}\end{aligned}$$

Use Ohm's law to calculate $\mathbf{V_2}$.

$$\begin{aligned}\mathbf{V_2} &= (12 + j16\ \Omega)(-2 - j21.14\ \text{A}) \\ &= 314.29 - j285.71\ \text{V}\end{aligned}$$

Calculate $\mathbf{V}$.

$$\begin{aligned}\mathbf{V} &= \mathbf{V_1} + \mathbf{V_2} \\ &= (14.29 + j28.57\ \text{V}) + (314.29 - j285.71\ \text{V}) \\ &= 328.57 - j257.14\ \text{V} \\ &= 417.23\ \text{V}\angle - 38.05^\circ\end{aligned}$$

Convert to time-varying form.

$$v(t) = 417.23\cos(2\pi 100t - 38.05\pi/180)\ \text{V}$$

19. Calculate $i(t)$ given the circuit diagram shown in figure 9.19. The frequency of operation is 2 kHz.

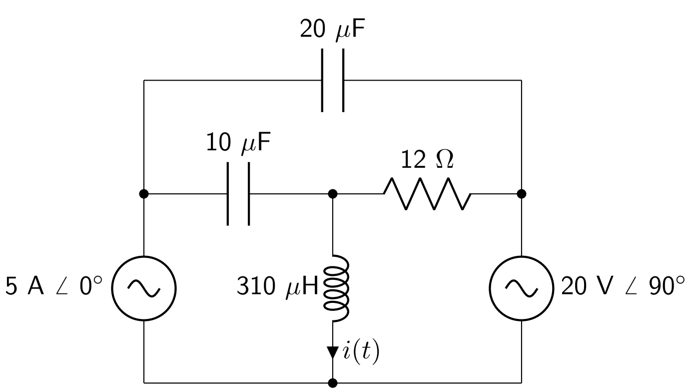
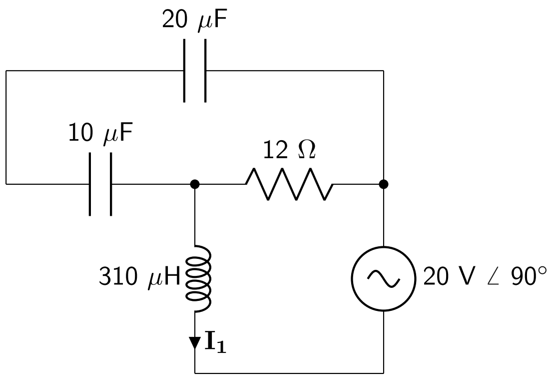
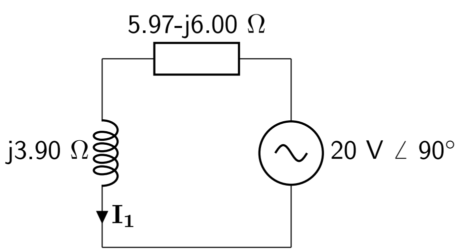


Figure 9.19: Circuit diagram for complex superposition question 19.

Calculate the current contributed by the voltage source.



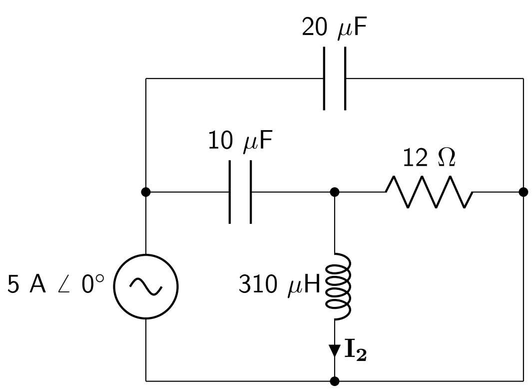
Calculate the equivalent impedance.



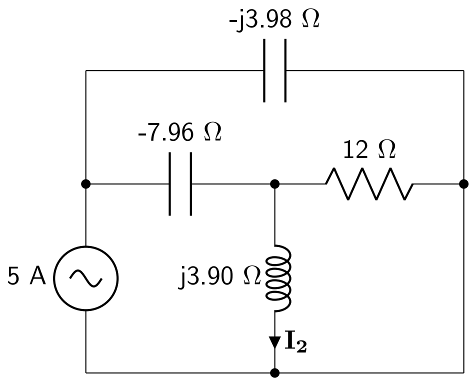
Use the Ohm's law to calculate $\mathbf{I_1}$.

$$\begin{aligned}\mathbf{I_1} &= \frac{j20\ \text{V}}{5.97 - j6.00\ \Omega + j3.90\ \Omega} \\ &= -1.05 + j2.98\ \text{A}\end{aligned}$$

Calculate the voltage drop contributed by the current source.



Convert to impedances.



Use the complex current divider rule to calculate $\mathbf{I_2}$.

$$
\begin{aligned}
\mathbf{I_2} &= (5\ \text{A})\left(\frac{(-j7.96\ \Omega + 12\ \Omega // j3.90\ \Omega) // -j3.98\ \Omega}{-j7.96\ \Omega + 12\ \Omega // j3.90\ \Omega}\right)\left(\frac{j3.90\ \Omega // 12\ \Omega}{j3.90\ \Omega}\right) \\
&= (5\ \text{A})\left(\frac{(1.14 - j4.43\ \Omega) // -j3.98\ \Omega}{1.14 - j4.43\ \Omega}\right)(0.90 - j0.29) \\
&= (5\ \text{A})\,(0.46 - j0.06)\,(0.90 - j0.29) \\
&= 2.01 - j0.97\ \text{A}
\end{aligned}
$$

Calculate $\mathbf{I}$.

$$
\begin{aligned}
\mathbf{I} &= \mathbf{I_1} + \mathbf{I_2} \\
&= (-1.05 + j2.98\ \text{A}) + (2.01 - j0.97\ \text{A}) \\
&= 0.96 + j2.01\ \text{A} \\
&= 2.23\ \text{A}\angle 64.57^\circ
\end{aligned}
$$

Convert to time-varying form.

$$i(t) = 2.23\cos(2\pi 2000t + 64.57\pi/180)\ \text{A}$$

20. Calculate $i(t)$ given the circuit diagram shown in figure 9.20. The frequency of operation is 20 Hz.

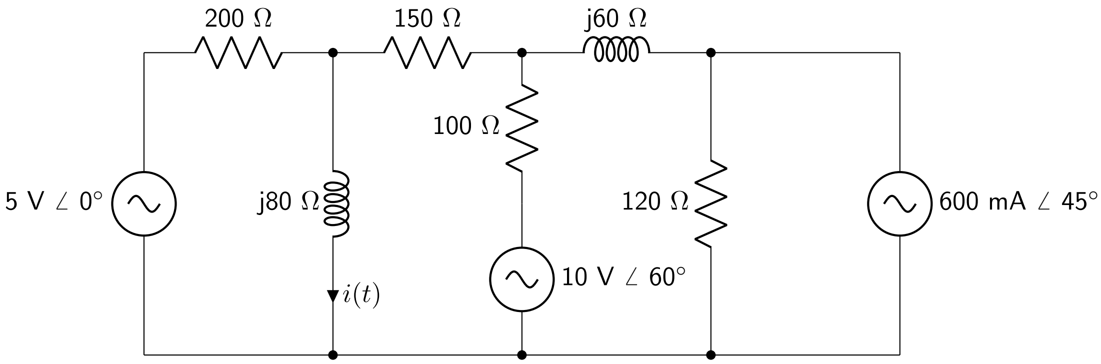
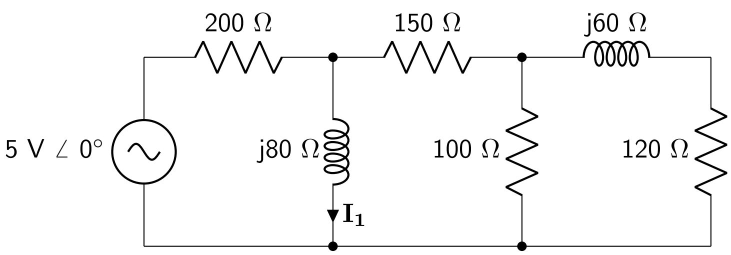
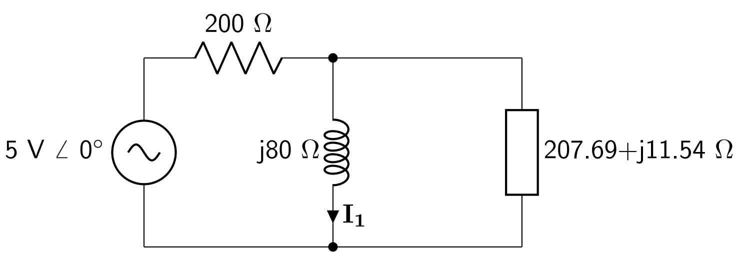


Figure 9.20: Circuit diagram for complex superposition question 20.

Calculate the current contributed by the 5 V source.



Calculate the equivalent impedance.



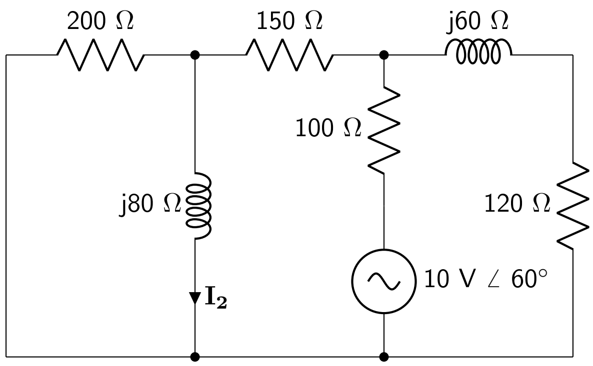
Use the complex voltage divider rule to calculate the voltage drop over the inductor.

$$\mathbf{V_L} = 5\text{ V}\left(\frac{(j80\ \Omega)//(207.69 + j11.54\ \Omega)}{200\ \Omega + (j80\ \Omega)//(207.69 + jj.54\ \Omega)}\right)$$
$$= 0.95 + j1.23\text{ V}$$

Use Ohm's law to calculate $\mathbf{I_1}$.

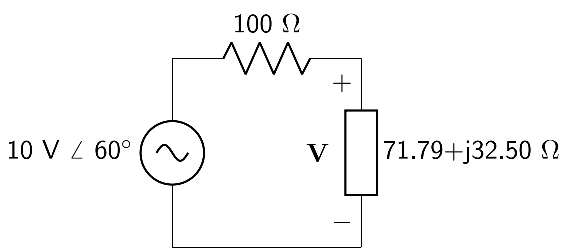
$$\mathbf{I_1} = \frac{0.95 + j1.23\text{ V}}{j80\ \Omega}$$
$$= 15.40 - j11.82\text{ mA}$$

Calculate the current contributed by the 10 V source.



Convert to a simpler circuit to take a voltage divider. Find the equivalent impedance of all but the 100 Ω resistor. All units are ohms.

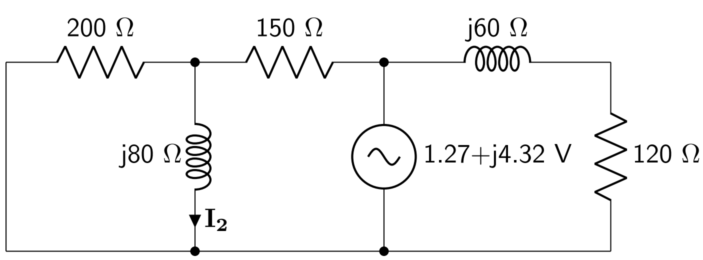
$$\begin{aligned}\mathbf{Z_{EQ}} &= ((200//j80) + 150)//(120 + j60)\\ &= (177.79 + j68.97)//(120 + j60)\\ &= 71.79 + j32.50\ \Omega\end{aligned}$$



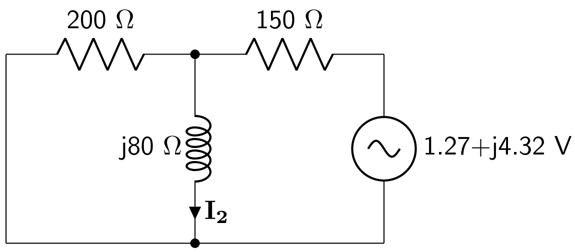
Use the complex voltage divider rule to calculate **V**.

$$\begin{aligned}\mathbf{V} &= (5 + j8.66\ \text{V})\left(\frac{71.79 + j32.50\ \Omega}{171.79 + j32.50\ \Omega}\right)\\ &= 1.27 + j4.32\ \text{V}\end{aligned}$$

Re-draw the reduced circuit.



The 120+j60 Ω branch, bering in parallel with the circuitry containing the inductor of interest, can be removed from the circuit in the interest of calculating voltage.



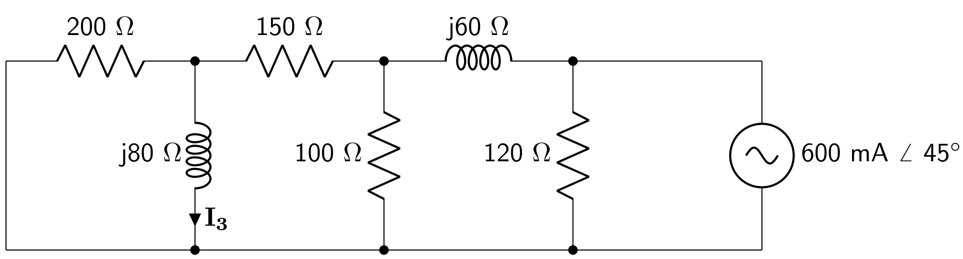
Use the complex voltage divider rule to calculate the voltage drop over the inductor. Units are volts and ohms.

$$\begin{aligned}\mathbf{V} &= (1.27 + j4.32)\left(\frac{200//j80}{150 + 200//j80}\right)\\ &= -0.90 + j1.51 \text{ V}\end{aligned}$$

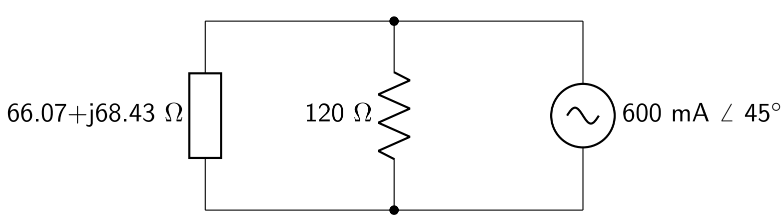
Use Ohm's law to calculate $\mathbf{I_2}$.

$$\begin{aligned}\mathbf{I_2} &= \frac{-0.90 + j1.51 \text{ V}}{j80 \ \Omega}\\ &= 18.90 + j11.19 \text{ mA}\end{aligned}$$

Calculate the current contributed by the current source.



Reduce to a circuit with two impedances.

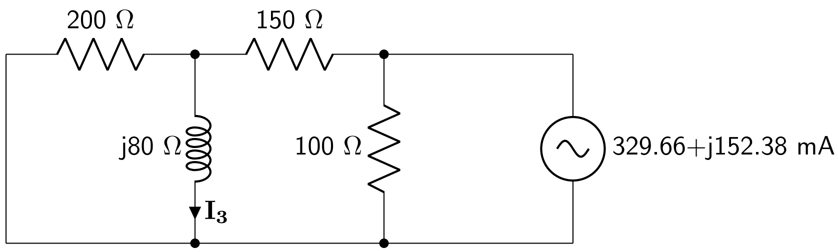


Use the complex current divider rule to calculate the current flowing through the 66.07+j68.43 Ω branch.

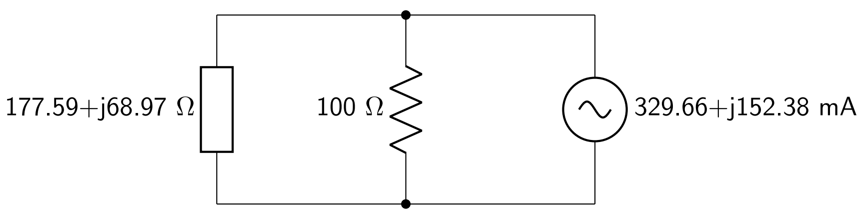
Units are mA and ohms.

$$\mathbf{I_X} = (424.26 + j424.26)\left(\frac{120//(66.07 + j68.43)}{66.07 + j68.43}\right)$$
$$= 329.66 + j152.38 \text{ mA}$$

Re-draw the circuit.



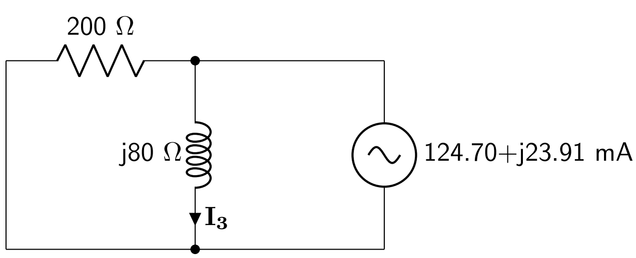
Reduce to a circuit with two impedances.



Use the complex current divider rule to calculate the current flowing through the 177.59+j68.97 Ω branch. Units are mA and ohms.

$$\mathbf{I_Y} = (329.66 + j152.38)\left(\frac{100//(177.59 + j68.97)}{177.59 + j68.97}\right)$$
$$= 124.70 + j23.91 \text{ mA}$$

Re-draw the circuit.



Use the complex current divider rule to calculate $\mathbf{I_3}$. Units are mA and ohms.

$$\begin{aligned}\mathbf{I_3} &= (124.70 + j23.91)\left(\frac{200//j80}{j80}\right)\\ &= 115.75 - j22.38 \text{ mA}\end{aligned}$$

Calculate $\mathbf{I}$.

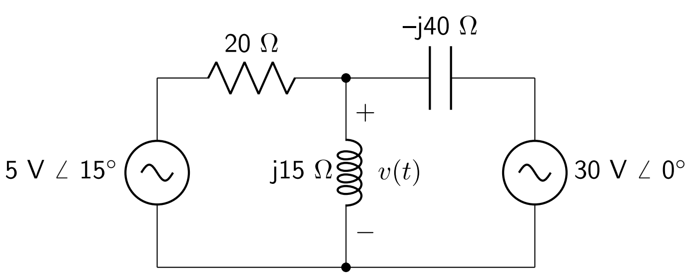
$$\begin{aligned}\mathbf{I} &= \mathbf{I_1} + \mathbf{I_2} + \mathbf{I_3}\\ &= (15.40 - j11.82 \text{ mA}) + (18.90 + j11.19 \text{ mA}) + (115.75 - j22.38 \text{ mA})\\ &= 150.05 - j23.02 \text{ mA}\\ &= 151.81 \text{ mA}\angle -8.72^\circ\end{aligned}$$

Convert to time-varying form.

$$i(t) = 151.81\cos(2\pi 20t - 8.72\pi/180) \text{ mA}$$

9.5 Complex Source Transformation

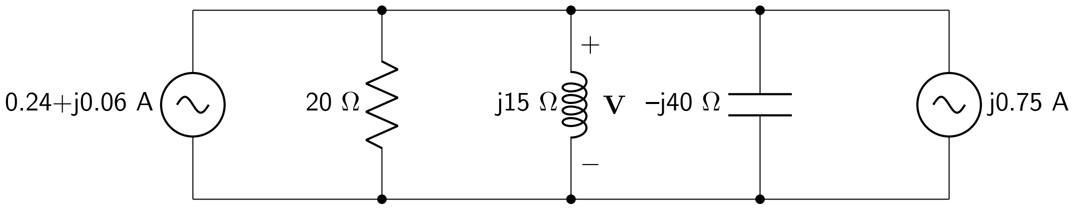
21. Use source transformation to calculate $v(t)$ in the circuit shown in figure 9.16 (in the complex superposition section).



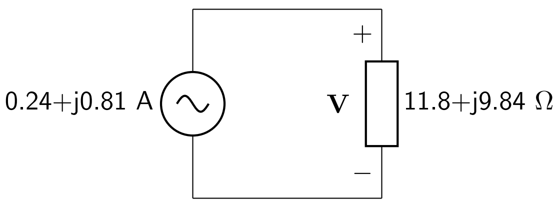
Convert both sources to current sources.

$$\begin{aligned}\mathbf{I_{S1}} &= \frac{5\cos(15^\circ) + j5\sin(15^\circ) \text{ V}}{20\ \Omega}\\ &= 0.24 + j0.06 \text{ A}\\ \mathbf{I_{S2}} &= \frac{30 \text{ V}}{-j40\ \Omega}\\ &= j0.75 \text{ A}\end{aligned}$$

Re-draw the circuit.



Combine sources and impedances.



Use Ohm's law to calculate **V**.

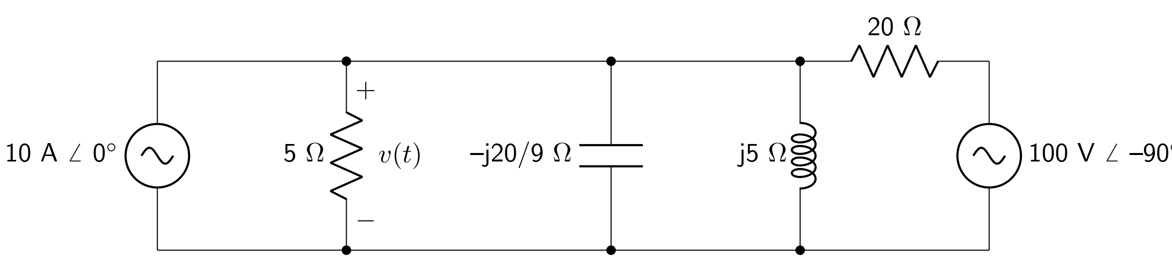
$$\begin{aligned}\mathbf{V} &= (0.24 + j0.81\text{ A})(11.80 + j9.84\ \Omega)\\ &= -5.16 + j11.99\text{ V}\\ &= 13.06\text{ V}\angle 113.30^\circ\end{aligned}$$

Convert to time-varying form.

$$v(t) = 13.06\cos(2\pi 10000t + 113.30\pi/180)\text{ V}$$

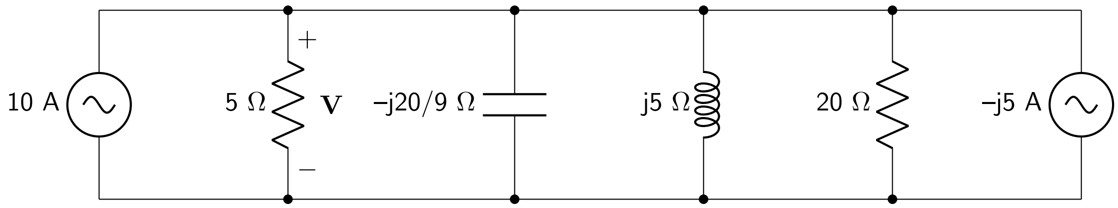
This answer should be and is identical to question 16 in the complex superposition section.

22. Use source transformation to calculate $v(t)$ in the circuit shown in figure 9.17 (in the complex superposition section).

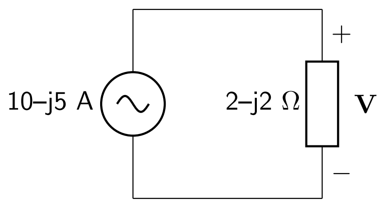


Convert the voltage source to a current source.

$$\begin{aligned}\mathbf{I_S} &= \frac{-j100\text{ V}}{20\ \Omega}\\ &= -j5\text{ A}\end{aligned}$$



Combine sources and impedances.



Use Ohm's law to calculate **V**.

$$\begin{aligned}\mathbf{V} &= (10 - j5 \text{ A})(2 - j2 \ \Omega) \\ &= 10 - j30 \text{ V} \\ &= 31.62 \text{ V}\angle - 71.57^\circ\end{aligned}$$

Convert to time-varying form.

$$v(t) = 31.62 \cos(2\pi 40t - 71.57\pi/180) \text{ V}$$

This answer should be and is identical to question 17 in the complex superposition section.

23. Use source transformation to calculate $i(t)$ for the circuit shown in figure 9.21. The frequency of operation is 10 Hz.

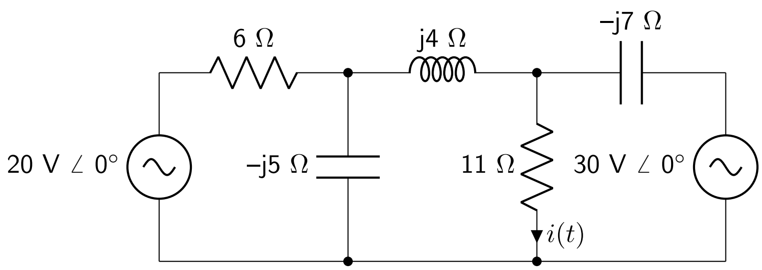
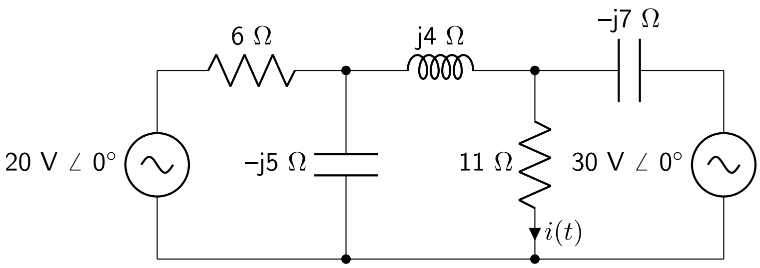


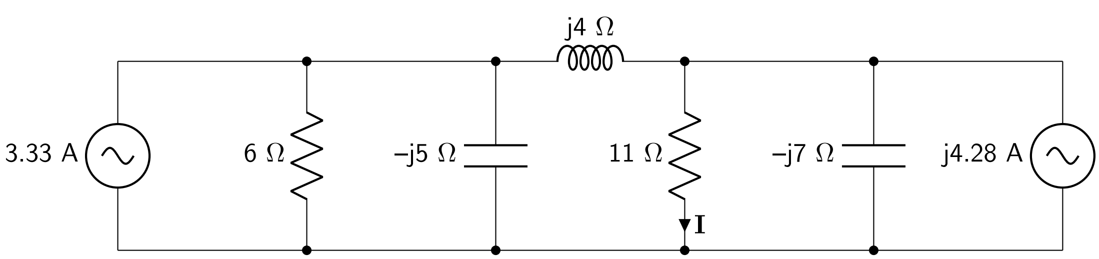
Figure 9.21: Circuit diagram for complex source transformation question 23.



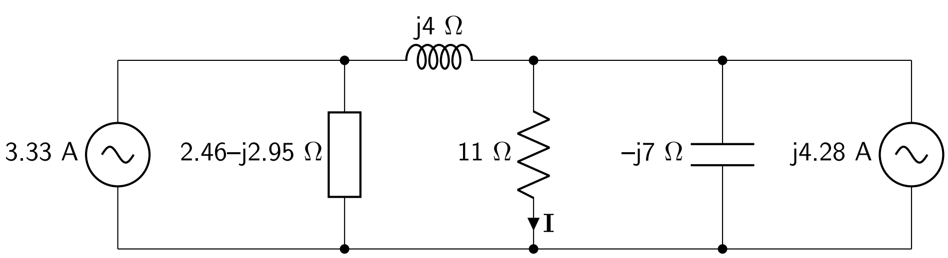
Convert both voltage sources into current sources.

$$\mathbf{I_1} = \frac{20\text{ V}}{6\ \Omega}$$
$$= 3.33\text{ A}$$
$$\mathbf{I_2} = \frac{30\text{ V}}{-j7\ \Omega}$$
$$= j4.28\text{ A}$$

Re-draw the circuit.



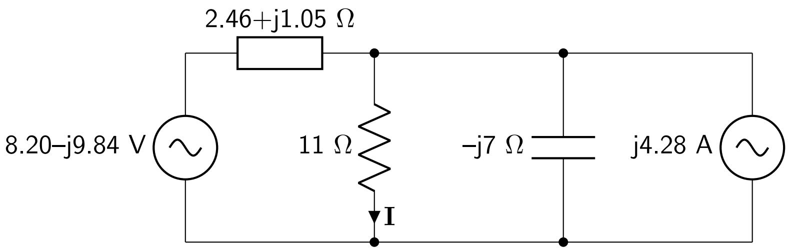
Combine impedances in parallel.



Convert the 3.33 A source to a voltage source.

$$\mathbf{V_S} = (3.33\text{ A})(2.46 - j2.95\ \Omega)$$
$$= 8.20 - j9.84\text{ V}$$

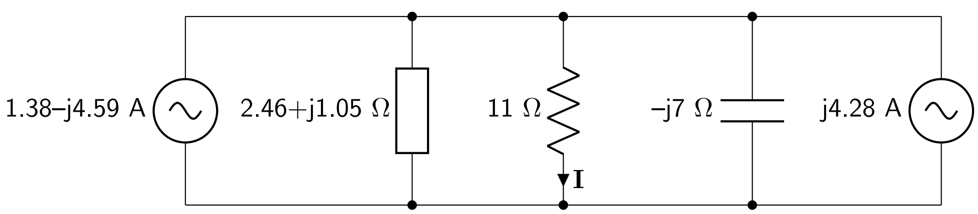
Re-draw the circuit. Combine series impedances.



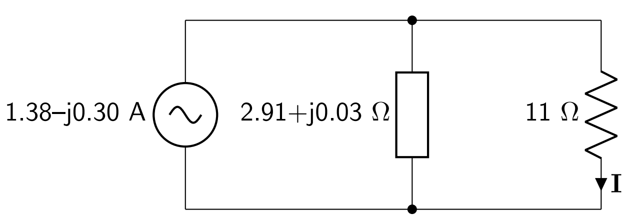
Convert the voltage source back to a current source.

$$\mathbf{I_S} = \frac{8.20 - j9.84 \text{ V}}{2.46 + j1.05 \ \Omega}$$
$$= 1.38 - j4.59 \text{ A}$$

Re-draw the circuit.



Combine current sources. Combine all impedances but the 11 Ω resistor.



Use the complex current divider rule to calculate **I**.

$$\mathbf{I} = (1.38 - j0.30 \text{ A}) \left(\frac{(2.91 + j0.03 \ \Omega)//(11 \ \Omega)}{11 \ \Omega} \right)$$
$$= 0.29 - j0.06 \text{ A}$$
$$= 288.18 - j60.40 \text{ mA}$$
$$= 294.44 \text{ mA}\angle - 11.84°$$

Convert to time-varying form.

$$i(t) = 294.44 \cos(2\pi 10t - 11.84\pi/180) \text{ mA}$$

24. Use source transformation to calculate $v(t)$ for the circuit shown in figure 9.22.

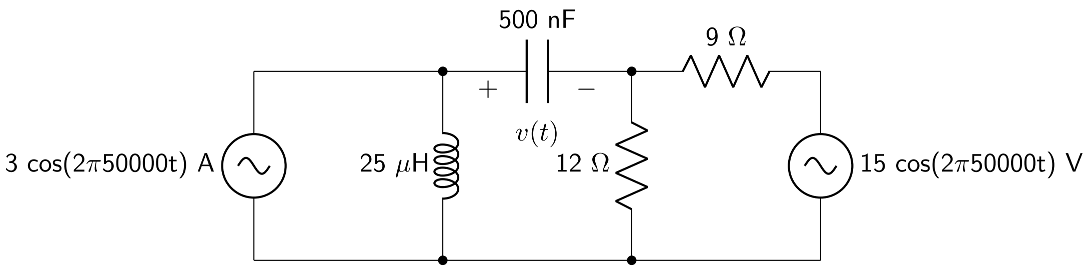
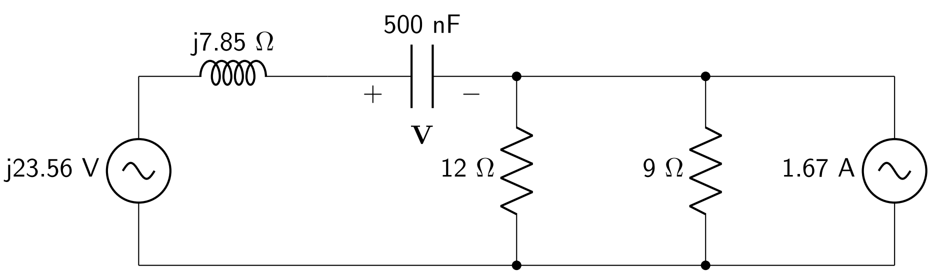


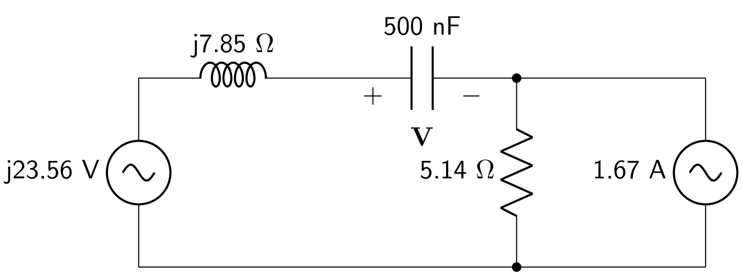
Figure 9.22: Circuit diagram for complex source transformation question 24.

Convert both sources.

$$\mathbf{I_S} = \frac{15 \text{ V}}{9 \text{ } \Omega}$$
$$= 1.67 \text{ A}$$
$$\mathbf{V_S} = (3 \text{ A})(j2\pi 50000)(25 \times 10^{-6}) \text{ } \Omega$$
$$= j23.56 \text{ V}$$



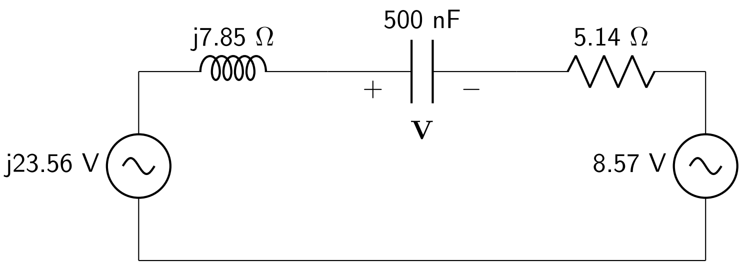
Combine parallel resistors.



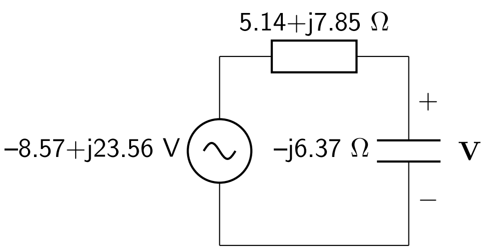
Convert the current source to a voltage source.

$$\mathbf{V_S} = (1.67 \text{ A})(5.14 \text{ } \Omega)$$
$$= 8.57 \text{ V}$$

Re-draw the circuit.



Combine the voltage sources and the inductor and resistor impedances.



Use the complex voltage divider rule to calculate **V**.

$$\begin{aligned}\mathbf{V} &= (-8.57 + j23.56 \text{ V})\left(\frac{-j6.37\ \Omega}{(-j6.37\ \Omega) + (5.14 + j7.85\ \Omega)}\right)\\ &= 29.75 + j2.00 \text{ V}\\ &= 29.81 \text{ V}\angle 3.86^\circ\end{aligned}$$

Convert to time-varying form.

$$v(t) = 29.81\cos(2\pi 50000t + 3.86\pi/180) \text{ V}$$

25. Use source transformation to calculate $v(t)$ for the circuit shown in figure 9.23.

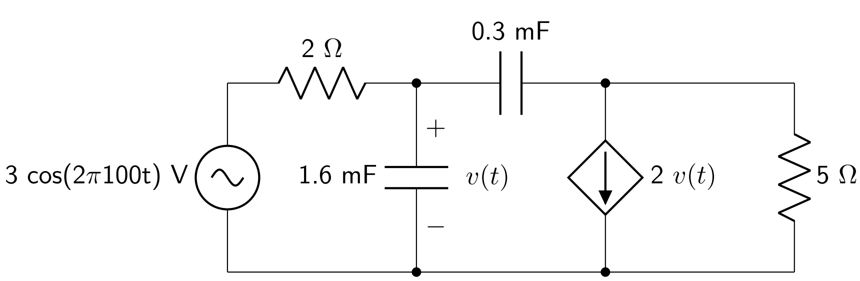
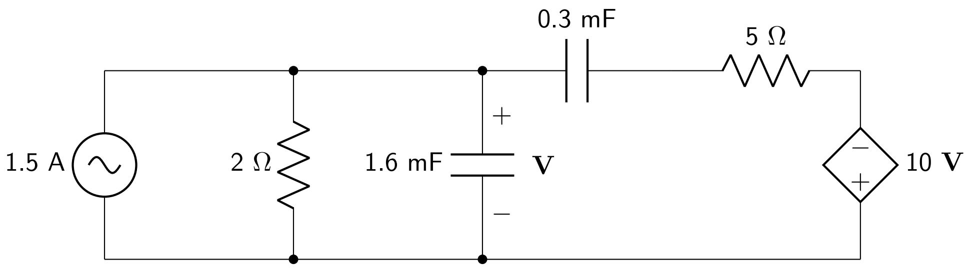


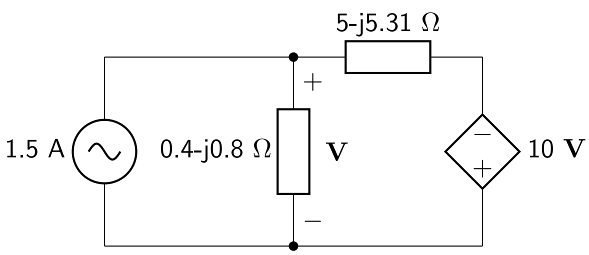
Figure 9.23: Circuit diagram for complex source transformation question 25.

Convert both sources.

$$\mathbf{I_S} = \frac{3 \text{ V}}{2 \ \Omega}$$
$$= 1.5 \text{ A}$$
$$\mathbf{V_S} = (-2\mathbf{V})(5)$$
$$= -10\mathbf{V}$$

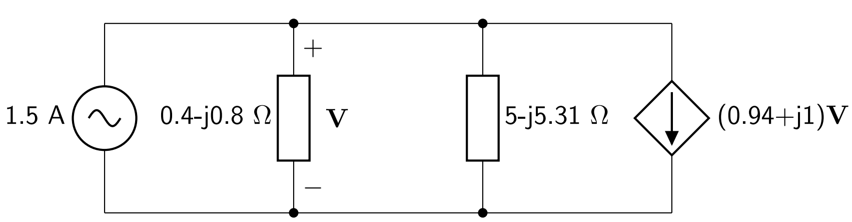


Combine impedances.

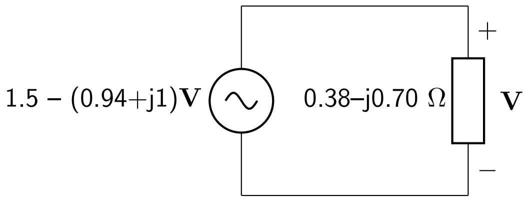


Convert the dependent source to a VCCS.

$$\mathbf{I_S} = \frac{10\mathbf{V}}{5 - j5.31}$$
$$= (0.94 + j1.00)\mathbf{V}$$



Combine sources and impedances.



Use Ohm's law to calculate **V**.

$$\begin{aligned}
\mathbf{V} &= (0.38 - j0.70)(1.5 - (0.94 + j1)\mathbf{V}) \\
&= (0.57 - j1.05) - (1.06 - j0.28)\mathbf{V} \\
(2.06 - j0.28)\mathbf{V} &= (0.57 - j1.05) \\
\mathbf{V} &= \frac{(0.57 - j1.05)}{(2.06 - j0.28)} \\
&= 0.34 - j0.47 \text{ V} \\
&= 339.57 - j465.34 \text{ mV} \\
&= 576.06 \text{ mV}\angle - 53.88^\circ
\end{aligned}$$

Convert to time-varying from.

$$v(t) = 576.06 \cos(2\pi 100t - 53.88\pi/180) \text{ mV}$$

9.6 Complex Thévenin and Norton's Theorems

26. Derive the Thévenin equivalent circuit between nodes a and b in the circuit shown in figure 9.24. The frequency of operation is 50 Hz.

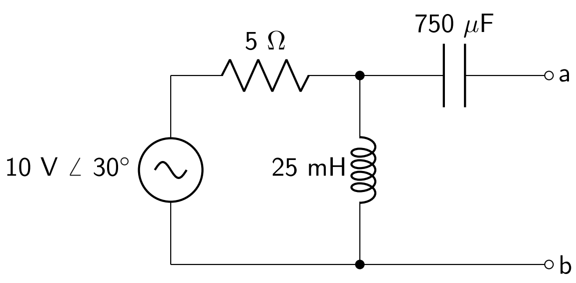
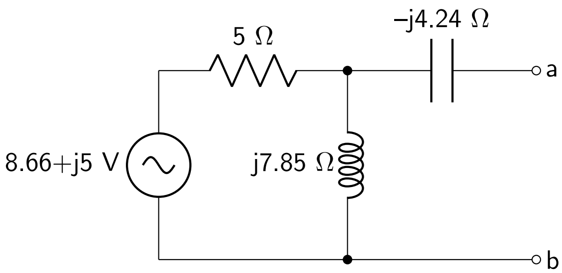


Figure 9.24: Circuit diagram for complex Thévenin and Norton theorems question 26.

Phasor transform the circuit.



Use the complex voltage divider rule to calculate $\mathbf{V_{TH}}$.

$$\begin{aligned}\mathbf{V_{TH}} &= (8.66 + j5 \text{ V})\left(\frac{j7.85 \ \Omega}{5 + j7.85 \ \Omega}\right) \\ &= 3.90 + j7.48 \text{ V} \\ &= 8.44 \text{ V}\angle 62.48°\end{aligned}$$

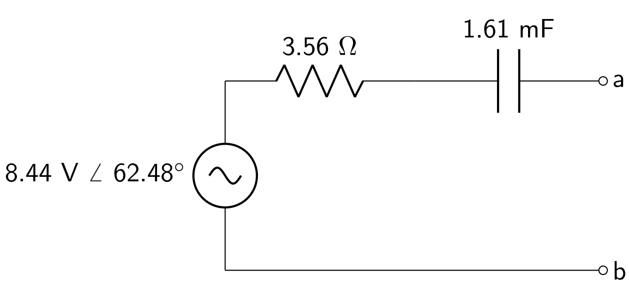
Deactivate the source and calculate the equivalent impedance of the circuit.

$$\begin{aligned}\mathbf{Z_{TH}} &= (-j4.24 \ \Omega) + (5 \ \Omega)//(j7.85 \ \Omega) \\ &= (-j4.24 \ \Omega) + (3.56 + j2.27 \ \Omega) \\ &= 3.56 - j1.98 \ \Omega\end{aligned}$$

Calculate the capacitance from the reactance.

$$\begin{aligned}C &= -\frac{1}{2\pi f X} \\ &= -\frac{1}{2\pi 50(-1.98)} \\ &= 1.61 \text{ mF}\end{aligned}$$

Draw the Thévenin equivalent circuit.



27. Derive the Thévenin equivalent circuit between nodes a and b in the circuit shown in figure 9.25.

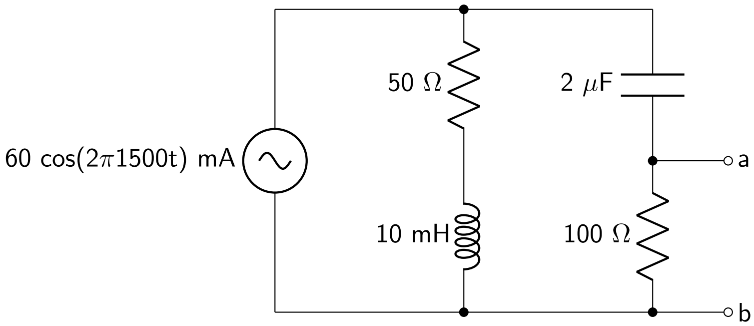
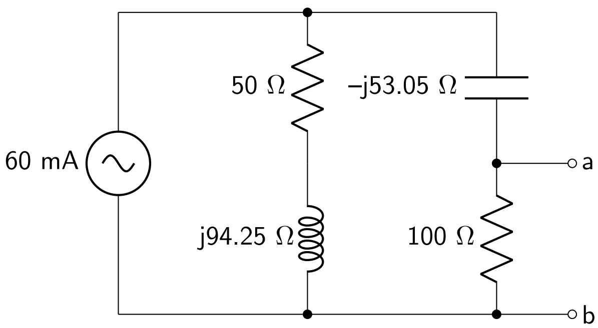


Figure 9.25: Circuit diagram for complex Thévenin and Norton theorems question 27.

Phasor transform the circuit.



Use the complex current divider rule to calculate the current flow through the 100 Ω resistor.

$$\mathbf{I_X} = 60 \text{ mA} \left(\frac{(50 + j94.25)//(100 - j53.05)}{100 - j53.05} \right)$$
$$= 28.22 + j29.95 \text{ mA}$$

Use Ohm's law to calculate the Thévenin equivalent voltage.

$$\mathbf{V_{TH}} = (28.22 + j29.95 \text{ mA})(0.1 \text{ k}\Omega)$$
$$= 2.82 + j2.99 \text{ V}$$
$$= 4.12 \text{ V}\angle 46.70°$$

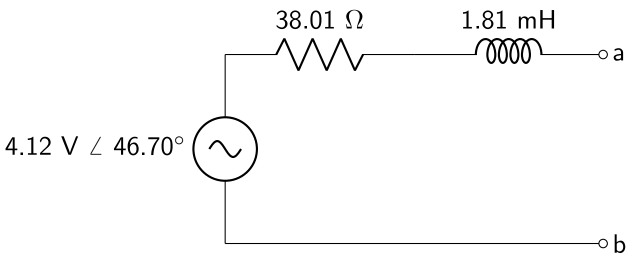
Deactivate the source and calculate the equivalent impedance.

$$\mathbf{Z_{TH}} = (100\ \Omega)//(50\ \Omega + j94.25\ \Omega - 53.05\ \Omega)$$
$$= 38.01 + j17.03\ \Omega$$

Calculate the inductance from the reactance.

$$L = \frac{X}{2\pi f}$$
$$= \frac{17.03}{2\pi 50}$$
$$= 1.81 \text{ mH}$$

Draw the Thévenin equivalent circuit.



28. Derive the Thévenin equivalent circuit between nodes a and b in the circuit shown in figure 9.26. The frequency of operation is 2 kHz.

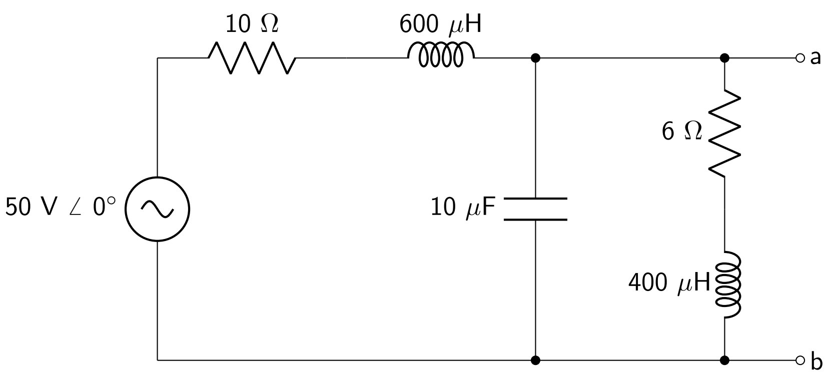
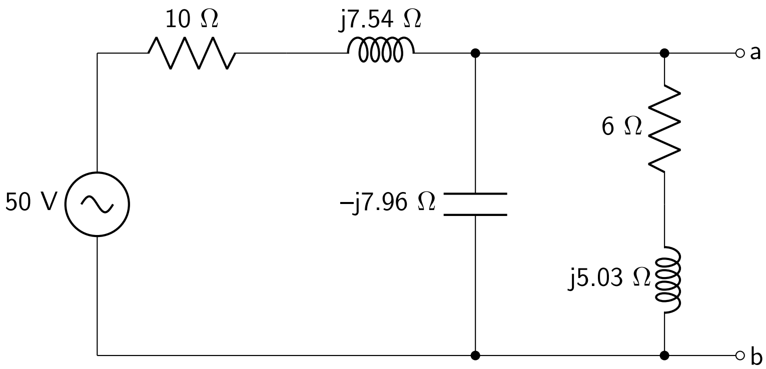
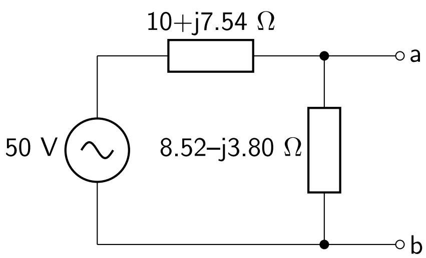


Figure 9.26: Circuit diagram for complex Thévenin and Norton theorems question 28.

Phasor transform the circuit.



Combine impedances.



Use the complex voltage divider rule to calculate $\mathbf{V_{TH}}$.

$$\begin{aligned}\mathbf{V_{TH}} &= 50\ \text{V}\left(\frac{8.52 - j3.80\ \Omega}{(10 + j7.54\ \Omega) + (8.52 - j3.80\ \Omega)}\right)\\ &= 20.11 - j14.31\ \text{V}\\ &= 24.68\ \text{V}\angle - 35.44^\circ\end{aligned}$$

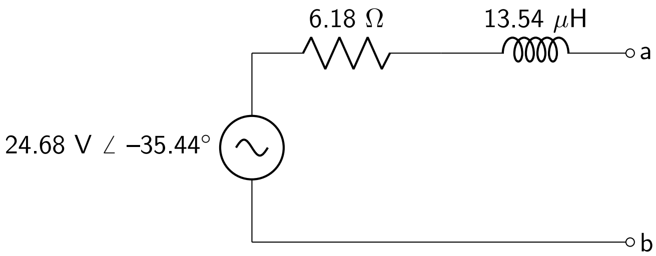
Deactivate the source and calculate the equivalent impedance.

$$\begin{aligned}\mathbf{Z_{TH}} &= (10 + j7.54\ \Omega)//(8.52 - j3.80\ \Omega)\\ &= 6.18 + j0.17\ \Omega\end{aligned}$$

Calculate the inductance from the reactance.

$$\begin{aligned}L &= \frac{X}{2\pi f}\\ &= \frac{0.17}{2\pi 2000}\\ &= 13.54\ \mu\text{H}\end{aligned}$$

Draw the Thévenin equivalent circuit.



29. Derive the Norton equivalent circuit between nodes a and b in the circuit shown in figure 9.27. The frequency of operation is 60 Hz.

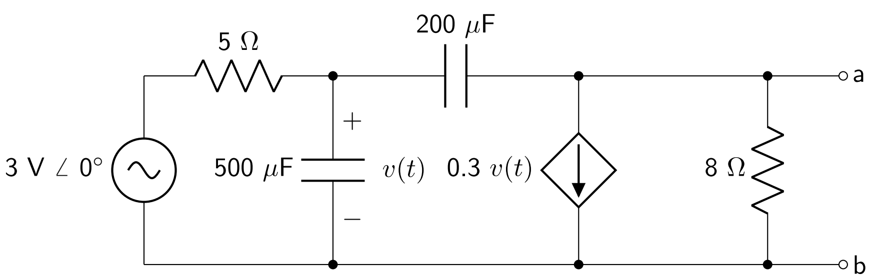
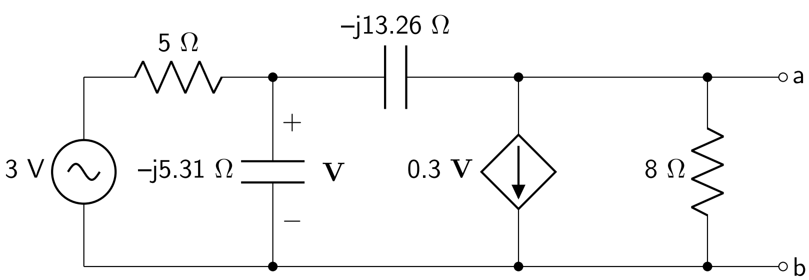
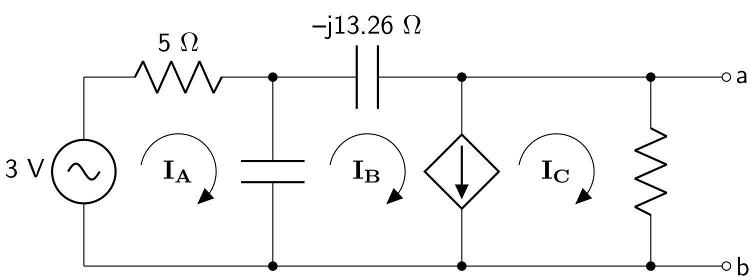


Figure 9.27: Circuit diagram for complex Thévenin and Norton theorems question 29.

Phasor transform the circuit.



Perform complex mesh analysis. The circuit drawn below may have component values hidden so that the mesh current labels can be read.



Derive the mesh equations.

$$3 = 5\mathbf{I_A} + -j5.31(\mathbf{I_A} - \mathbf{I_B})$$
$$0 = -j5.31(\mathbf{I_B} - \mathbf{I_A}) - j13.26\mathbf{I_B} + 8\mathbf{I_C}$$

Perform KCL at the supermesh.

$$0 = \mathbf{I_B} - 0.3\mathbf{V} - \mathbf{I_C}$$

Derive a dependent source equation.

$$\mathbf{Z} = -j5.31(\mathbf{I_A} - \mathbf{I_B})$$

Place the equations into form $\alpha\mathbf{I_A} + \beta\mathbf{I_B} + \gamma\mathbf{I_C} + \delta\mathbf{V} = \mathbf{c}$, and then place each coefficient into a matrix.

$$\begin{bmatrix} 5 - j5.31 & j5.31 & 0 & 0 & 3 \\ j5.31 & -j18.57 & 8 & 0 & 0 \\ 0 & 1 & -1 & -0.3 & 0 \\ j5.31 & -j5.31 & 0 & 1 & 0 \end{bmatrix}$$

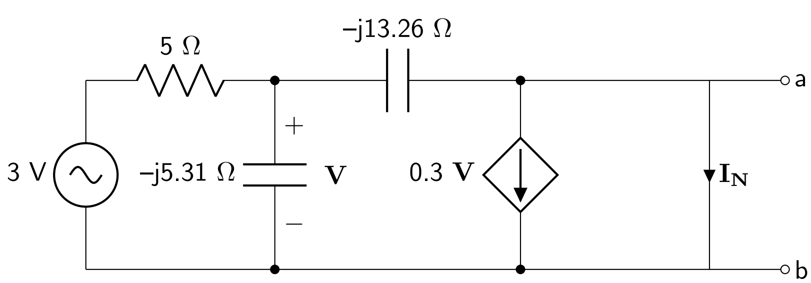
Solve the matrix for $\mathbf{I_C}$.

$$\mathbf{I_B} = 0.03 + j0.32 \text{ A}$$

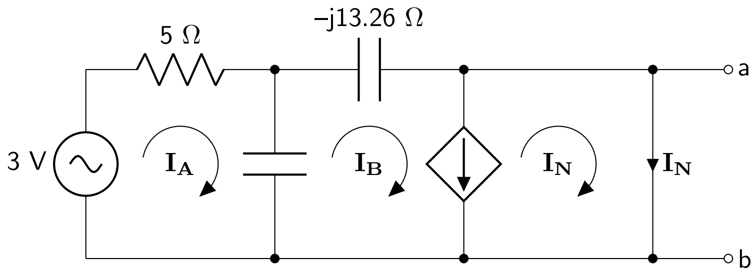
Calculate the open-circuit voltage.

$$\mathbf{V_{OC}} = (8\ \Omega)(0.03 + j0.32 \text{ A})$$
$$= 0.25 + j2.58 \text{ V}$$

Short terminals a and b to calculate the Norton equivalent current.



Perform complex mesh analysis. The circuit drawn below may have component values hidden so that the mesh current labels can be read.



Derive the mesh equations.

$$3 = 5\mathbf{I_A} + -j5.31(\mathbf{I_A} - \mathbf{I_B})$$
$$0 = -j5.31(\mathbf{I_B} - \mathbf{I_A}) - j13.26\mathbf{I_B}$$

Perform KCL at the supermesh.

$$0 = \mathbf{I_B} - 0.3\mathbf{V} - \mathbf{I_N}$$

Derive a dependent source equation.

$$\mathbf{Z} = -j5.31(\mathbf{I_A} - \mathbf{I_B})$$

Place the equations into form $\alpha\mathbf{I_A} + \beta\mathbf{I_B} + \gamma\mathbf{I_N} + \delta\mathbf{V} = \mathbf{c}$, and then place each coefficient into a matrix.

$$\begin{bmatrix} 5 - j5.31 & j5.31 & 0 & 0 & 3 \\ j5.31 & -j18.57 & 0 & 0 & 0 \\ 0 & 1 & -1 & -0.3 & 0 \\ j5.31 & -j5.31 & 0 & 1 & 0 \end{bmatrix}$$

Solve the matrix for $\mathbf{I_N}$.

$$\mathbf{I_N} = -0.22 + j0.52 \text{ A}$$
$$= 560.52 \text{ mA}\angle 113.05^\circ$$

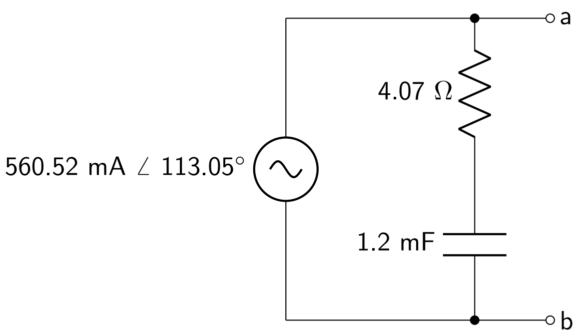
Calculate the norton equivalent impedance.

$$\mathbf{Z_N} = \frac{0.25 + j2.58 \text{ V}}{-0.22 + j0.52 \text{ A}}$$
$$= 4.07 - j2.21\ \Omega$$

Calculate the capacitance from the reactance.

$$\begin{aligned} C &= -\frac{1}{2\pi f X} \\ &= -\frac{1}{2\pi(60)(-2.21)} \\ &= 1.2 \text{ mF} \end{aligned}$$

Draw the Norton equivalent circuit.



30. Derive the Norton equivalent circuit between nodes a and b in the circuit shown in figure 9.28. The frequency of operation is 300 Hz.

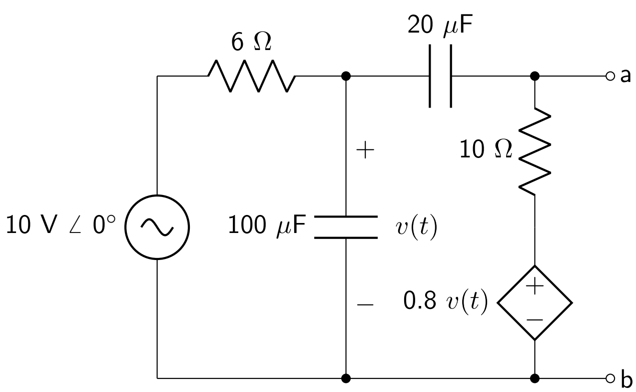
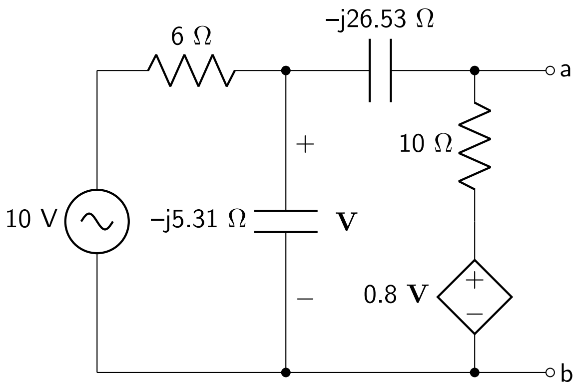


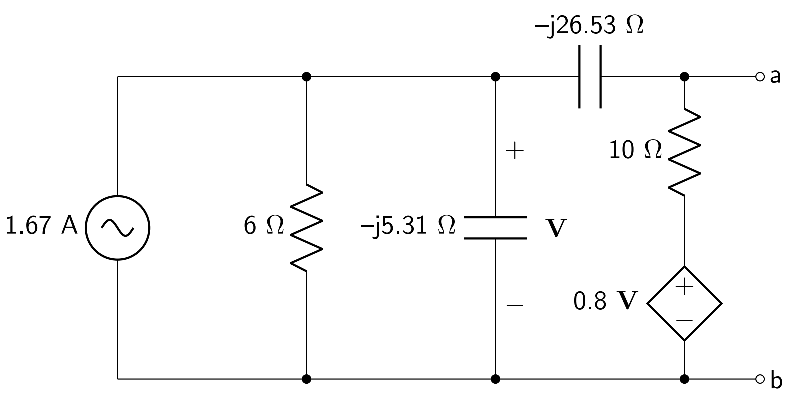
Figure 9.28: Circuit diagram for complex Thévenin and Norton theorems question 30.

Phasor transform the circuit.

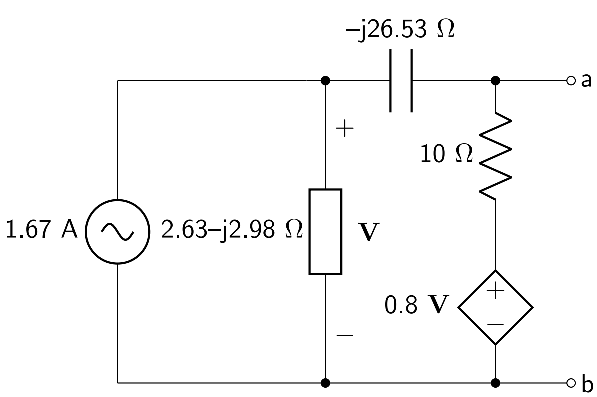


Transform the voltage source to a current source.

$$\mathbf{I_S} = \frac{10\text{ V}}{6\ \Omega}$$
$$= 1.67\text{ A}$$

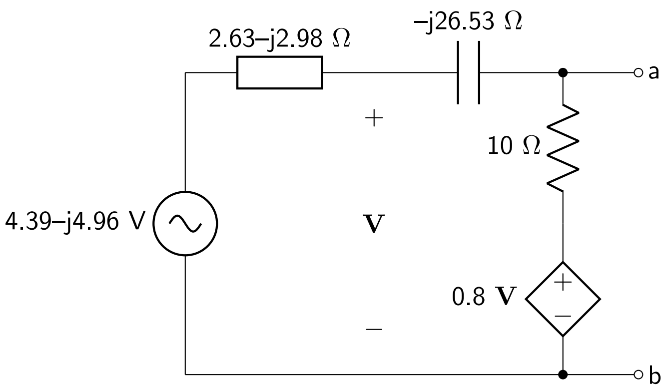


Combine parallel impedances.



Convert the current source to a voltage source.

$$\begin{aligned}\mathbf{V_S} &= (1.67\text{ A})(2.63 - j2.98\ \Omega)\\ &= 4.39 - j4.96\text{ V}\end{aligned}$$



Perform KVL around the entire loop. Then perform KVL around the left side to calculate $\mathbf{V}$. Obtain two equations with two unknowns. All units are V, A, and Ω.

$$\begin{aligned}(4.39 - j4.96) &= (2.63 - j2.98 - j26.53 + 10)\mathbf{I} + 0.8\mathbf{V}\\ &= (2.63 - j2.98)\mathbf{I} + \mathbf{V}\end{aligned}$$

Place the equations into form $\alpha\mathbf{V} + \beta\mathbf{I} = \mathbf{c}$, and then place each coefficient into a matrix.

$$\begin{bmatrix} 0.8 & 12.63 - j29.50 & 4.39 - j4.96\\ 1 & 2.63 - j2.98 & 4.39 - j4.96 \end{bmatrix}$$

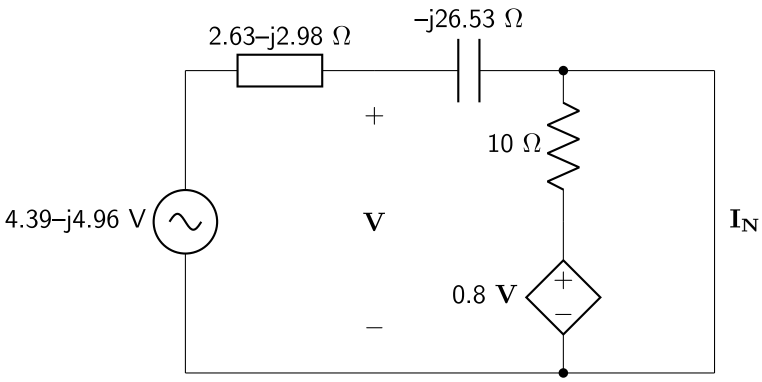
Solve the matrix for $\mathbf{V}$ and $\mathbf{I}$.

$$\begin{aligned}\mathbf{V} &= 4.23 - j4.88\text{ V}\\ \mathbf{I} &= 0.04 + j0.016\text{ A}\end{aligned}$$

Calculate the open-circuit voltage.

$$\begin{aligned}\mathbf{V_{OC}} &= 0.8\mathbf{V} + 10\mathbf{I}\\ &= 0.8(4.23 - j4.88\text{ V}) + 10(0.04 + j0.016\text{ A})\\ &= 3.81 - j3.74\text{ V}\end{aligned}$$

Short nodes a and b to calculate the Norton equivalent current.



Perform KVL three times: around the left half of the left loop, around the right half of the left loop, and around the right loop. All units are V, A, and Ω.

$$(4.39 - j4.96) = (2.63 - j2.98)\mathbf{I} + \mathbf{V}$$
$$0.8\mathbf{V} = 10(\mathbf{I_N} - \mathbf{I})$$
$$\mathbf{V} = -j26.53\mathbf{I}$$

Place the equations into form $\alpha\mathbf{I} + \beta\mathbf{I_N} + \gamma\mathbf{V} = \mathbf{c}$, and then place each coefficient into a matrix.

$$\begin{bmatrix} -j26.53 & 0 & -1 & 0 \\ 10 & -10 & 0.8 & 0 \\ (2.63 - j2.98) & 0 & 1 & (4.39 - j4.96) \end{bmatrix}$$

Solve the matrix for $\mathbf{I_N}$.

$$\mathbf{I_N} = 461.54 - j249.39 \text{ mA}$$
$$= 524.61 \text{ mA}\angle - 28.38°$$

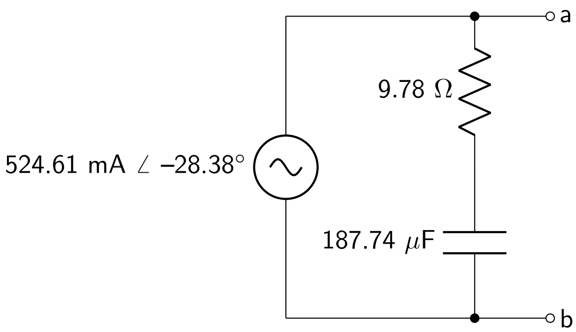
Calculate the Norton equivalent impedance.

$$\mathbf{Z_N} = \frac{3.81 - j3.74 \text{ V}}{0.46 - j0.25 \text{ A}}$$
$$= 9.78 - j2.23\ \Omega$$

Calculate the capacitance from the reactance.

$$
\begin{aligned}
C &= -\frac{1}{2\pi f X} \\
&= -\frac{1}{2\pi(300)(-2.23)} \\
&= 187.74\ \mu\text{F}
\end{aligned}
$$

Draw the Norton equivalent circuit.



9.7 Filters

31. Given the circuit shown in figure 9.29, determine the filter type, calculate the center frequency, bandwidth, and quality factor.

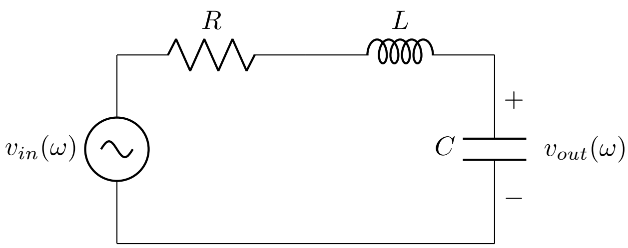
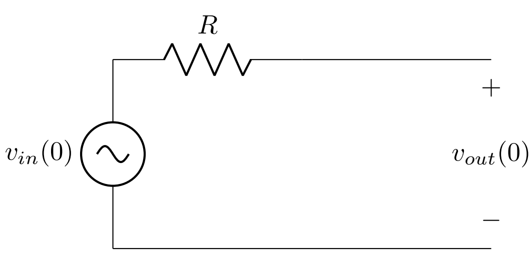
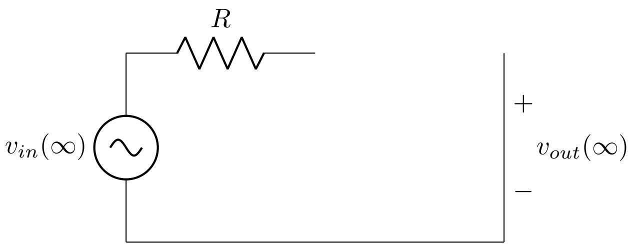


Figure 9.29: Circuit diagram for filters question 31.

Draw the circuit at $\omega = 0$.



Draw the circuit at $\omega = 0$.



This is an LPF. This is a series RLC circuit with a homogeneous differential equation shown below.

$$0 = \frac{d^2v(t)}{dt^2} + \frac{R}{L}\frac{dv(t)}{dt} + \frac{1}{LC}v(t)$$

The center frequency is equal to the resonant frequency.

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

The bandwidth equals a_1.

$$\beta = \frac{R}{L}$$

Calculate the quality factor.

$$\begin{aligned} Q &= \frac{\omega_0}{a_1} \\ &= \frac{\frac{1}{\sqrt{LC}}}{\frac{R}{L}} \\ &= \frac{L}{R\sqrt{LC}} \\ &= \frac{\sqrt{L}}{R\sqrt{C}} \end{aligned}$$

32. Given the circuit shown in figure 9.30, determine the filter type, calculate the center frequency, bandwidth, and quality factor.

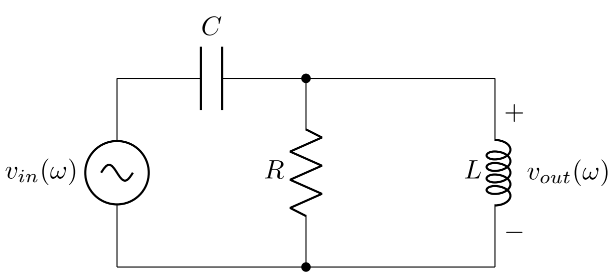
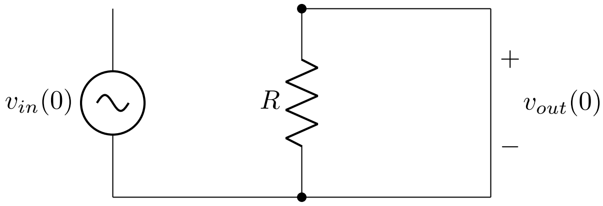
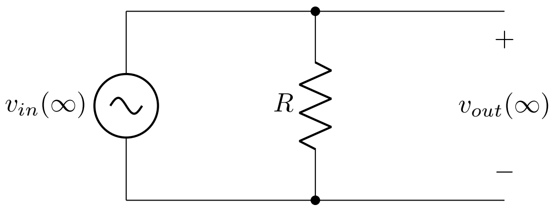


Figure 9.30: Circuit diagram for filters question 32.

Draw the circuit at $\omega = 0$.



Draw the circuit at $\omega \to \infty$.



This is an HPF. This is a parallel RLC circuit with a homogeneous differential equation shown below.

$$0 = \frac{d^2 i(t)}{dt^2} + \frac{1}{RC}\frac{di(t)}{dt} + \frac{1}{LC} i(t)$$

The center frequency is equal to the resonant frequency.

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

The bandwidth equals a_1.

$$\beta = \frac{1}{RC}$$

Calculate the quality factor.

$$\begin{aligned} Q &= \frac{\omega_0}{a_1} \\ &= \frac{\frac{1}{\sqrt{LC}}}{\frac{1}{RC}} \\ &= \frac{RC}{\sqrt{LC}} \\ &= \frac{R\sqrt{C}}{\sqrt{L}} \end{aligned}$$

33. Given the circuit shown in figure 9.31, determine the filter type, calculate the center frequency, bandwidth, and quality factor.

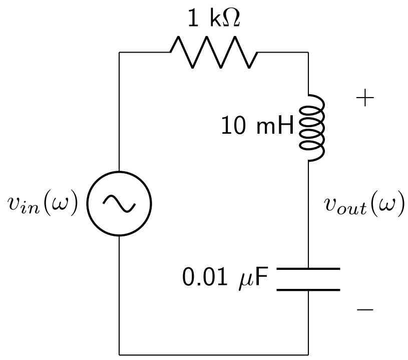
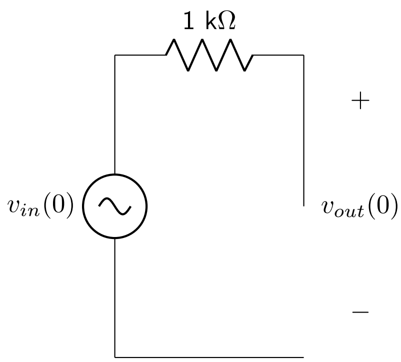
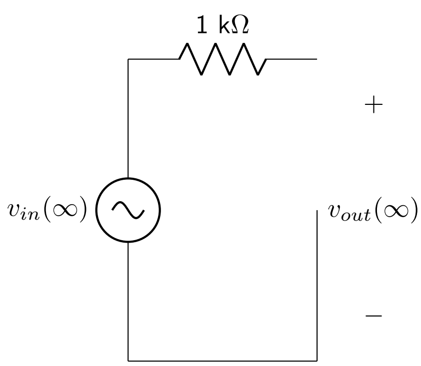


Figure 9.31: Circuit diagram for filters question 33.

Draw the circuit at $\omega = 0$.



Draw the circuit at $\omega \to \infty$.



This is a a BSF. This is a series RLC circuit with a homogeneous differential equation shown below.

$$0 = \frac{d^2v(t)}{dt^2} + \frac{R}{L}\frac{dv(t)}{dt} + \frac{1}{LC}v(t)$$

Plug in component values.

$$0 = \frac{d^2v(t)}{dt^2} + 100000\frac{dv(t)}{dt} + 1 \times 10^{10}v(t)$$

The center frequency is equal to the resonant frequency.

$$\omega_0 = 100000 \text{ rad/s}$$

The bandwidth equals a_1.

$$\beta = 100000 \text{ rad/s}$$

Calculate the quality factor.

$$\begin{aligned} Q &= \frac{\omega_0}{a_1} \\ &= \frac{100000 \text{ rad/s}}{100000 \text{ rad/s}} \\ &= 1 \end{aligned}$$

34. Given the circuit shown in figure 9.32, determine the filter type and calculate the cutoff frequency.

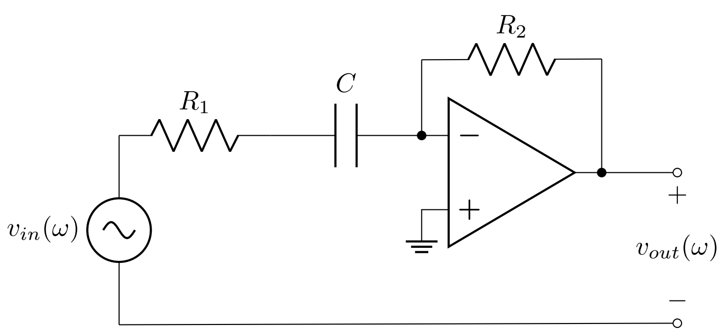
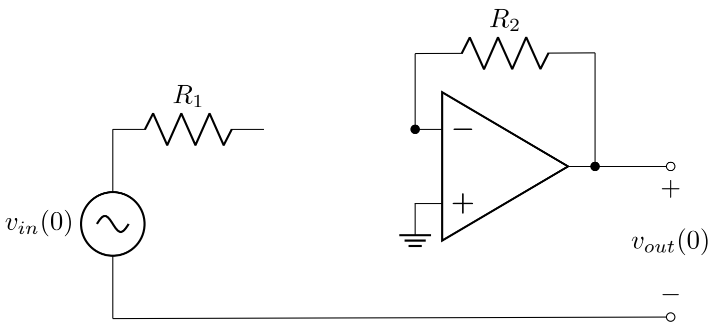
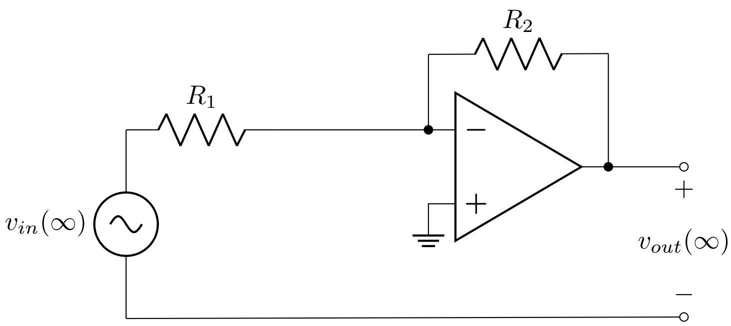


Figure 9.32: Circuit diagram for filters question 34.

Draw the circuit at $\omega = 0$.



Draw the circuit at $\omega \to \infty$.



This is a an HPF. Perform KVL in the loop including the source, R_1 and the capacitor. The voltage between the resistor and capacitor is labeled $v(t)$.

$$v_{in}(t) = R_1 C \frac{d}{dt} v(t) + v(t)$$

Perform KCL at the inverting node and solve for $v(t)$.

$$\begin{aligned} C\frac{d}{dt}v(t) &= -\frac{1}{R_2}v_{out}(t) \\ v(t) &= -\frac{1}{R_2 C}\int v_{out}(t) \end{aligned}$$

Plug $v(t)$ into the KVL equation and normalize the first order differential equation.

$$\begin{aligned} v_{in}(t) &= R_1 C\frac{d}{dt}\left[-\frac{1}{R_2 C}\int v_{out}(t)\right] + \left[-\frac{1}{R_2 C}\int v_{out}(t)\right] \\ &= -\frac{R_1}{R_2}v_{out}(t) - \frac{1}{R_2 C}\int v_{out}(t) \\ \frac{d}{dt}v_{in}(t) &= -\frac{R_1}{R_2}\frac{d}{dt}v_{out}(t) - \frac{1}{R_2 C} \\ -\frac{R_2}{R_1}\frac{d}{dt}v_{in}(t) &= \frac{d}{dt}v_{out}(t) + \frac{1}{R_1 C}v_{out}(t) \end{aligned}$$

The cutoff frequency is equal to a_0.

$$\omega_0 = \frac{1}{R_1 C}$$

35. Given the circuit shown in figure 9.33, determine the filter type and calculate the cutoff frequency.

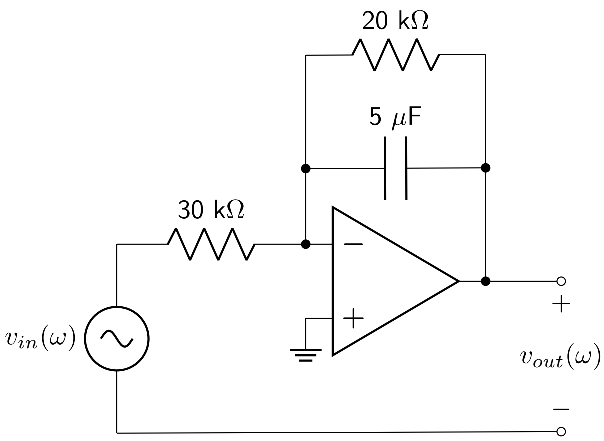
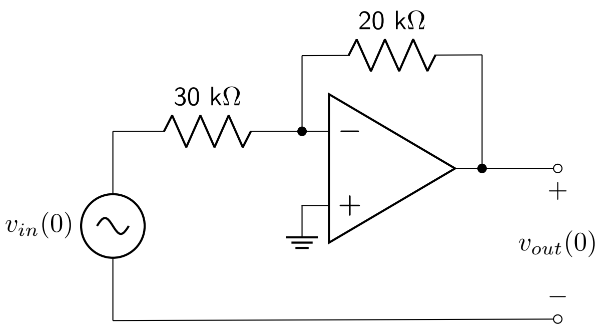
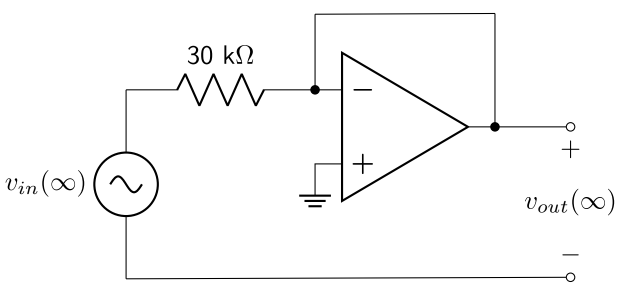


Figure 9.33: Circuit diagram for filters question 35.

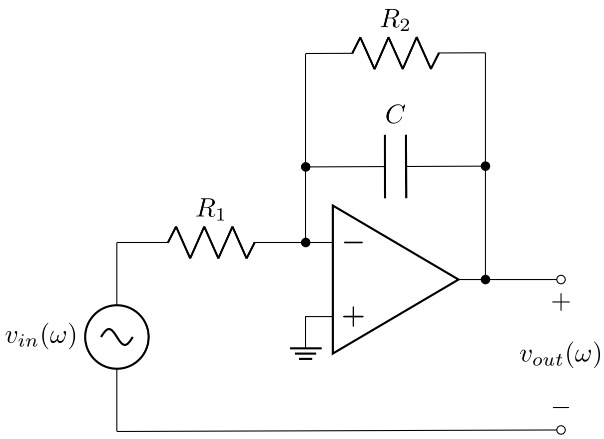
Draw the circuit at $\omega = 0$.



Draw the circuit at $\omega \to \infty$.



This is a an LPF. Replace component values with symbols.



Perform KCL at the inverting node and normalize the first order differential equation.

$$\frac{1}{R_1}v_{in}(t) = -C\frac{d}{dt}v_{out}(t) - \frac{1}{R_2}v_{out}(t)$$
$$-\frac{1}{R_1C}\frac{d}{dt}v_{in}(t) = \frac{d}{dt}v_{out}(t) + \frac{1}{R_2C}v_{out}(t)$$

The cutoff frequency is equal to a_0. Plug in component values.

$$\begin{aligned}\omega_0 &= \frac{1}{R_2C}\\ &= \frac{1}{(20000\ \Omega)(5\times10^{-6}\ \text{F})}\\ &= 10\ \text{rad/s}\end{aligned}$$

9.8 Transfer Functions

36. Derive a transfer function for the circuit shown in figure 9.34. Then, determine the filter type.

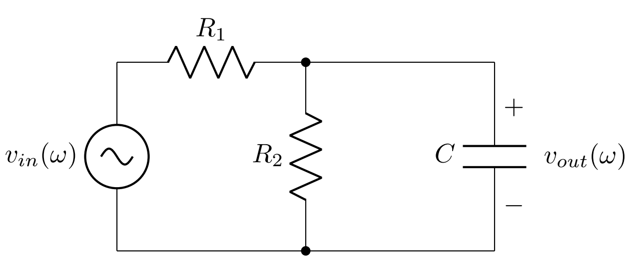


Figure 9.34: Circuit diagram for transfer functions question 36.

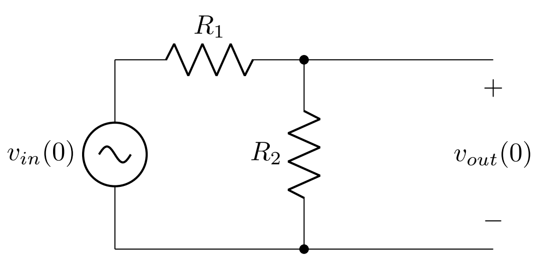
Calculate the equivalent impedance of R_2 in parallel with C.

$$\begin{aligned} Z_{EQ} &= R_2//C \\ &= R_2//\frac{1}{j\omega C} \\ &= \frac{\frac{R_2}{j\omega C}}{R_2+\frac{1}{j\omega C}} \\ &= \frac{R_2}{j\omega C R_2+1} \end{aligned}$$

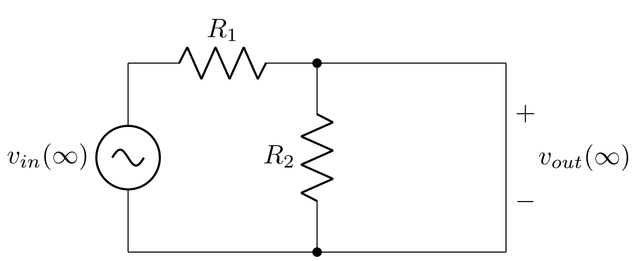
Use the complex voltage divider rule to calculate $H(\omega)$ and normalize.

$$\begin{aligned} H(\omega) &= \frac{\frac{R_2}{j\omega C R_2+1}}{R_1+\frac{R_2}{j\omega C R_2+1}} \\ &= \frac{R_2}{j\omega C R_2 R_1 + R_1 + R_2} \\ &= \frac{\frac{1}{CR_1}}{j\omega + \frac{R_1+R_2}{CR_1R_2}} \end{aligned}$$

Analyze the circuit at $\omega = 0$.



Analyze the circuit at $\omega \to \infty$.



This circuit is an LPF.

37. Derive a transfer function for the circuit shown in figure 9.35. Then, determine the filter type.

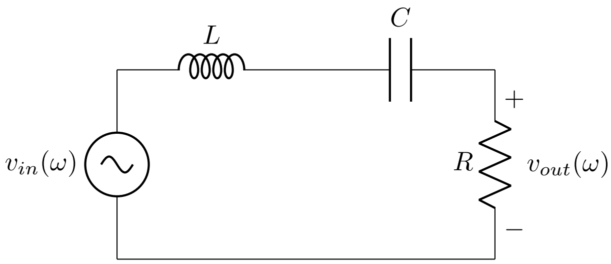
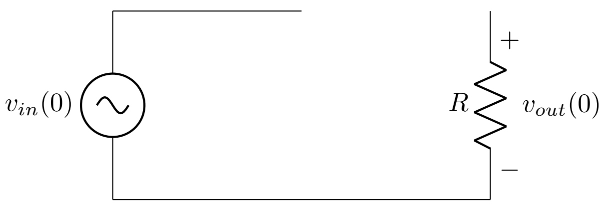


Figure 9.35: Circuit diagram for transfer functions question 37.

Use the complex voltage divider rule to calculate $H(\omega)$.

$$\begin{aligned} H(\omega) &= \frac{R}{R + j\omega L + \frac{1}{j\omega C}} \\ &= \frac{j\omega CR}{j\omega CR - \omega^2 LC + 1} \\ &= \frac{j\omega \frac{R}{L}}{-\omega^2 + j\omega \frac{R}{L} + \frac{1}{LC}} \end{aligned}$$

Analyze the circuit at $\omega = 0$.



Analyze the circuit at $\omega \to \infty$.

This circuit is a BPF.

38. Derive a transfer function for the circuit shown in figure 9.36. Then, determine the filter type.

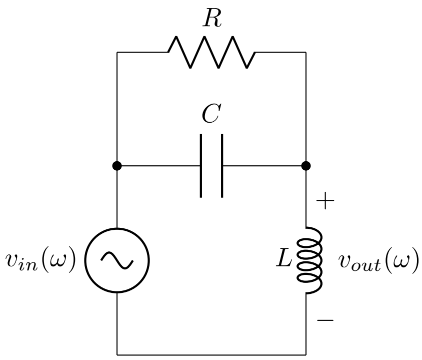


Figure 9.36: Circuit diagram for transfer functions question 38.

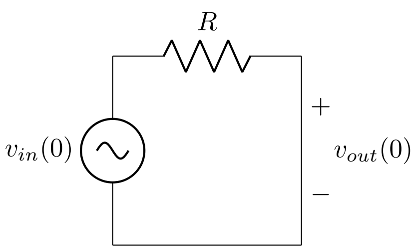
Calculate the equivalent impedance of the resistor and capacitor in parallel.

$$\begin{aligned} Z_{EQ} &= R//C \\ &= R//\frac{1}{j\omega C} \\ &= \frac{\frac{R}{j\omega C}}{R+\frac{1}{j\omega C}} \\ &= \frac{R}{j\omega CR+1} \end{aligned}$$

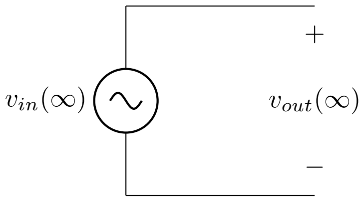
Use the complex voltage divider rule to calculate $H(\omega)$.

$$\begin{aligned} H(\omega) &= \frac{j\omega L}{j\omega L+\frac{R}{j\omega CR+1}} \\ &= \frac{-\omega^2 LCR + j\omega L}{-\omega^2 LCR + j\omega L + R} \\ &= \frac{-\omega^2 + j\omega\frac{1}{RC}}{-\omega^2 + j\omega\frac{1}{RC}+\frac{1}{LC}} \end{aligned}$$

Analyze the circuit at $\omega = 0$.



Analyze the circuit at $\omega \to \infty$.



This circuit is an HPF.

39. Derive a transfer function for the circuit shown in figure 9.37. Then, determine the filter type.

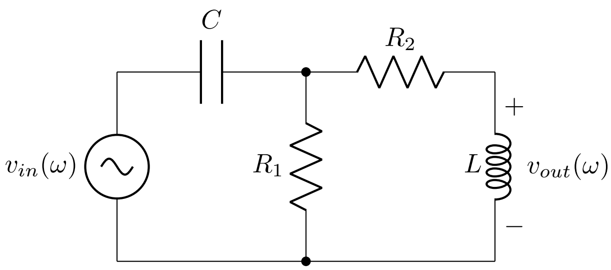
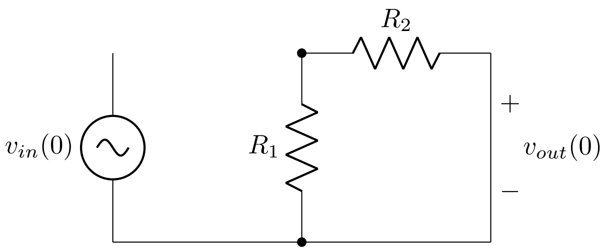


Figure 9.37: Circuit diagram for transfer functions question 39.

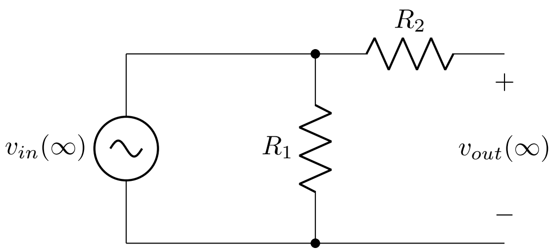
Use the complex voltage divider rule to calculate $H(\omega)$.

$$\begin{aligned}
H(\omega) &= \left(\frac{R_1//(j\omega L + R_2)}{\frac{1}{j\omega C} + R_1//(j\omega L + R_2)}\right)\left(\frac{j\omega L}{R_2 + j\omega L}\right) \\
&= \left(\frac{\frac{(j\omega L+R_2)R_1}{j\omega L+R_1+R_2}}{\frac{1}{j\omega C} + \frac{(j\omega L+R_2)R_1}{j\omega L+R_1+R_2}}\right)\left(\frac{j\omega L}{R_2 + j\omega L}\right) \\
&= \left(\frac{\frac{R_1}{j\omega L+R_1+R_2}}{\frac{1}{j\omega C} + \frac{(j\omega L+R_2)R_1}{j\omega L+R_1+R_2}}\right)(j\omega L) \\
&= \frac{\frac{j\omega L R_1}{j\omega L+R_1+R_2}}{\frac{1}{j\omega C} + \frac{(j\omega L+R_2)R_1}{j\omega L+R_1+R_2}} \\
&= \frac{j\omega L R_1}{j\omega L R_1 + R_1 R_2 + \frac{L}{C} + \frac{R_2+R_1}{j\omega C}} \\
&= \frac{-\omega^2 L R_1 C}{-\omega^2 L R_1 C + j\omega C R_1 R_2 + j\omega L + R_1 + R_2} \\
&= \frac{-\omega^2}{-\omega^2 + j\omega\left(\frac{R_2}{L} + \frac{1}{R_1 C}\right) + \frac{R_1+R_2}{R_1 R_2}}
\end{aligned}$$

Analyze the circuit at $\omega = 0$.



Analyze the circuit at $\omega \to \infty$.



This circuit is an HPF.

40. Derive a transfer function for the circuit shown in figure 9.38. Then, determine the filter type.

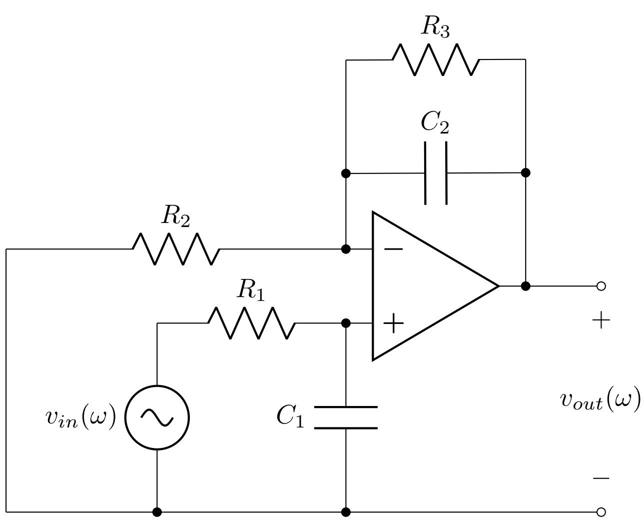


Figure 9.38: Circuit diagram for transfer functions question 40.

Use the complex voltage divider rule to calculate the voltage at the non-inverting node, denoted as $v_x(\omega)$.

$$\frac{v_x(\omega)}{v_{in}(\omega)} = \frac{\frac{1}{j\omega C_1}}{R_1 + \frac{1}{j\omega C_1}}$$
$$= \frac{1}{j\omega C R_1 + 1}$$

Perform KCL at the inverting node. Everything will be expressed as generic phasor for now.

$$-\frac{1}{R_2}\mathbf{V_X} = \frac{\mathbf{V_X} - \mathbf{V_O}}{R_3//Z_{C2}}$$
$$-\frac{R_3//Z_{C2}}{R_2}\mathbf{V_X} = \mathbf{V_X} - \mathbf{V_O}$$
$$\mathbf{V_O} = \left(1 + \frac{R_3//Z_{C2}}{R_2}\right)\mathbf{V_X}$$
$$\frac{\mathbf{V_O}}{\mathbf{V_I}} = \left(1 + \frac{R_3//Z_{C2}}{R_2}\right)\frac{\mathbf{V_X}}{\mathbf{V_I}}$$

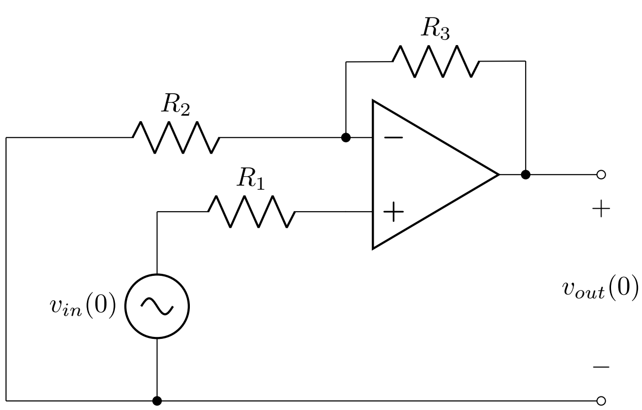
Reduce the impedance defined in the KCL equation.

$$\begin{aligned}\mathbf{Z} &= 1 + \frac{R_3//Z_{C2}}{R_2} \\ &= 1 + \frac{\frac{R_3}{R_2 j\omega C_2}}{R_3 + \frac{1}{j\omega C_2}} \\ &= 1 + \frac{\frac{R_3}{R_2}}{j\omega C_2 R_3 + 1} \\ &= \frac{j\omega C_2 R_3 + 1 + \frac{R_3}{R_2}}{j\omega C_2 R_3 + 1}\end{aligned}$$

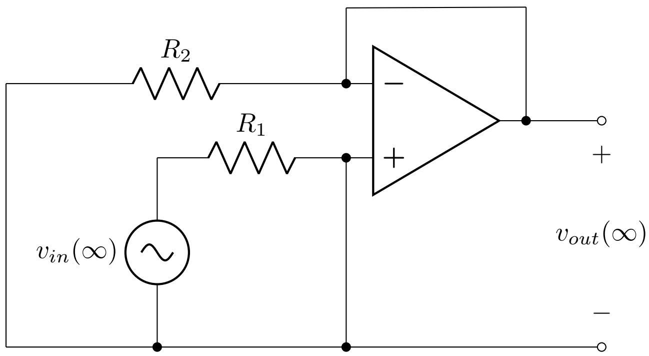
Plug the value of v_x/v_{in} into the KCL equation. Then normalize the transfer function.

$$\begin{aligned}H(\omega) &= \left(\frac{j\omega C_2 R_3 + 1 + \frac{R_3}{R_2}}{j\omega C_2 R_3 + 1}\right)\left(\frac{1}{j\omega C R_1 + 1}\right) \\ &= \frac{j\omega C_2 R_3 + 1 + \frac{R_3}{R_2}}{-\omega^2 C_2 R_3 C_1 R_1 + j\omega C_1 R_1 + j\omega C_2 R_3 + 1} \\ &= \frac{j\omega \frac{1}{C_1 R_1} + \frac{R_2 + R_3}{C_1 C_2 R_1 R_2 R_3}}{-\omega^2 + j\omega\left(\frac{1}{C_2 R_3} + \frac{1}{C_1 R_1}\right) + \frac{1}{C_1 C_2 R_1 R_3}}\end{aligned}$$

Analyze the circuit at $\omega = 0$.



Analyze the circuit at $\omega \to \infty$.



This circuit is an LPF.

10 Chapter 10 Solutions

10.1 Phase and Root Mean Square

1. Calculate the RMS value of the voltage and current signals, as well as the phase difference.
$v(t) = 120\cos(2\pi 60t + 120\pi/180)$ **V,** $i(t) = 5\cos(2\pi 60t + 60\pi/180)$ **A**

Calculate the RMS value of the voltage and current.

$$\begin{aligned} |\mathbf{V_{RMS}}| &= \frac{120\ \text{V}}{\sqrt{2}} \\ &= 84.85\ \text{VRMS} \\ |\mathbf{I_{RMS}}| &= \frac{5\ \text{A}}{\sqrt{2}} \\ &= 3.54\ \text{ARMS} \end{aligned}$$

Calculate the phase difference.

$$\begin{aligned} \theta &= \phi_v - \phi_i \\ &= 120^\circ - 60^\circ \\ &= 60^\circ \end{aligned}$$

2. Calculate the RMS value of the voltage and current signals, as well as the phase difference.
$v(t) = 150\cos(2\pi 60t + 60\pi/180)$ **V,** $i(t) = 3\cos(2\pi 60t + 35\pi/180)$ **A**

Calculate the RMS value of the voltage and current.

$$\begin{aligned} |\mathbf{V_{RMS}}| &= \frac{150\ \text{V}}{\sqrt{2}} \\ &= 106.07\ \text{VRMS} \\ |\mathbf{I_{RMS}}| &= \frac{3\ \text{A}}{\sqrt{2}} \\ &= 2.12\ \text{ARMS} \end{aligned}$$

Calculate the phase difference.

$$\begin{aligned} \theta &= \phi_v - \phi_i \\ &= 60^\circ - 35^\circ \\ &= 25^\circ \end{aligned}$$

3. Calculate the RMS value of the voltage and current signals, as well as the phase difference.
$v(t) = 220\cos(2\pi 50t + 120\pi/180)$ **V,** $i(t) = 6\cos(2\pi 50t - 40\pi/180)$ **A**

Calculate the RMS value of the voltage and current.

$$|\mathbf{V_{RMS}}| = \frac{220\text{ V}}{\sqrt{2}}$$
$$= 155.56\text{ VRMS}$$
$$|\mathbf{I_{RMS}}| = \frac{6\text{ A}}{\sqrt{2}}$$
$$= 4.24\text{ ARMS}$$

Calculate the phase difference.

$$\theta = \phi_v - \phi_i$$
$$= 120^\circ + 40^\circ$$
$$= 160^\circ$$

4. Calculate the RMS value of the voltage and current signals, as well as the phase difference, given the circuit shown in figure 10.1.

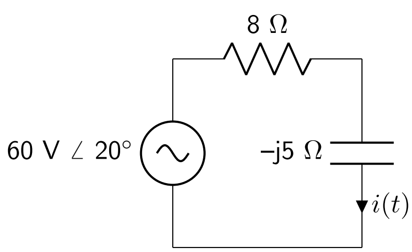


Figure 10.1: Circuit diagram for phase and root mean square question 4.

Calculate the RMS value of the voltage.

$$|\mathbf{V_{RMS}}| = \frac{60\text{ V}}{\sqrt{2}}$$
$$= 42.43\text{ VRMS}$$

Use Ohm's law to calculate the current.

$$\begin{aligned}\mathbf{I} &= \frac{39.87 + 14.51 \text{ VRMS}}{8 - j5 \ \Omega} \\ &= 2.77 + j3.54 \text{ ARMS} \\ &= 4.50 \text{ ARMS}\angle 52.01^\circ\end{aligned}$$

Calculate the phase difference.

$$\begin{aligned}\theta &= \phi_v - \phi_i \\ &= 20^\circ - 52.01^\circ \\ &= -32.01^\circ\end{aligned}$$

5. Calculate the RMS value of the voltage and current signals, as well as the phase difference, given the circuit shown in figure 10.2.

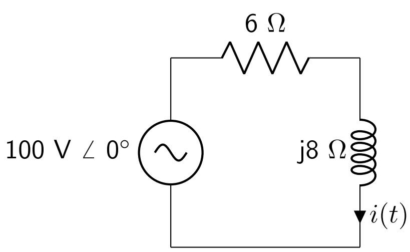


Figure 10.2: Circuit diagram for phase and root mean square question 5.

Calculate the RMS value of the voltage.

$$\begin{aligned}|\mathbf{V_{RMS}}| &= \frac{100 \text{ V}}{\sqrt{2}} \\ &= 70.71 \text{ VRMS}\end{aligned}$$

Use Ohm's law to calculate the current.

$$\begin{aligned}\mathbf{I} &= \frac{70.71 \text{ VRMS}}{6 + j8 \ \Omega} \\ &= 4.24 - j5.66 \text{ ARMS} \\ &= 7.07 \text{ ARMS}\angle -53.13^\circ\end{aligned}$$

Calculate the phase difference.

$$\begin{aligned}\theta &= \phi_v - \phi_i \\ &= 0^\circ + 53.13^\circ \\ &= 53.13^\circ\end{aligned}$$

10.2 Complex Power

6. Calculate the power consumed by the load (indicated with dashed lines), as well as the power factor, for the circuit given in figure 10.3. The frequency of operation is 50 Hz.

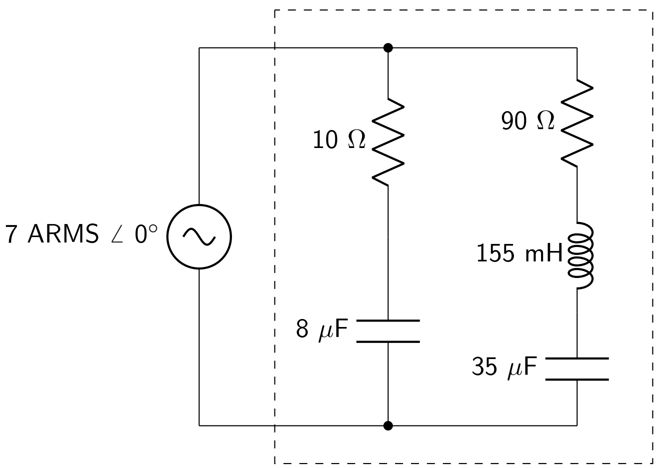
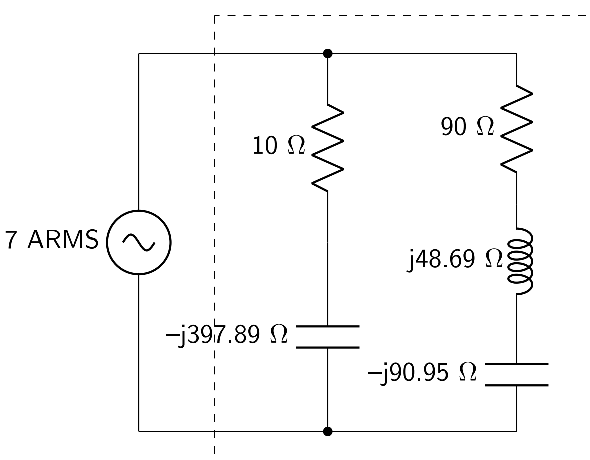


Figure 10.3: Circuit diagram for complex power question 6.

Phasor transform the circuit.



Calculate the equivalent impedance of the load.

$$\begin{aligned}\mathbf{Z_{LOAD}} &= (10 - j397.89\ \Omega)//(90 - j42.25\ \Omega) \\ &= 70.47 - j52.16\ \Omega\end{aligned}$$

Calculate the power consumed by the load.

$$\begin{aligned}\mathbf{S_{LOAD}} &= |\mathbf{I_{LOAD,RMS}}|^2\mathbf{Z_{LOAD}} \\ &= (7\ \text{ARMS})^2(70.47 - j52.16\ \Omega) \\ &= 3453.00 - j2555.89\ \text{VA}\end{aligned}$$

Calculate the power factor.

$$\begin{aligned}pf &= \frac{P}{|\mathbf{S}|} \\ &= \frac{3453.00}{4296.02} \\ &= 0.804\end{aligned}$$

7. Calculate the power consumed by the load (indicated with dashed lines), as well as the power factor, for the circuit given in figure 10.4. The frequency of operation is 60 Hz.

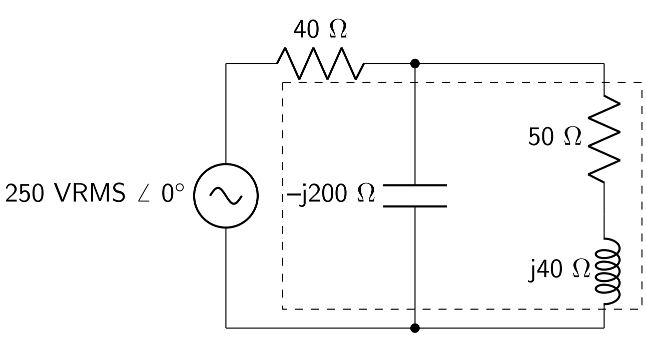


Figure 10.4: Circuit diagram for complex power question 7.

Calculate the equivalent impedance of the load.

$$\begin{aligned}\mathbf{Z_{LOAD}} &= (50 + j40\ \Omega)//(-j200\ \Omega) \\ &= 71.17 + j27.76\ \Omega\end{aligned}$$

Use Ohm's law to calculate the current flowing through the load.

$$\begin{aligned}\mathbf{I_{LOAD}} &= \frac{250 \text{ VRMS}}{111.17 + j27.76\ \Omega} \\ &= 2.12 - j0.53 \text{ ARMS} \\ &= 2.18 \text{ ARMS}\angle - 14.02°\end{aligned}$$

Calculate the power consumed by the load.

$$\begin{aligned}\mathbf{S_{LOAD}} &= |\mathbf{I_{LOAD,RMS}}|^2 \mathbf{Z_{LOAD}} \\ &= (2.18 \text{ ARMS})^2(71.17 + j27.76\ \Omega) \\ &= 338.79 + j132.13 \text{ VA}\end{aligned}$$

Calculate the power factor.

$$\begin{aligned}pf &= \frac{P}{|\mathbf{S}|} \\ &= \frac{338.79}{363.64} \\ &= 0.932\end{aligned}$$

8. Calculate the power consumed by the load (indicated with dashed lines), as well as the power factor, for the circuit given in figure 10.5. The frequency of operation is 60 Hz.

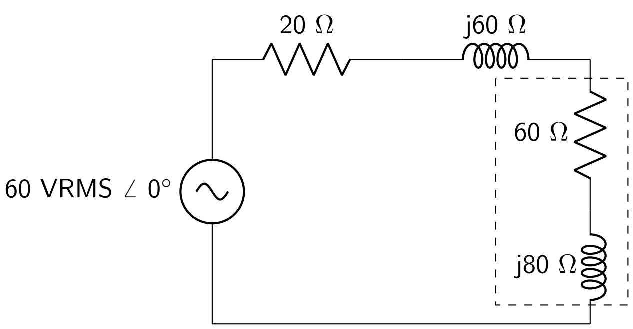


Figure 10.5: Circuit diagram for complex power question 8.

Use Ohm's law to calculate the current flowing through the load.

$$\begin{aligned}\mathbf{I_{LOAD}} &= \frac{60 \text{ VRMS}}{80 + j140\ \Omega} \\ &= 0.18 - j0.32 \text{ ARMS} \\ &= 0.37 \text{ ARMS}\angle - 60.26°\end{aligned}$$

Calculate the power consumed by the load.

$$\begin{aligned}\mathbf{S_{LOAD}} &= |\mathbf{I_{LOAD,RMS}}|^2\mathbf{Z_{LOAD}} \\ &= (0.37 \text{ ARMS})^2(60 + j80\ \Omega) \\ &= 8.31 + j11.08 \text{ VA}\end{aligned}$$

Calculate the power factor.

$$\begin{aligned}pf &= \frac{P}{|\mathbf{S}|} \\ &= \frac{8.31}{13.85} \\ &= 0.600\end{aligned}$$

9. Calculate the power consumed by the load (indicated with dashed lines), as well as the power factor, for the circuit given in figure 10.6. The frequency of operation is 60 Hz.

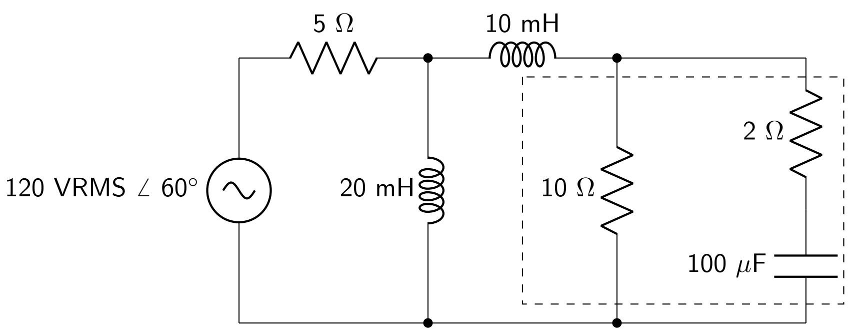
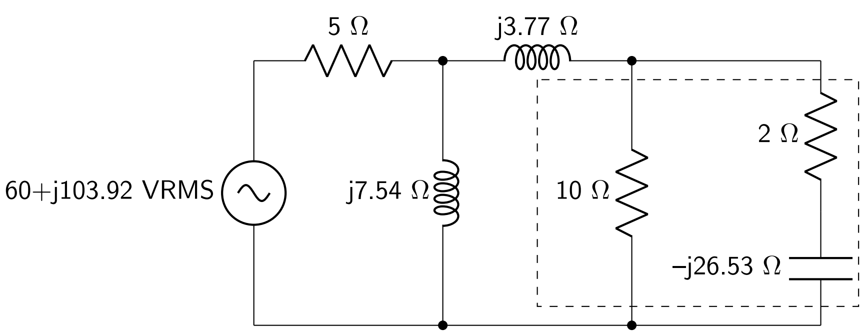


Figure 10.6: Circuit diagram for complex power question 9.

Phasor transform the circuit.



Calculate the impedance of the load.

$$\begin{aligned}\mathbf{Z_{LOAD}} &= (10\ \Omega)//(2 - j26.53\ \Omega) \\ &= 8.58 + j3.13\ \Omega\end{aligned}$$

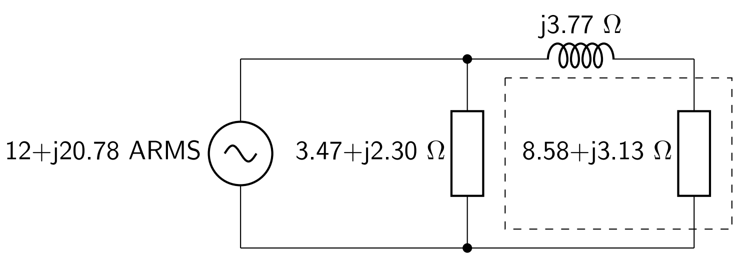
Transform the voltage source to a current source.

$$\begin{aligned}\mathbf{I_S} &= \frac{60 + j103.92\text{ VRMS}}{5\ \Omega} \\ &= 12 + j20.78\text{ ARMS}\end{aligned}$$

Combine the 5 Ω and 20 mH impedances in parallel.

$$\begin{aligned}\mathbf{Z} &= (5\ \Omega)//(j7.54\ \Omega) \\ &= 3.47 + j2.30\ \Omega\end{aligned}$$

Re-draw the circuit.



Use the complex current divider rule to calculate the current flowing through the load.

$$\begin{aligned}\mathbf{I_{LOAD}} &= (12 + j20.78\text{ ARMS})\left(\frac{(3.47 + j2.30\ \Omega)//(8.58 + j6.90\ \Omega)}{8.58 + j6.90\ \Omega}\right) \\ &= 1.42 + j7.93\text{ ARMS} \\ &= 8.06\text{ ARMS}\angle 79.83^\circ\end{aligned}$$

Calculate the power consumed by the load.

$$\begin{aligned}\mathbf{S_{LOAD}} &= |\mathbf{I_{LOAD,RMS}}|^2\mathbf{Z_{LOAD}} \\ &= (8.06\text{ ARMS})^2(8.58 + j3.13\ \Omega) \\ &= 557.38 - j203.20\text{ VA}\end{aligned}$$

Calculate the power factor.

$$\begin{aligned} pf &= \frac{P}{|\mathbf{S}|} \\ &= \frac{557.38}{593.26} \\ &= 0.940 \end{aligned}$$

10. Calculate the power consumed by the load (indicated with dashed lines), as well as the power factor, for the circuit given in figure 10.7. The frequency of operation is 50 Hz.

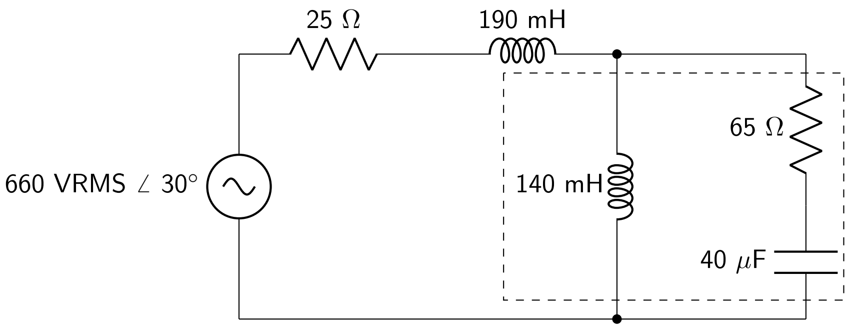
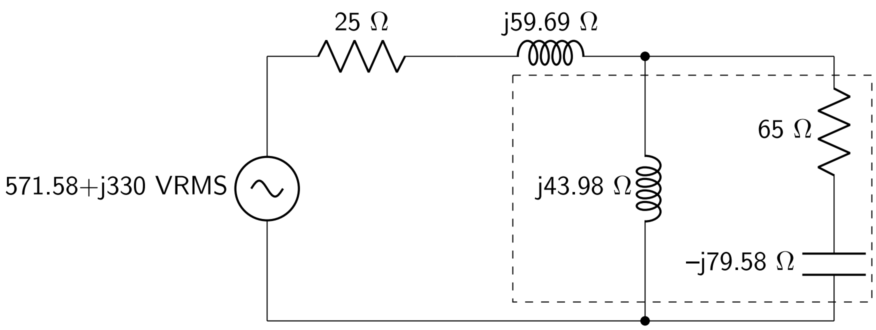


Figure 10.7: Circuit diagram for complex power question 10.

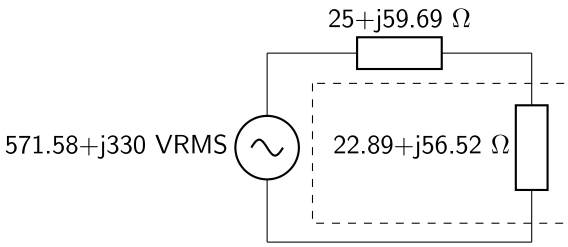
Phasor transform the circuit.



Calculate the impedance of the load.

$$\begin{aligned} \mathbf{Z_{LOAD}} &= (j43.98\ \Omega)//(65 - j79.58\ \Omega) \\ &= 22.89 + j56.52\ \Omega \end{aligned}$$

Re-draw the circuit.



Use Ohm's law to calculate the current flowing through the load.

$$\mathbf{I_{LOAD}} = \frac{571.58 + j330 \text{ VRMS}}{(22.89 + j56.52\ \Omega) + (25 + j59.69\ \Omega)}$$
$$= 4.16 - j3.20 \text{ ARMS}$$
$$= 5.25 \text{ ARMS}\angle - 37.60^\circ$$

Calculate the power consumed by the load.

$$\mathbf{S_{LOAD}} = |\mathbf{I_{LOAD,RMS}}|^2 \mathbf{Z_{LOAD}}$$
$$= (5.25 \text{ ARMS})^2 (22.89 + j56.52\ \Omega)$$
$$= 631.25 + j1558.36 \text{ VA}$$

Calculate the power factor.

$$pf = \frac{P}{|\mathbf{S}|}$$
$$= \frac{631.25}{1681.36}$$
$$= 0.375$$

10.3 Maximum Power Transfer

11. Determine the circuit elements that must be placed on the load (shown as a generic circuit element) for maximum power transfer given the circuit in figure 10.8. Then, calculate the power consumed by that load. The frequency of operation is 60 Hz.

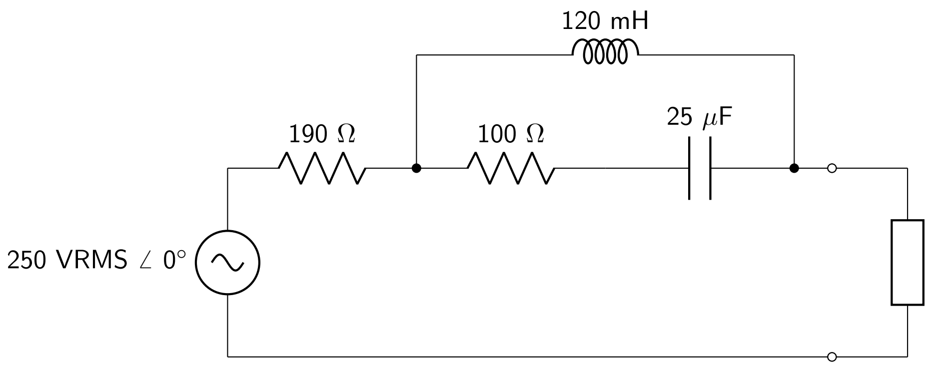
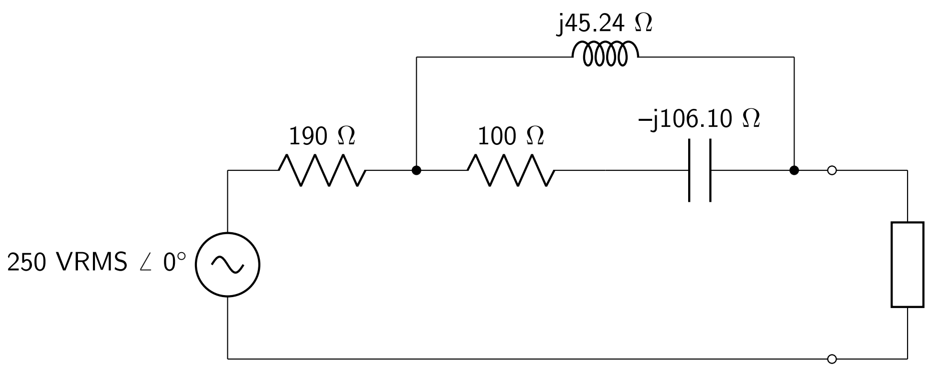


Figure 10.8: Circuit diagram for maximum power transfer question 11.

Phasor transform the circuit.



Deactivate the source and calculate the Thévenin equivalent impedance.

$$\begin{aligned}\mathbf{Z_{TH}} &= 190\ \Omega + (100 - j106.10\ \Omega)//(j45.24\ \Omega) \\ &= 204.93 + j54.33\ \Omega\end{aligned}$$

For maximum power transfer, the load must be equal to $\mathbf{Z_{TH}}^*$.

$$\mathbf{Z_{LOAD}} = 204.93 - j54.33\ \Omega$$

Calculate the capacitance from the reactance.

$$C = -\frac{1}{2\pi f X}$$
$$= -\frac{1}{2\pi(60)(-54.33)}$$
$$= 48.83\ \mu\text{F}$$

Calculate the power transferred to the load.

$$P_{LOAD,MAX} = \frac{|\mathbf{V_{TH,RMS}}|^2}{4R_{TH}}$$
$$= \frac{|250\ \text{VRMS}|^2}{4(204.93\ \Omega)}$$
$$= 76.24\ \text{W}$$

12. Determine the circuit elements that must be placed on the load (shown as a generic circuit element) for maximum power transfer given the circuit in figure 10.9. Then, calculate the power consumed by that load. The frequency of operation is 60 Hz.

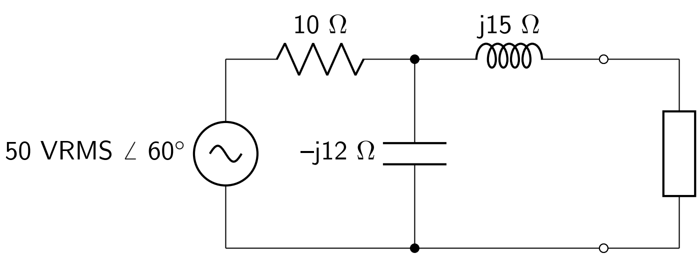


Figure 10.9: Circuit diagram for maximum power transfer question 12.

Deactivate the source and calculate the Thévenin equivalent impedance.

$$\mathbf{Z_{TH}} = j15\ \Omega + (-j12\ \Omega)//(10\ \Omega)$$
$$= 5.90 + j10.08\ \Omega$$

Use the complex voltage divider rule to calculate $\mathbf{V_{TH}}$.

$$\mathbf{V_{TH}} = (25 + j43.30\ \text{VRMS})\left(\frac{-j12\ \Omega}{10 - j12\ \Omega}\right)$$
$$= 36.05 + j13.26\ \text{VRMS}$$
$$= 38.41\ \text{VRMS}\angle 20.19°$$

For maximum power transfer, the load must be equal to $\mathbf{Z_{TH}}^*$.

$$\mathbf{Z_{LOAD}} = 5.90 - j10.08\ \Omega$$

Calculate the capacitance from the reactance.

$$\begin{aligned} C &= -\frac{1}{2\pi f X} \\ &= -\frac{1}{2\pi(60)(-10.08)} \\ &= 263.10\ \mu\text{F} \end{aligned}$$

Calculate the power transferred to the load.

$$\begin{aligned} P_{LOAD,MAX} &= \frac{|\mathbf{V_{TH,RMS}}|^2}{4R_{TH}} \\ &= \frac{|38.41\ \text{VRMS}|^2}{4(5.90\ \Omega)} \\ &= 62.50\ \text{W} \end{aligned}$$

13. Determine the circuit elements that must be placed on the load (shown as a generic circuit element) for maximum power transfer given the circuit in figure 10.10. Then, calculate the power consumed by that load. The frequency of operation is 50 Hz.

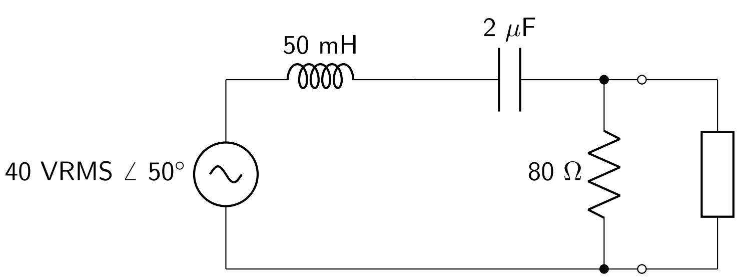
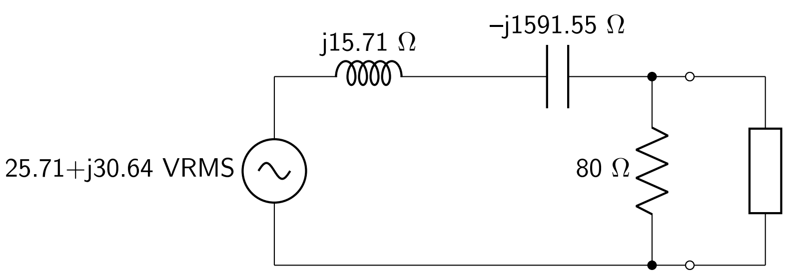


Figure 10.10: Circuit diagram for maximum power transfer question 13.

Phasor transform the circuit.



Deactivate the source and calculate the Thévenin equivalent impedance.

$$\begin{aligned}\mathbf{Z_{TH}} &= (-j1575.84\ \Omega)//(80\ \Omega)\\ &= 79.79 - j4.05\ \Omega\end{aligned}$$

Use the complex voltage divider rule to calculate $\mathbf{V_{TH}}$.

$$\begin{aligned}\mathbf{V_{TH}} &= (25.71 + j30.64\ \text{VRMS})\left(\frac{80\ \Omega}{80 - j1575.84\ \Omega}\right)\\ &= -1.49 + j1.38\ \text{VRMS}\\ &= 20.3\ \text{VRMS}\angle 137.09^\circ\end{aligned}$$

For maximum power transfer, the load must be equal to $\mathbf{Z_{TH}}^*$.

$$\mathbf{Z_{LOAD}} = 79.79 + j4.05\ \Omega$$

Calculate the inductance from the reactance.

$$\begin{aligned}L &= \frac{X}{2\pi f}\\ &= \frac{4.05}{2\pi 50}\\ &= 12.89\ \text{mH}\end{aligned}$$

Calculate the power transferred to the load.

$$\begin{aligned}P_{LOAD,MAX} &= \frac{|\mathbf{V_{TH,RMS}}|^2}{4R_{TH}}\\ &= \frac{|20.3\ \text{VRMS}|^2}{4(79.79\ \Omega)}\\ &= 12.89\ \text{mW}\end{aligned}$$

14. Determine the circuit elements that must be placed on the load (shown as a generic circuit element) for maximum power transfer given the circuit in figure 10.11. Then, calculate the power consumed by that load. The frequency of operation is 50 Hz.

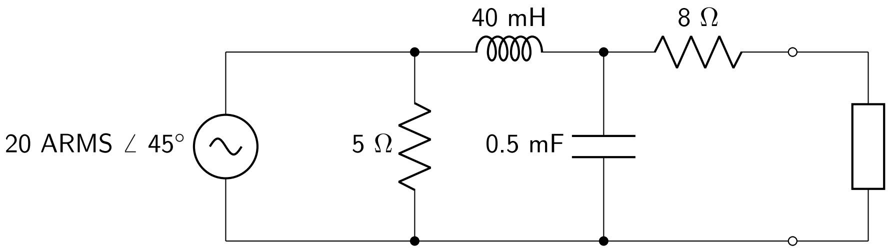
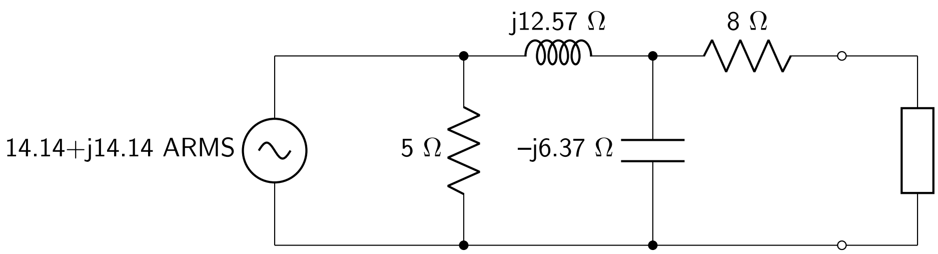


Figure 10.11: Circuit diagram for maximum power transfer question 14.

Phasor transform the circuit.

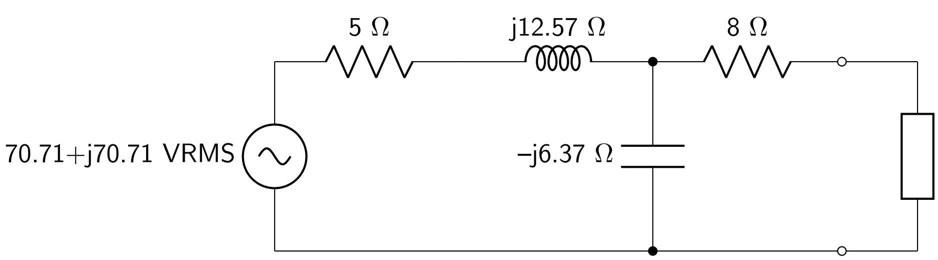


Deactivate the source and calculate the Thévenin equivalent impedance.

$$\mathbf{Z_{TH}} = 8\ \Omega + (-j6.37\ \Omega)//(5 + j12.57\ \Omega)$$
$$= 11.19 - j10.33\ \Omega$$

Convert the source to a voltage source.

$$\mathbf{V_S} = (14.14 + j14.14\ \text{ARMS})(5\ \Omega)$$
$$= 70.71 + j70.71\ \text{VRMS}$$



Use the complex voltage divider rule to calculate $\mathbf{V_{TH}}$.

$$\begin{aligned}\mathbf{V_{TH}} &= (70.71 + j70.71 \text{ VRMS}) \left(\frac{-j6.37\ \Omega}{5 + j12.57 - j6.37\ \Omega} \right) \\ &= -8.52 - j79.47 \text{ VRMS} \\ &= 79.93 \text{ VRMS}\angle - 96.12°\end{aligned}$$

For maximum power transfer, the load must be equal to $\mathbf{Z_{TH}}^*$.

$$\mathbf{Z_{LOAD}} = 11.19 + j10.33\ \Omega$$

Calculate the inductance from the reactance.

$$\begin{aligned}L &= \frac{X}{2\pi f} \\ &= \frac{10.33}{2\pi 50} \\ &= 32.87 \text{ mH}\end{aligned}$$

Calculate the power transferred to the load.

$$\begin{aligned}P_{LOAD,MAX} &= \frac{|\mathbf{V_{TH,RMS}}|^2}{4R_{TH}} \\ &= \frac{|79.93 \text{ VRMS}|^2}{4(11.19\ \Omega)} \\ &= 142.67 \text{ W}\end{aligned}$$

15. Determine the circuit elements that must be placed on the load (shown as a generic circuit element) for maximum power transfer given the circuit in figure 10.12. Then, calculate the power consumed by that load. The frequency of operation is 60 Hz.

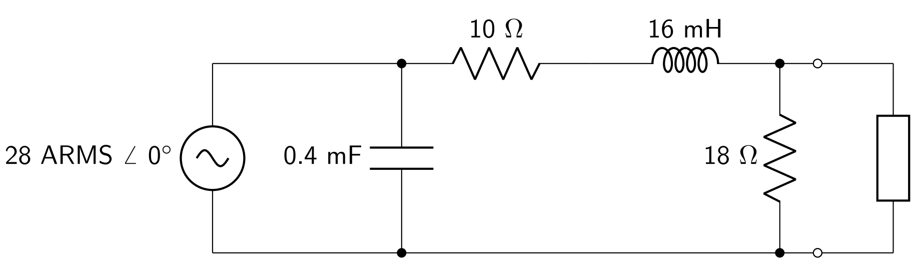
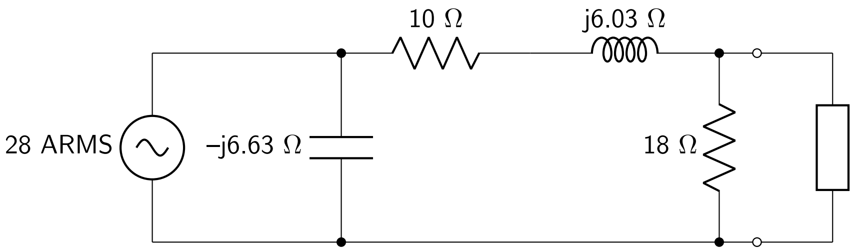


Figure 10.12: Circuit diagram for maximum power transfer question 15.

Phasor transform the circuit.

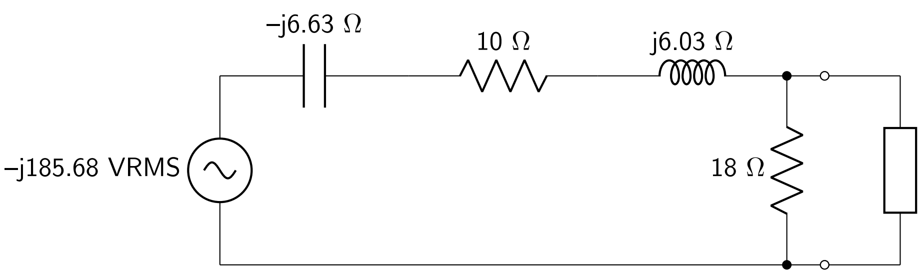


Deactivate the source and calculate the Thévenin equivalent impedance.

$$\begin{aligned}\mathbf{Z_{TH}} &= (18\ \Omega)//(10 + j6.03 - j6.63\ \Omega)\\ &= 6.43 - j0.25\ \Omega\end{aligned}$$

Convert the source to a voltage source.

$$\begin{aligned}\mathbf{V_S} &= (28\ \text{ARMS})(-j6.63\ \Omega)\\ &= -j185.68\ \text{VRMS}\end{aligned}$$



Use the complex voltage divider rule to calculate $\mathbf{V_{TH}}$.

$$\begin{aligned}\mathbf{V_{TH}} &= (-j185.68\ \text{VRMS})\left(\frac{18\ \Omega}{28 + j6.03 - j6.63\ \Omega}\right)\\ &= 2.55 - j119.31\ \text{VRMS}\\ &= 119.34\ \text{VRMS}\angle -88.77^\circ\end{aligned}$$

For maximum power transfer, the load must be equal to $\mathbf{Z_{TH}}^*$.

$$\mathbf{Z_{LOAD}} = 6.43 + j0.25\ \Omega$$

Calculate the inductance from the reactance.

$$L = \frac{X}{2\pi f} = \frac{0.25}{2\pi 50} = 656.99\ \mu\text{H}$$

Calculate the power transferred to the load.

$$P_{LOAD,MAX} = \frac{|\mathbf{V_{TH,RMS}}|^2}{4R_{TH}} = \frac{|119.34\ \text{VRMS}|^2}{4(6.43\ \Omega)} = 553.39\ \text{W}$$

10.4 Power Factor Correction

16. Calculate the initial power factor consumed by the load (indicated with dashed lines) in the circuit shown in figure 10.13. Then, determine the circuit element that must be placed in parallel with the load to increase the power factor to 0.800. The frequency of operation is 60 Hz.

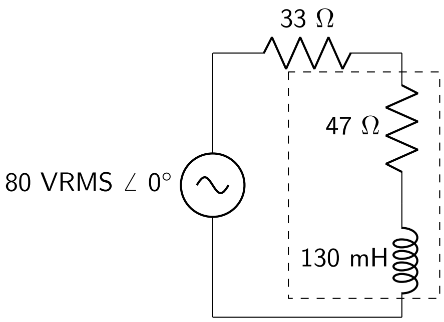
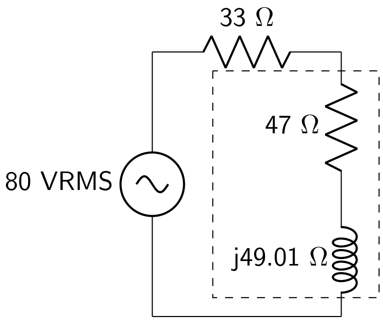


Figure 10.13: Circuit diagram for power factor correction question 16.

Phasor transform the circuit.



Calculate the load impedance.

$$\mathbf{Z_{LOAD}} = 47 + j49.01\ \Omega$$

Calculate the current through the load.

$$\begin{aligned}\mathbf{I_{LOAD}} &= \frac{80\ \text{VRMS}}{80 + j49.01\ \Omega} \\ &= 0.73 - j0.45\ \text{ARMS} \\ &= 0.85\ \text{ARMS}\angle -31.49^\circ\end{aligned}$$

Calculate the voltage dropped over the load.

$$\begin{aligned}\mathbf{V_{LOAD}} &= (0.73 - j0.45\ \text{ARMS})(47 + j49.01\ \Omega) \\ &= 56.01 + j14.70\ \text{VRMS} \\ &= 57.90\ \text{VRMS}\angle 14.71^\circ\end{aligned}$$

Calculate the power consumed by the load.

$$\begin{aligned}\mathbf{S_{LOAD}} &= |\mathbf{I_{LOAD,RMS}}|^2 \mathbf{Z_{LOAD}} \\ &= (0.85\ \text{ARMS})^2 (47 + j49.01\ \Omega) \\ &= 34.17 + j35.64\ \text{VA}\end{aligned}$$

Calculate the initial power factor.

$$\begin{aligned}pf &= \frac{P}{|\mathbf{S}|} \\ &= \frac{34.17}{49.37} \\ &= 0.692\end{aligned}$$

Calculate the final reactive power required to increase the power factor to the desired level.

$$\begin{aligned}Q_f &= P\sqrt{\frac{1}{pf_f^2} - 1} \\ &= 34.17\sqrt{\frac{1}{0.800^2} - 1} \\ &= 25.63\ \text{VAR}\end{aligned}$$

Calculate Q_C.

$$\begin{aligned} Q_C &= Q_f - Q \\ &= 25.63 \text{ VAR} - 35.64 \text{ VAR} \\ &= -10.00 \text{ VAR} \end{aligned}$$

Calculate the compensating capacitance.

$$\begin{aligned} C &= \frac{-Q_C}{|\mathbf{V_{LOAD,RMS}}|^2\omega} \\ &= \frac{10.00}{|57.90|^2(2\pi 60)} \\ &= 7.92\ \mu\text{F} \end{aligned}$$

17. Calculate the initial power factor consumed by the load (indicated with dashed lines) in the circuit shown in figure 10.14. Then, determine the circuit element that must be placed in parallel with the load to increase the power factor to 0.860. The frequency of operation is 60 Hz.

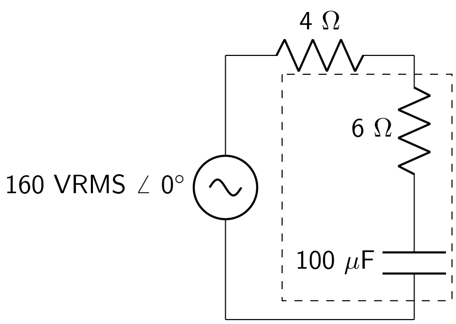
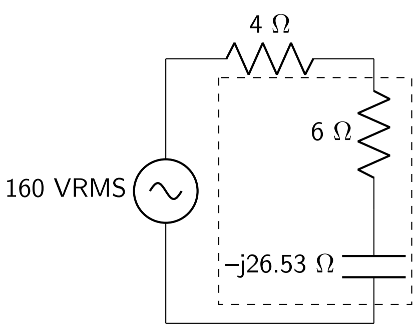


Figure 10.14: Circuit diagram for power factor correction question 17.

Phasor transform the circuit.



Calculate the load impedance.

$$\mathbf{Z_{LOAD}} = 6 - j26.53\ \Omega$$

Calculate the current through the load.

$$\begin{aligned}\mathbf{I_{LOAD}} &= \frac{160\text{ VRMS}}{10 - j26.53\ \Omega} \\ &= 1.99 + j5.28\text{ ARMS} \\ &= 5.64\text{ ARMS}\angle 69.34^\circ\end{aligned}$$

Calculate the voltage dropped over the load.

$$\begin{aligned}\mathbf{V_{LOAD}} &= (1.99 + j5.28\text{ ARMS})(6 - j26.53\ \Omega) \\ &= 152.04 - j21.13\text{ VRMS} \\ &= 153.50\text{ VRMS}\angle - 7.91^\circ\end{aligned}$$

Calculate the power consumed by the load.

$$\begin{aligned}\mathbf{S_{LOAD}} &= |\mathbf{I_{LOAD,RMS}}|^2\mathbf{Z_{LOAD}} \\ &= (5.64\text{ ARMS})^2(6 - j26.53\ \Omega) \\ &= 191.14 - j845.00\text{ VA}\end{aligned}$$

Calculate the initial power factor.

$$\begin{aligned}pf &= \frac{P}{|\mathbf{S}|} \\ &= \frac{191.14}{866.35} \\ &= 0.221\end{aligned}$$

Calculate the final reactive power required to increase the power factor to the desired level.

$$\begin{aligned}Q_f &= -P\sqrt{\frac{1}{pf_f^2} - 1} \\ &= -191.14\sqrt{\frac{1}{0.860^2} - 1} \\ &= -113.41\text{ VAR}\end{aligned}$$

Calculate Q_L.

$$\begin{aligned} Q_L &= Q_f - Q \\ &= -113.41 \text{ VAR} + 845.00 \text{ VAR} \\ &= 731.59 \text{ VAR} \end{aligned}$$

Calculate the compensating inductance.

$$\begin{aligned} L &= \frac{|\mathbf{V_{LOAD,RMS}}|^2}{\omega Q_L} \\ &= \frac{|153.50|^2}{2\pi(60)(731.59)} \\ &= 85.43 \text{ mH} \end{aligned}$$

18. Calculate the initial power factor consumed by the load (indicated with dashed lines) in the circuit shown in figure 10.15. Then, determine the circuit element that must be placed in parallel with the load to increase the power factor to 0.990. The frequency of operation is 60 Hz.

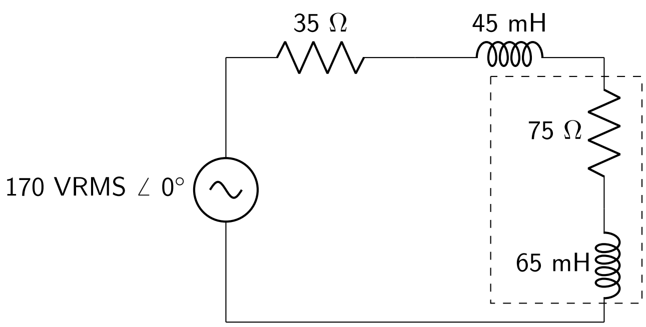
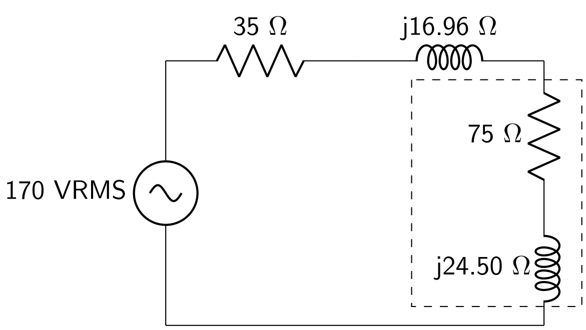


Figure 10.15: Circuit diagram for power factor correction question 18.

Phasor transform the circuit.



Calculate the load impedance.

$$\mathbf{Z_{LOAD}} = 75 + j24.50\ \Omega$$

Calculate the current through the load.

$$\begin{aligned}\mathbf{I_{LOAD}} &= \frac{170\ \text{VRMS}}{110 + j16.96 + j24.50\ \Omega}\\ &= 1.35 - j0.51\ \text{ARMS}\\ &= 1.45\ \text{ARMS}\angle -20.66^\circ\end{aligned}$$

Calculate the voltage dropped over the load.

$$\begin{aligned}\mathbf{V_{LOAD}} &= (1.35 - j0.51\ \text{ARMS})(75 + j24.50\ \Omega)\\ &= 113.99 - j5.10\ \text{VRMS}\\ &= 114.10\ \text{VRMS}\angle -2.56^\circ\end{aligned}$$

Calculate the power consumed by the load.

$$\begin{aligned}\mathbf{S_{LOAD}} &= |\mathbf{I_{LOAD,RMS}}|^2\mathbf{Z_{LOAD}}\\ &= (1.45\ \text{ARMS})^2(75 + j24.50\ \Omega)\\ &= 156.84 + j51.24\ \text{VA}\end{aligned}$$

Calculate the initial power factor.

$$\begin{aligned}pf &= \frac{P}{|\mathbf{S}|}\\ &= \frac{156.84}{165.00}\\ &= 0.951\end{aligned}$$

Calculate the final reactive power required to increase the power factor to the desired level.

$$\begin{aligned}Q_f &= P\sqrt{\frac{1}{pf_f^2} - 1}\\ &= 156.84\sqrt{\frac{1}{0.990^2} - 1}\\ &= 22.35\ \text{VAR}\end{aligned}$$

Calculate Q_C.

$$\begin{aligned} Q_C &= Q_f - Q \\ &= 22.35 \text{ VAR} - 51.24 \text{ VAR} \\ &= -28.90 \text{ VAR} \end{aligned}$$

Calculate the compensating capacitance.

$$\begin{aligned} C &= \frac{-Q_C}{|\mathbf{V_{LOAD,RMS}}|^2 \omega} \\ &= \frac{28.90}{|114.10|^2 (2\pi 60)} \\ &= 5.89 \ \mu\text{F} \end{aligned}$$

19. Calculate the initial power factor consumed by the load (indicated with dashed lines) in the circuit shown in figure 10.16. Then, determine the circuit element that must be placed in parallel with the load to increase the power factor to 0.980. The frequency of operation is 60 Hz.

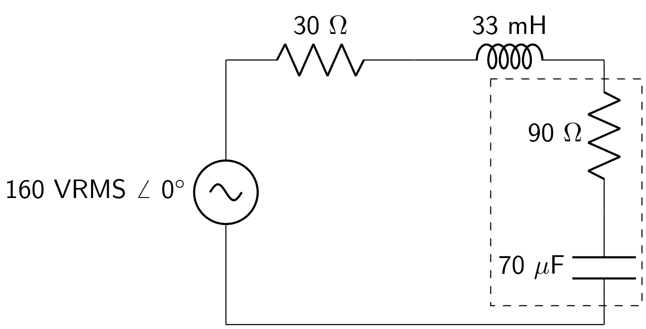
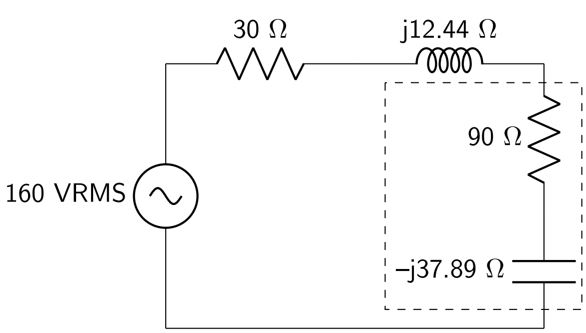


Figure 10.16: Circuit diagram for power factor correction question 19.

Phasor transform the circuit.



Calculate the load impedance.

$$\mathbf{Z_{LOAD}} = 90 - j37.89\ \Omega$$

Calculate the current through the load.

$$\begin{aligned}\mathbf{I_{LOAD}} &= \frac{160\text{ VRMS}}{120 + j12.44 - j37.89\ \Omega} \\ &= 1.28 + j0.27\text{ ARMS} \\ &= 1.30\text{ ARMS}\angle 11.98^\circ\end{aligned}$$

Calculate the voltage dropped over the load.

$$\begin{aligned}\mathbf{V_{LOAD}} &= (1.28 + j0.27\text{ ARMS})(90 - j37.89\ \Omega) \\ &= 125.09 - j23.99\text{ VRMS} \\ &= 127.37\text{ VRMS}\angle - 10.86^\circ\end{aligned}$$

Calculate the power consumed by the load.

$$\begin{aligned}\mathbf{S_{LOAD}} &= |\mathbf{I_{LOAD,RMS}}|^2\mathbf{Z_{LOAD}} \\ &= (1.30\text{ ARMS})^2(90 - j37.89\ \Omega) \\ &= 153.11 - j64.47\text{ VA}\end{aligned}$$

Calculate the initial power factor.

$$\begin{aligned}pf &= \frac{P}{|\mathbf{S}|} \\ &= \frac{153.11}{166.13} \\ &= 0.922\end{aligned}$$

Calculate the final reactive power required to increase the power factor to the desired level.

$$\begin{aligned}Q_f &= -P\sqrt{\frac{1}{pf_f^2} - 1} \\ &= -153.11\sqrt{\frac{1}{0.980^2} - 1} \\ &= -31.09\text{ VAR}\end{aligned}$$

Calculate Q_L.

$$\begin{aligned} Q_L &= Q_f - Q \\ &= -31.09\text{ VAR} + 64.47\text{ VAR} \\ &= 33.38\text{ VAR} \end{aligned}$$

Calculate the compensating inductance.

$$\begin{aligned} L &= \frac{|\mathbf{V_{LOAD,RMS}}|^2}{\omega Q_L} \\ &= \frac{|127.37|^2}{2\pi(60)(33.38)} \\ &= 1.29\text{ H} \end{aligned}$$

20. Calculate the initial power factor consumed by the load (indicated with dashed lines) in the circuit shown in figure 10.17. Then, determine the circuit element that must be placed in parallel with the load to increase the power factor to 0.960. The frequency of operation is 60 Hz.

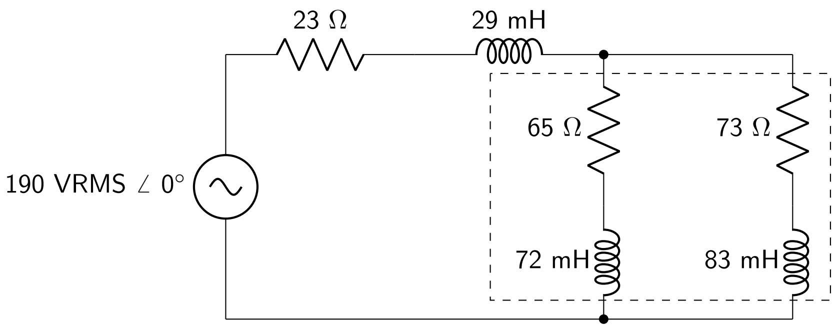
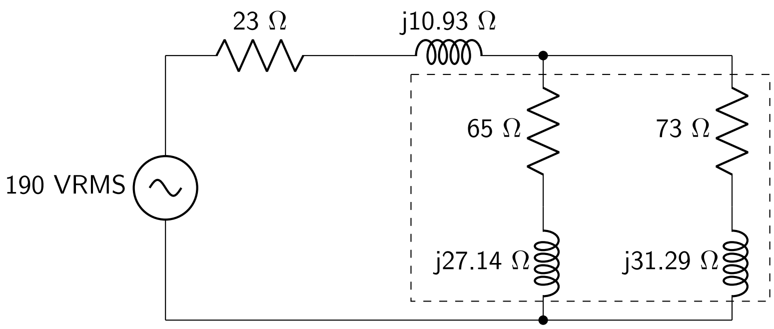


Figure 10.17: Circuit diagram for power factor correction question 20.

Phasor transform the circuit.



Calculate the load impedance.

$$\begin{aligned}\mathbf{Z_{LOAD}} &= (65 + j27.14\ \Omega)//(73 + j31.29\ \Omega)\\ &= 34.38 + j14.54\ \Omega\end{aligned}$$

Calculate the current through the load.

$$\begin{aligned}\mathbf{I_{LOAD}} &= \frac{190\ \text{VRMS}}{23 + j10.93 + 34.38 + j14.54\ \Omega}\\ &= 2.77 - j1.23\ \text{ARMS}\\ &= 3.03\ \text{ARMS}\angle - 23.93^\circ\end{aligned}$$

Calculate the voltage dropped over the load.

$$\begin{aligned}\mathbf{V_{LOAD}} &= (2.77 - j1.23\ \text{ARMS})(34.38 + j14.54\ \Omega)\\ &= 112.96 - j2.00\ \text{VRMS}\\ &= 112.98\ \text{VRMS}\angle - 1.02^\circ\end{aligned}$$

Calculate the power consumed by the load.

$$\begin{aligned}\mathbf{S_{LOAD}} &= |\mathbf{I_{LOAD,RMS}}|^2\mathbf{Z_{LOAD}}\\ &= (3.03\ \text{ARMS})^2(34.38 + j14.54\ \Omega)\\ &= 314.91 + j133.13\ \text{VA}\end{aligned}$$

Calculate the initial power factor.

$$\begin{aligned}pf &= \frac{P}{|\mathbf{S}|}\\ &= \frac{314.91}{341.90}\\ &= 0.921\end{aligned}$$

Calculate the final reactive power required to increase the power factor to the desired level.

$$\begin{aligned}Q_f &= P\sqrt{\frac{1}{pf_f^2} - 1}\\ &= 314.91\sqrt{\frac{1}{0.960^2} - 1}\\ &= 91.85\ \text{VAR}\end{aligned}$$

Calculate Q_C.

$$\begin{aligned} Q_C &= Q_f - Q \\ &= 91.85 \text{ VAR} - 133.13 \text{ VAR} \\ &= -41.29 \text{ VAR} \end{aligned}$$

Calculate the compensating capacitance.

$$\begin{aligned} C &= \frac{-Q_C}{|\mathbf{V_{LOAD,RMS}}|^2 \omega} \\ &= \frac{41.29}{|112.98|^2 (2\pi 60)} \\ &= 8.58\ \mu\text{F} \end{aligned}$$

11 Chapter 11 Solutions

11.1 Laplace Transforms

1. Derive the Laplace transform of $f(t) = 2\ \mathbf{e}^{-5t}\ u(t)$.

Use the Laplace transform describing the exponential function as well as the multiplication property.

$$F(s) = \frac{2}{s+5}$$

2. Derive the Laplace transform of $f(t) = \left[6\ t\ \mathbf{e}^{-4t} + 2\ \mathbf{e}^{-4t} + 5\right]\ u(t)$.

Use the Laplace transforms describing the exponential function, exponential function times time, and constant.

$$F(s) = \frac{6}{(s+4)^2} + \frac{2}{(s+4)} + \frac{5}{s}$$

Multiply all terms so they share a common denominator, and simplify to obtain $F(s)$.

$$\begin{aligned} F(s) &= \frac{6s + 2s(s+4) + 5(s+4)^2}{s(s+4)^2} \\ &= \frac{7s^2 + 54s + 80}{s(s^2 + 8s + 16)} \end{aligned}$$

3. Derive the Laplace transform of $f(t) = \mathbf{e}^{-t}\left[2\cos(10t) + 4\sin(10t)\right]\ u(t)$.

Use the Laplace transforms describing exponential functions multiplied by sine and cosine. Simplify to obtain $F(s)$.

$$\begin{aligned} F(s) &= \frac{2(s+1)}{(s+1)^2 + 100} + \frac{40}{(s+1)^2 + 100} \\ &= \frac{2s + 42}{s^2 + 2s + 101} \end{aligned}$$

4. Derive the Laplace transform of $v(t)$. The differential equation describing $v(t)$ is given in equation 11.1. Note that $v(0) = 3$ V.

$$\frac{dv(t)}{dt} + 0.25v(t) = 0 \tag{11.1}$$

Use the differentiation property of the Laplace transform, then solve for $V(s)$. All units are V, Ω, A, F, and H.

$$\begin{aligned} 0 &= sV(s) - v(0) + 0.25V(s) \\ &= sV(s) - 3 + 0.25V(s) \\ V(s) &= \frac{3}{s+0.25} \end{aligned}$$

5. Derive the Laplace transform of $i(t)$. The differential equation describing $i(t)$ is given in equation 11.2. Note that $i(0) = 10$ A and $i'(0) = -5$ A/s.

$$\frac{d^2i(t)}{dt^2} + 6\frac{di(t)}{dt} + 109i(t) = 0 \tag{11.2}$$

Use the differentiation property of the Laplace transform, then solve for $I(s)$. All units are V, Ω, A, F, and H.

$$\begin{aligned} 0 &= s^2I(s) - si(0) - i'(0) + 6\left(sI(s) - i(0)\right) + 109I(s) \\ &= s^2I(s) - 10s + 5 + 6sI(s) - 60 + 109I(s) \\ 10s + 55 &= s^2I(s) + 6sI(s) + 109I(s) \\ I(s) &= \frac{10s+55}{s^2+6s+109} \end{aligned}$$

11.2 Inverse Laplace Transforms

6. Derive $f(t)$ by calculating the Laplace transform of $F(s)$, given in equation 11.3. Identify the terms that contribute to the transient response, then identify the terms that contribute to the steady-state response.

$$F(s) = \frac{10}{s+10} \tag{11.3}$$

Use the Laplace transform describing the exponential function as well as the multiplication property. Because the term is a decaying exponential, it contributes to the transient response. There are no steady-state terms.

$$f(t) = 10\ \mathrm{e}^{-10t}\ u(t)$$

7. Derive $f(t)$ by calculating the Laplace transform of $F(s)$, given in equation 11.4. Identify the terms that contribute to the transient response, then identify the terms that contribute to the steady-state response.

$$F(s) = \frac{10s^2 + 10}{s(s^2 + 10s + 25)} \tag{11.4}$$

Perform partial fraction expansion on $F(s)$, solve for the coefficients, and then use the Laplace transforms describing constants, exponential functions, and exponential times time.

$$\begin{aligned} F(s) &= \frac{K_1}{s} + \frac{K_2}{(s+5)^2} + \frac{K_3}{(s+5)} \\ &= \frac{0.4}{s} + \frac{-52}{(s+5)^2} + \frac{9.6}{(s+5)} \\ f(t) &= \left[0.4 - 52\ t\ \mathrm{e}^{-5t} + 9.6\ \mathrm{e}^{-5t}\right]\ u(t) \end{aligned}$$

The steady-state solution is described by terms without decaying exponentials:

$$f_{ss} = 0.4\ u(t)$$

The transient solution is described by terms with decaying exponentials:

$$f_{tr} = \left[-52\ t\ \mathrm{e}^{-5t} + 9.6\ \mathrm{e}^{-5t}\right]\ u(t)$$

8. Derive $f(t)$ by calculating the Laplace transform of $F(s)$, given in equation 11.5. Identify the terms that contribute to the transient response, then identify the terms that contribute to the steady-state response.

$$F(s) = \frac{s^2 + 4}{(s^2 + 16)(s^2 + 5s + 4)} \tag{11.5}$$

Perform partial fraction expansion on $F(s)$, solve for the coefficients, and then use the Laplace transforms describing exponential functions and decaying sinusoidal functions.

$$\begin{aligned} F(s) &= \frac{K_1}{(s+j4)} + \frac{K_2}{(s-j4)} + \frac{K_3}{(s+1)} + \frac{K_4}{(s+4)} \\ &= \frac{0.055 + j0.033}{(s+j4)} + \frac{0.055 - j0.033}{(s-j4)} + \frac{0.098}{(s+1)} + \frac{-0.208}{(s+4)} \\ f(t) &= \left[0.11\ \cos(4t) + 0.066\ \sin(4t) + 0.098\ \mathrm{e}^{-t} - 0.208\ \mathrm{e}^{-4t}\right]\ u(t) \end{aligned}$$

The steady-state solution is described by terms without decaying exponentials:

$$f_{ss} = [0.11\ \cos(4t) + 0.066\ \sin(4t)]\ u(t)$$

The transient solution is described by terms with decaying exponentials:

$$f_{tr} = \left[0.098\ \mathrm{e}^{-t} - 0.208\ \mathrm{e}^{-4t}\right]\ u(t)$$

9. Derive $f(t)$ by calculating the Laplace transform of $F(s)$, given in equation 11.6. Identify the terms that contribute to the transient response, then identify the terms that contribute to the steady-state response.

$$F(s) = \frac{5s^2 + 10s + 3}{(s+2)^2(s^2 + 5s + 64)} \tag{11.6}$$

Perform partial fraction expansion on $F(s)$, solve for the coefficients, and then use the Laplace transforms describing exponential functions and decaying sinusoidal functions.

$$\begin{aligned} F(s) &= \frac{K_1}{(s+2)^2} + \frac{K_2}{(s+1)} + \frac{K_3}{(s+2.5-j7.6)} + \frac{K_4}{(s+2.5+j7.6)} \\ &= \frac{0.052}{(s+2)^2} + \frac{-0.173}{(s+1)} + \frac{0.087-j0.331}{(s+2.5-j7.6)} + \frac{0.087+j0.331}{(s+2.5+j7.6)} \\ f(t) &= \left[0.052\ t\ \mathrm{e}^{-2t} - 0.173\ \mathrm{e}^{-2t} + \mathrm{e}^{-2.5t}\left(0.173\ \cos(7.6t) + 0.663\ \sin(7.6t)\right)\right]\ u(t) \end{aligned}$$

There are no steady-state terms in the output response, as all terms are multiplied by a decaying exponential.

10. Derive $f(t)$ by calculating the Laplace transform of $F(s)$, given in equation 11.7. Identify the terms that contribute to the transient response, then identify the terms that contribute to the steady-state response.

$$F(s) = \frac{-4s^2 + 10}{s(s^2 + 25)(s^2 + 20s + 100)} \tag{11.7}$$

Perform partial fraction expansion on $F(s)$, solve for the coefficients, and then use the Laplace transforms describing constant terms, sinusoidal functions, and exponential functions.

$$\begin{aligned} F(s) &= \frac{K_1}{s} + \frac{K_2}{(s+j5)} + \frac{K_3}{(s-j5)} + \frac{K_4}{(s+10)^2} + \frac{K_5}{(s+10)} \\ &= \frac{0.004}{s} + \frac{-0.011-j0.014}{(s+j5)} + \frac{-0.011+j0.014}{(s-j5)} + \frac{0.312}{(s+10)^2} + \frac{0.017}{(s+10)} \\ f(t) &= \left[0.004 - 0.021\ \cos(5t) + 0.028\ \sin(5t) + 0.312\ t\ \mathrm{e}^{-10t} + 0.017\ \mathrm{e}^{-10t}\right]\ u(t) \end{aligned}$$

The steady-state solution is described by terms without decaying exponentials:

$$f_{ss} = [0.004 - 0.021\ \cos(5t) + 0.028\ \sin(5t)]\ u(t)$$

The transient solution is described by terms with decaying exponentials:

$$f_{tr} = \left[0.312\ t\ \mathrm{e}^{-10t} + 0.017\ \mathrm{e}^{-10t}\right]\ u(t)$$

11.3 s-Domain Analysis

11. Use s-domain analysis to calculate $V(s)$ and $v(t)$ of the circuit shown in figure 11.1. The switch moves from position a to b at a time of zero seconds. The component values are: $V_{S1} = 3$ V, $V_{S2} = 2$ V, $R_1 = 50\ \Omega$, $R_2 = 100\ \Omega$, $C = 250\ \mu$F.

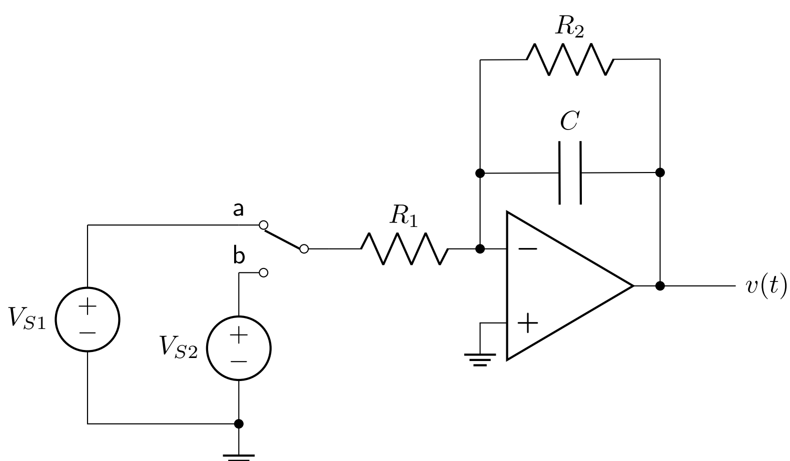
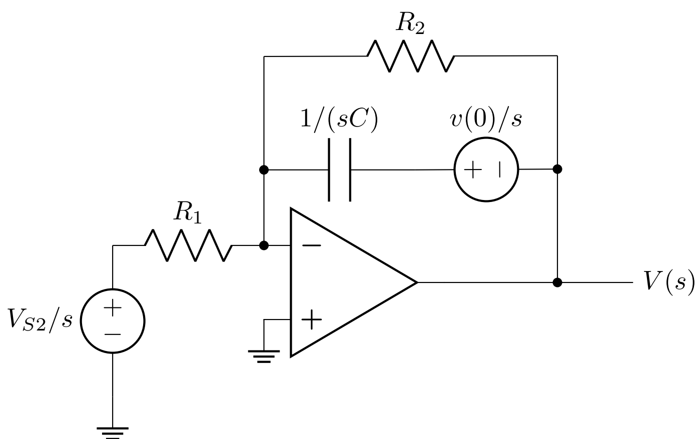


Figure 11.1: Circuit diagram for s-domain analysis question 11.

Calculate the initial conditions of the circuit. At $t > 0$ the switch is in position a and the capacitor can be treated as an open.

$$\begin{aligned} v(0) &= -V_{S1}\left(\frac{R_2}{R_1}\right) \\ &= -3\ \text{V}\left(\frac{100\ \Omega}{50\ \Omega}\right) \\ &= -6\ \text{V} \end{aligned}$$

Transform the circuit into the s-domain.



Use KCL at the inverting node of the op-amp to derive an equation in terms of $V(s)$, then solve for $V(s)$ and normalize.

$$\begin{aligned} \frac{V_{S2}}{sR_1} &= \frac{-V(s)}{R_2} + \frac{-V(s) + \frac{v(0)}{s}}{\frac{1}{sC}} \\ &= \frac{-V(s)}{R_2} - sCV(s) + Cv(0) \\ -\frac{V_{S2}}{sR_1} + Cv(0) &= \frac{V(s)}{R_2} + sCV(s) \\ -\frac{V_{S2}}{sCR_1} + v(0) &= \frac{V(s)}{CR_2} + sV(s) \\ V(s) &= \frac{sv(0) - \frac{V_{S2}}{CR_1}}{s(s + \frac{1}{CR_2})} \end{aligned}$$

Plug in component values, then perform a partial fraction expansion. Inverse Laplace transform to obtain an equation for $v(t)$.

$$\begin{aligned} V(s) &= \frac{-6s - 160}{s(s+40)} \\ &= \frac{K_1}{s} + \frac{K_2}{(s+40)} \\ &= \frac{-4}{s} + \frac{-2}{(s+40)} \\ v(t) &= \left[-4 \text{ V} - 2 \text{ V } \mathrm{e}^{-40t}\right] u(t) \end{aligned}$$

12. Use s-domain analysis to calculate $V(s)$ of the circuit shown in figure 11.2. The switch closes at a time of zero seconds.

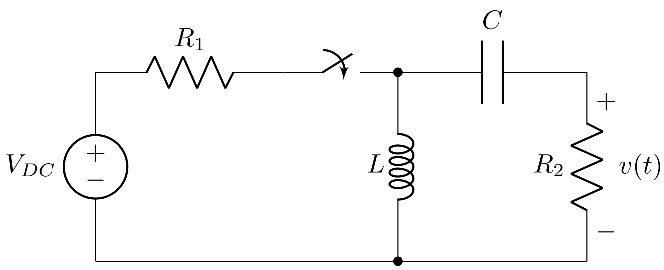
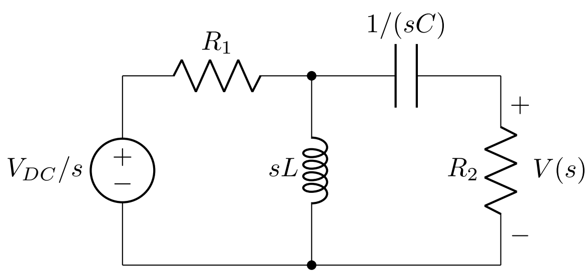


Figure 11.2: Circuit diagram for s-domain analysis question 12.

The initial current flow through the inductor and initial voltage drop over the capacitor are both zero, as no source is connected before the switch closes. Transform the circuit into the s-domain.



Perform KCL and KVL to obtain an equation in terms of $V(s)$, then solve for $V(s)$ and normalize. (Note:

V_X is the voltage drop over the inductor.)

$$\begin{aligned}
\frac{\frac{V_{DC}}{s} - V_X}{R_1} &= \frac{V_X}{sL} + \frac{V(s)}{R_2} \\
\frac{V_{DC}}{R_1 s} &= V_X \left[\frac{1}{R_1} + \frac{1}{sL}\right] + \frac{V(s)}{R_2} \\
V_X &= \frac{V(s)}{sCR_2} + V(s) \\
\frac{V_{DC}}{R_1 s} &= \left[\frac{1}{R_1} + \frac{1}{sL}\right]\left[\frac{V(s)}{sCR_2} + V(s)\right] + \frac{V(s)}{R_2} \\
&= \frac{V(s)}{sCR_1R_2} + \frac{V(s)}{R_1} + \frac{V(s)}{s^2LCR_2} + \frac{V(s)}{sL} + \frac{V(s)}{R_2} \\
\frac{sV_{DC}}{R_1} &= \left[\frac{1}{R_1} + \frac{1}{R_2}\right] s^2V(s) + \left[\frac{1}{CR_1R_2} + \frac{1}{L}\right] sV(s) + \left[\frac{1}{LCR_2}\right] V(s) \\
\frac{sR_2V_{DC}}{R_1 + R_2} &= s^2V(s) + \left[\frac{L + CR_1R_2}{R_1 + R_2}\right] sV(s) + \left[\frac{R_1}{LC(R_1 + R_2)}\right] V(s) \\
V(s) &= \frac{\left[\frac{V_{DC}R_2}{R_1+R_2}\right] s}{s^2 + \left[\frac{L+CR_1R_2}{LC(R_1+R_2)}\right] s + \left[\frac{R_1}{LC(R_1+R_2)}\right]}
\end{aligned}$$

No component values have been provided, therefore, this is as far as the solution can go.

13. Use s-domain analysis to calculate $V(s)$ and $v(t)$ of the circuit shown in figure 11.3. The switch moves from position a to b at a time of zero seconds. The component values are: $V_{S1} = 6$ V, $V_{S2} = 20$ V, $R_1 = 4\ \Omega$, $R_2 = 2\ \Omega$, $C = 50\ \mu$F, $L = 1$ mH.

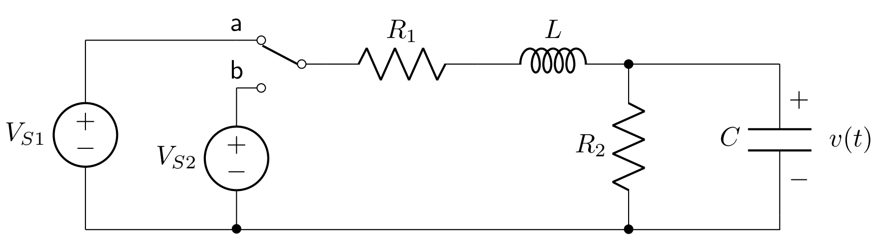
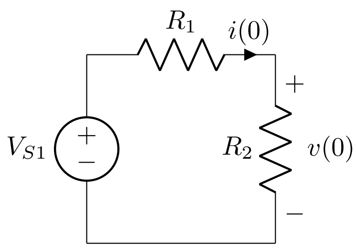


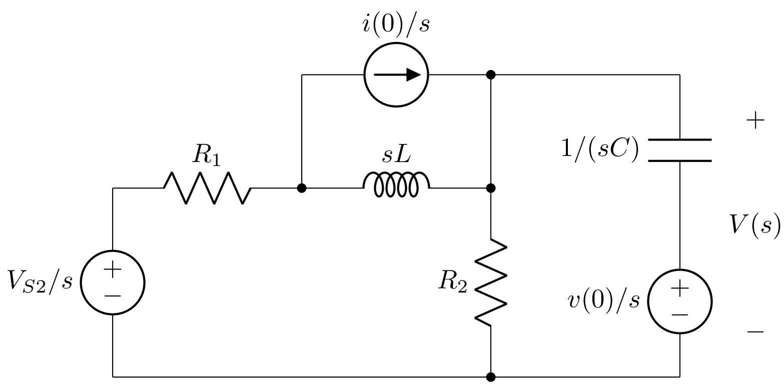
Figure 11.3: Circuit diagram for s-domain analysis question 13.

Calculate the initial conditions of the circuit.



$$i(0) = \frac{V_{S1}}{R_1 + R_2}$$
$$= 1 \text{ A}$$
$$v(0) = V_{S1}\left(\frac{R_2}{R_1 + R_2}\right)$$
$$= 2 \text{ V}$$

Transform the circuit into the s-domain.



Perform KCL to find an equation in terms of $V(s)$. Then, solve for $V(s)$ and normalize.

$$
\begin{aligned}
\frac{\frac{V_{S2}}{s}+\frac{LV_{S1}}{R_1+R_2}-V(s)}{R_1+sL} &= \frac{V(s)}{R_2}+\frac{V(s)-\frac{V_{S1}R_2}{s(R_1+R_2)}}{\frac{1}{sC}} \\
\frac{V_{S2}}{s}+\frac{LV_{S1}}{R_1+R_2} &= V(s)+(R_1+sL)\left[\frac{V(s)}{R_2}+sCV(s)-\frac{CV_{S1}R_2}{R_1+R_2}\right] \\
&= V(s)+\frac{R_1}{R_2}V(s)+\frac{sL}{R_2}V(s)+sCR_1V(s)+s^2LCV(s) \\
&\quad -\frac{CV_{S1}R_1R_2}{R_1+R_2}-\frac{sLCV_{S1}R_2}{R_1+R_2} \\
\frac{V_{S2}}{s}+\frac{LV_{S1}}{R_1+R_2}+\frac{CV_{S1}R_1R_2}{R_1+R_2}+\frac{sLCV_{S1}R_2}{R_1+R_2} &= (LC)s^2V(s)+\left[\frac{L}{R_2}+CR_1\right]sV(s)+\left[\frac{R_1+R_2}{R_2}\right]V(s) \\
\left[\frac{V_{S1}R_2}{R_1+R_2}\right]s+\left[\frac{LV_{S1}+CV_{S1}R_1R_2}{LC(R_1+R_2)}\right]+\left[\frac{V_{S2}}{LC}\right]\frac{1}{s} &= s^2V(s)+\left[\frac{1}{CR_2}+\frac{R_1}{L}\right]sV(s)+\left[\frac{R_1+R_2}{LCR_2}\right]V(s) \\
\left[\frac{V_{S1}R_2}{R_1+R_2}\right]s^2+\left[\frac{LV_{S1}+CV_{S1}R_1R_2}{LC(R_1+R_2)}\right]s+\left[\frac{V_{S2}}{LC}\right] &= s\left(s^2V(s)+\left[\frac{1}{CR_2}+\frac{R_1}{L}\right]sV(s)+\left[\frac{R_1+R_2}{LCR_2}\right]V(s)\right) \\
V(s) &= \frac{\left[\frac{V_{S1}R_2}{R_1+R_2}\right]s^2+\left[\frac{LV_{S1}+CV_{S1}R_1R_2}{LC(R_1+R_2)}\right]s+\left[\frac{V_{S2}}{LC}\right]}{s\left(s^2+\left[\frac{1}{R_2C}+\frac{R_1}{L}\right]s+\left[\frac{R_1+R_2}{LCR_2}\right]\right)}
\end{aligned}
$$

Plug in component values, then perform a partial fraction expansion. Inverse Laplace transform to obtain an equation for $v(t)$.

$$
\begin{aligned}
V(s) &= \frac{2s^2+28000s+4\text{E}8}{s\left(s^2+14000s+6\text{E}7\right)} \\
&= \frac{K_1}{s}+\frac{K_2}{(s+7000-j3166.6)}+\frac{K_3}{(s+7000+j3166.6)} \\
&= \frac{6.67}{s}+\frac{-2.333+j4.925}{(s+7000-j3166.6)}+\frac{-2.333-j4.925}{(s+7000+j3166.6)} \\
v(t) &= \left[6.67\text{ V}+\text{e}^{-7000t}\left(-4.67\text{ V }\cos(3316.63t)+9.85\text{ V }\sin(3316.63t)\right)\right]\,u(t)
\end{aligned}
$$

14. Use s-domain analysis to calculate $I(s)$ and $i(t)$ of the circuit shown in figure 11.4. The switch moves from position b to a at a time of zero seconds. The component values are: $V_{S1}=40$ V, $\alpha=4$ rad/s, $V_{S2}=10$ V, $R=50\ \Omega$, $C=10\ \mu$F, $L=400$ mH.

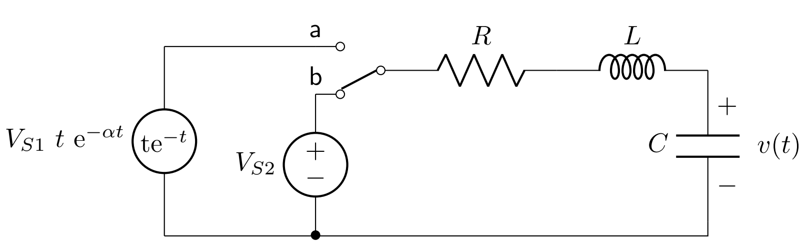
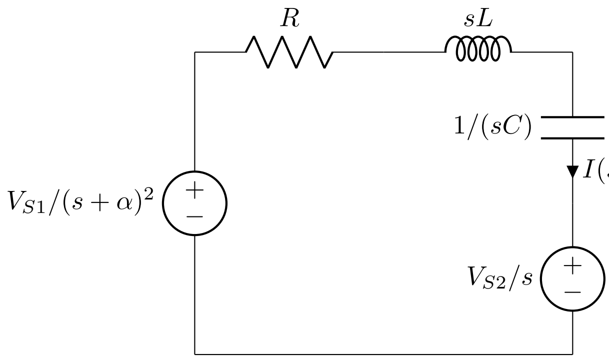


Figure 11.4: Circuit diagram for s-domain analysis question 14.

Calculate the initial conditions of the circuit. The inductor acts like a short and the capacitor acts like an open. For this reason, $i(0)$ is zero, and $v(0)$ is equal to V_{S2}. Transform the circuit into the s-domain.



Perform KVL around the loop to obtain an equation in terms of $I(s)$. Then, solve for $I(s)$ and normalize.

$$\begin{aligned}
\frac{V_{S1}}{(s+\alpha)^2} - \frac{V_{S2}}{s} &= RI(s) + sLI(s) + \frac{I(s)}{sC} \\
\frac{sCV_{S1}}{(s+\alpha)^2} - CV_{S2} &= sCRI(s) + s^2LCI(s) + I(s) \\
\frac{sV_{S1}}{L(s+\alpha)^2} - \frac{V_{S2}}{L} &= s^2I(s) + \frac{R}{L}sI(s) + \frac{1}{LC}I(s) \\
\frac{sV_{S1} - V_{S2}(s+\alpha)^2}{L(s+\alpha)^2} &= s^2I(s) + \frac{R}{L}sI(s) + \frac{1}{LC}I(s) \\
\frac{sV_{S1} - V_{S2}s^2 - s2\alpha V_{S2} - V_{S2}\alpha^2}{L(s+\alpha)^2} &= s^2I(s) + \frac{R}{L}sI(s) + \frac{1}{LC}I(s) \\
I(s) &= \frac{\left[\frac{-V_{S2}}{L}\right]s^2 + \left[\frac{V_{S1}-2\alpha V_{S2}}{L}\right]s + \left[\frac{-V_{S2}\alpha^2}{L}\right]}{(s+\alpha)^2\left(s^2 + \left[\frac{R}{L}\right]s + \left[\frac{1}{LC}\right]\right)}
\end{aligned}$$

Plug in component values, then perform a partial fraction expansion. Inverse Laplace transform to obtain an equation for $i(t)$. (Note: the equation for $I(s)$ is in terms of A, V, etc., but the coefficients are mA once the partial fraction expansion is derived.)

$$\begin{aligned}
I(s) &= \frac{-25s^2 - 100s - 400}{(s+4)^2\,(s^2 + 125s + 2.5\text{E}5)} \\
&= \frac{K_1}{(s+4)^2} + \frac{K_2}{(s+4)} + \frac{K_3}{(s+62.5+j496.08)} + \frac{K_4}{(s+62.5-j496.08)} \\
&= \frac{-1.603}{(s+4)^2} + \frac{0.4}{(s+4)} + \frac{-0.201-j25.220}{(s+62.5+j496.08)} + \frac{-0.201+j25.220}{(s+62.5-j496.08)} \\
i(t) &= [-1.6 \text{ mA/s } t\, \mathrm{e}^{-4t} + 0.4 \text{ mA } \mathrm{e}^{-4t} \\
&\quad + \mathrm{e}^{-62.5t}\left(-0.4 \text{ mA } \cos(496.08t) + 50.44 \text{ mA } \sin(496.08t)\right)]\, u(t)
\end{aligned}$$

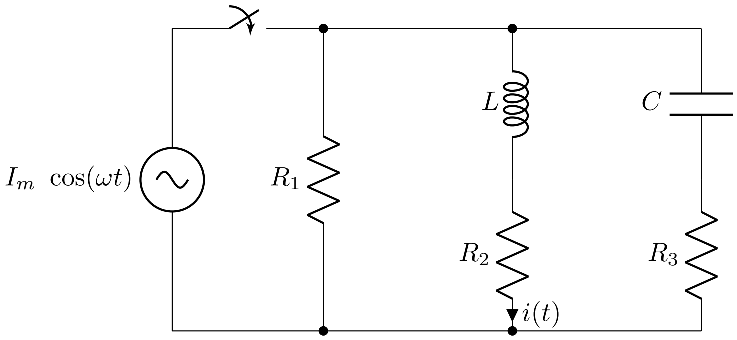
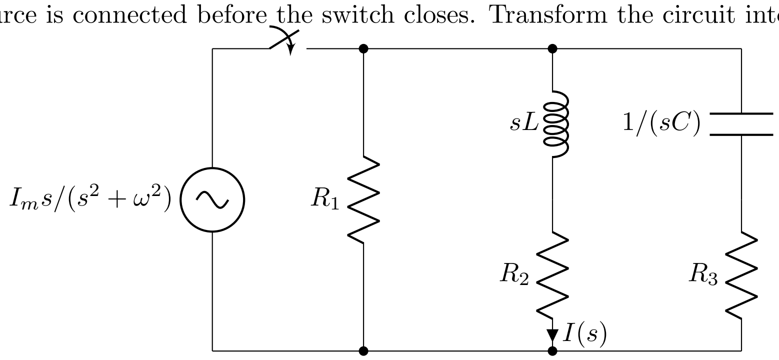


Figure 11.5: Circuit diagram for s-domain analysis question 15.

15. Use s-domain analysis to calculate $I(s)$ and $i(t)$ of the circuit shown in figure 11.5. The switch closes at a time of zero seconds. The component values are: $I_m = 500$ mA, $\omega = 60$ rad/s, $R_1 = 200\ \Omega$, $R_2 = 2\ \Omega$, $R_3 = 1\ \Omega$, $C = 20\ \mu$F, $L = 50$ mH.

The initial current flow through the inductor and initial voltage drop over the capacitor are both zero, as no source is connected before the switch closes. Transform the circuit into the s-domain.



Perform KCL and KVL to obtain an equation in terms of $I(s)$. Then, solve for $I(s)$ and normalize.

(Note: $V(s)$ is defined as the voltage drop over resistor R_1.)

$$\begin{aligned}
\frac{I_m s}{s^2+\omega^2} &= \frac{V(s)}{R_1} + I(s) + \frac{V(s)}{R_3+\frac{1}{sC}} \\
V(s) &= sLI(s) + R_2 I(s) \\
\frac{I_m s}{s^2+\omega^2} &= \frac{sLI(s)}{R_1} + \frac{R_2 I(s)}{R_1} + I(s) + \frac{sLI(s)+R_2I(s)}{R_3+\frac{1}{sC}} \\
\left(\frac{I_m s}{s^2+\omega^2}\right)\left(R_3+\frac{1}{sC}\right) &= \frac{sLI(s)}{R_1}\left(R_3+\frac{1}{sC}\right) + \frac{R_2I(s)}{R_1}\left(R_3+\frac{1}{sC}\right) + I(s)\left(R_3+\frac{1}{sC}\right) \\
&\quad + sLI(s) + R_2I(s) \\
\frac{I_m R_3 s}{s^2+\omega^2} + \frac{I_m}{C(s^2+\omega^2)} &= \frac{sLR_3I(s)}{R_1} + \frac{L}{R_1C}I(s) + \frac{R_2R_3I(s)}{R_1} + \frac{R_2}{sCR_1}I(s) \\
&\quad + R_3I(s) + \frac{1}{sC}I(s) + sLI(s) + R_2I(s) \\
\frac{CI_mR_3s^2 + I_m s}{C(s^2+\omega^2)} &= \left[\frac{LR_3}{R_1}+L\right]s^2I(s) + \left[\frac{L}{R_1C}+\frac{R_2R_3}{R_1}+R_3+R_2\right]sI(s) + \left[\frac{R_2}{CR_1}+\frac{1}{C}\right]I(s) \\
\frac{CI_mR_1R_3s^2 + R_1I_m s}{LC(R_1+R_3)(s^2+\omega^2)} &= s^2I(s) + \left[\frac{1}{C(R_1+R_3)} + \frac{R_2R_3+R_1R_3+R_1R_2}{L(R_1+R_3)}\right]sI(s) + \left[\frac{R_1+R_2}{LC(R_1+R_3)}\right]I(s) \\
I(s) &= \frac{\left[\frac{CR_1R_3I_m}{LC(R_1+R_3)}\right]s^2 + \left[\frac{R_1I_m}{LC(R_1+R_3)}\right]s}{(s^2+\omega^2)\left(s^2+\left[\frac{1}{C(R_1+R_3)}+\frac{R_2R_3+R_1R_3+R_1R_2}{L(R_1+R_3)}\right]s+\left[\frac{R_1+R_2}{LC(R_1+R_3)}\right]\right)}
\end{aligned}$$

Plug in component values, then perform a partial fraction expansion. Inverse Laplace transform to obtain an equation for $i(t)$.

$$\begin{aligned}
I(s) &= \frac{9.95s^2 + 497512.44s}{(s^2+3600)(s^2+308.66s+1004975.12)} \\
&= \frac{K_1}{(s+j60)} + \frac{K_2}{(s-j60)} + \frac{K_3}{(s+154.33+j990.53)} + \frac{K_4}{(s+154.33-j990.53)} \\
&= \frac{0.248+j0.004}{(s+j60)} + \frac{0.248-j0.004}{(s-j60)} + \frac{-0.248-j0.034}{(s+154.33+j990.53)} + \frac{-0.248+j0.034}{(s+154.33-j990.53)} \\
i(t) &= [0.5\text{ A }\cos(60t) - 0.01\text{ A }\sin(60t) \\
&\quad + \mathrm{e}^{-154.33t}\left(-0.5\text{ A }\cos(990.5t) + 0.07\text{ A }\sin(990.5t)\right)]\,u(t)
\end{aligned}$$