# $\odot$

Alyssa J. Pasquale, Ph.D. College of DuPage

# Circuit Analysis I Lab Manual

Spring 2025 Edition

this lab manual belongs to

# Table of Contents

License and Attribution Information $\ldots \ldots \mathbf{iv}$
Lab 1: Introduction to Analog Concepts and Measurement Tools
Lab 2: Ohm's Law and Kirchhoff's Circuit Laws 17
Lab 3: Transformations and Measurement Circuits
Lab 4: Superposition
Lab 5: Source Transformation and Equivalent Circuits
Lab 6: Operational Amplifiers
Lab 7: Capacitors
Lab 8: RC Circuits
Lab 9: Inductors and RL Circuits
Lab 10: Second Order Circuits
Lab 11: Filters
Lab 12: Phasors and Impedance
Lab 13: Diodes
Lab 14: AC Power
Lab 15: Course Review

# License and Attribution Information

This lab manual is licensed under creative commons as CC-BY-SA-NC. This license allows reusers to distribute, remix, adapt, and build upon the material in any medium or format for noncommercial purposes only, and only so long as attribution is given to the creator. If you remix, adapt, or build upon the material, you must license the modified material under identical terms. For more information, visit https://creativecommons.org.

This license (CC-BY-SA-NC) includes the following elements:

() BY – Credit must be given to the creator

S NC – Only noncommercial uses of the work are permitted

• SA – Adaptations must be shared under the same terms

The suggested attribution for this book is "Circuit Analysis I Lab Manual" by Alyssa J. Pasquale, Ph.D., College of DuPage, is licensed under CC BY-NC-SA 4.0.

The entirety of this work was created by Alyssa J. Pasquale, Ph.D. All circuit diagrams and figures in this text were created by the author using LAT<sub>F</sub>X libraries.

# Pre-Lab 1

Carefully read the entirety of Lab 1, then answer the following questions. Attach a separate sheet of paper, if necessary, to show all work and calculations.

1. What three things can be measured with a multimeter?

(a)

(b)

(c)

2. Can resistance measurements be made in the presence of an external voltage source? Why or why not?

3. What electrical connections will be made with...

(a) ...red wires?

(b) ...black wires?

- 4. How must a multimeter be hooked up to measure...
  - (a) ...voltage?

(b) ...current?

- 5. What is the internal resistance of a...
  - (a) ...voltmeter?

(b) ...ammeter?

6. Given the answers to the previous question, explain why precautions must be taken when measuring current in a circuit.

7. If the diameter of each cylindrical resistivity rod is 0.1 cm, what is the cross-sectional area of the rods? Ensure that your answer has units of  $\rm cm^2$ .

# Lab 1: Introduction to Analog Concepts and Measurement Tools

In this introductory lab, you will explore the concepts of power, current, voltage, resistivity, and resistance at several experimental stations in lab. In the process, you will start to become familiar with lab equipment such as tabletop multimeters and power supplies.

For lab resources and information, go to the following URL or scan the QR code. doctor-pasquale.com/circuit-analysis-lab-1



# 1.1 Analog Circuit Components

The analog circuit components that you will be working with in this lab include a voltage source, current source, resistor, variable resistor, and wire. Each of these components can be described with a symbol when drawn in a circuit diagram, as shown in figure 1.1.



Figure 1.1: Circuit symbols for a voltage source, current source, resistor, variable resistor, and wire.

# 1.2 Measuring Voltage, Current and Resistance

It is necessary to measure various properties of a circuit using a multimeter. Among other things, a multimeter can measure voltage, current, and resistance. Individually, a voltmeter is used to measure voltage, an ammeter is used to measure current, and an ohmmeter is used to measure resistance.

### 1.2.1 Measuring Voltage

As described in the textbook, voltage must be measured in parallel across the area of interest, as shown in figure 1.2. In order to maintain the characteristics of the circuit, a voltmeter must have a very high internal resistance to discourage current from flowing through it rather than the other circuit components. This high internal resistance makes voltage measurements rather safe to perform.



**Figure 1.2:** Circuit diagram depicting a voltmeter (left) and the correct configuration of a voltmeter in a circuit (right).

#### 1.2.2 Measuring Resistance

Resistance measurements also need to be performed in parallel across the component(s) of interest, as shown in figure 1.3. An ohmmeter functions by sending a small voltage over the resistors and measuring the current that results. Because of the fact that it supplies its own voltage, resistance measurements must be performed in the absence of an external voltage source.



**Figure 1.3:** Circuit diagram depicting an ohmmeter (left) and the correct configuration of an ohmmeter in a series resistance circuit (center) and parallel resistance circuit (right).

### 1.2.3 Measuring Current

As described in the textbook, current must be measured in series with the area of interest, as shown in figure 1.4. Because of the fact that all current in the circuit must flow through the ammeter, it has a very low internal resistance. Thus, it is imperative to use caution when hooking up a multimeter to measure current! If current is accidentally measured in parallel with a voltage source, it will result in a short circuit and can blow a fuse.



**Figure 1.4:** Circuit diagram depicting an ammeter (left) and the correct configuration of an ammeter in a circuit (right).

### 1.3 Voltage

Voltage (symbol: V), also known as potential difference or electromotive force (EMF), corresponds to the amount of energy it takes to move a unit of positive charge from the point of low potential to the point of high potential in a circuit. It can be thought of as the "push" in a circuit that causes current to flow. It is measured in units of volts (V).

**Circuit 1:** Build each of the circuits shown in figures 1.5-1.7, and record the voltage that is measured by the voltmeter in each circuit. The polarity of the voltmeter is indicated with a + sign pointing to where the high potential lead should be connected. Each measurement should be slightly different. Record your data in table 1.1. When you have finished recording your data, show it to your instructor to receive a stamp.



Figure 1.5: Circuit for voltage configuration 1 (left) and 2 (right).



Figure 1.6: Circuit for voltage configuration 3 (left) and 4 (right).



Figure 1.7: Circuit for voltage configuration 5 (left) and 6 (right).

Configuration	Voltage
1	
2	
3	
4	
5	
6	

Table 1.1: Table of voltage data.

Instructor Stamp: \_

#### 1.4 Current

Current (symbol: I) is the rate of flow of electric charge in a circuit past a given point. It is measured in units of amperes (A). The relationship between current and voltage can give useful information about the contents of a circuit. Figure 1.8 shows the voltage and current relationships for a resistor (left), a diode (center), and an incandescent lightbulb (right).



**Figure 1.8:** Voltage and current relationships for a resistor (left), a diode (center), and an incandescent lightbulb (right).

**Circuit 2:** Connect one of the mystery boxes (a, b, or c) to a variable power supply and ammeter, as shown in the figure 1.9 (left). Each mystery box contains one of the items depicted in the voltage and current relationship graphs shown in Figure 1.8 above. Vary the voltage from 5 V to 0 V in increments of half of a volt.

Then, reverse the connections from the power supply to apply voltages between -5 V to 0 V, as shown in figure 1.9 (left). Do **not** change the orientation of the ammeter, or you will wind up with the same set of conditions as you had in the positive voltage case!



Figure 1.9: Circuit diagram for positive voltages (left) and negative voltages (right).

Record the current for each of these voltages, for each of the three mystery boxes, in tables 1.2–1.4. When you have finished recording your data, show it to your instructor to receive a stamp.

V (V)	I (mA)	V (V)	l (mA)	V (V)	I (mA)	V (V)	I (mA)	V (V)	l (mA)
-5.0		-2.5		-0.5		1.5		3.5	
-4.5		-2.0		0.0		2.0		4.0	
-4.0		-1.5		0.5		2.5		4.5	
-3.5		-1.0		1.0		3.0		5.0	
-3.0									

Table 1.2: Data table for mystery box a.

V (V)	I (mA)	V (V)	l (mA)	V (V)	I (mA)	V (V)	I (mA)	V (V)	I (mA)
-5.0		-2.5		-0.5		1.5		3.5	
-4.5		-2.0		0.0		2.0		4.0	
-4.0		-1.5		0.5		2.5		4.5	
-3.5		-1.0		1.0		3.0		5.0	
-3.0									

Table 1.3: Data table for mystery box b.

V (V)	l (mA)	V (V)	l (mA)	V (V)	l (mA)	V (V)	I (mA)	V (V)	l (mA)
-5.0		-2.5		-0.5		1.5		3.5	
-4.5		-2.0		0.0		2.0		4.0	
-4.0		-1.5		0.5		2.5		4.5	
-3.5		-1.0		1.0		3.0		5.0	
-3.0									

Table 1.4: Data table for mystery box c.

Instructor Stamp: \_\_\_\_\_

# 1.5 Resistivity

Resistivity (symbol:  $\rho$ ) is a property inherent to all materials. It quantifies how much a material inhibits the flow of electric current. The units of resistivity are ohm-meter ( $\Omega$ m).

**Circuit 3:** Use the Pasco resistivity experiment that has been set up. Open the file saved on the desktop called **ResistivityExperiment**; this will initialize the software. Then, carry out the following instructions for each of the five metal rods (each of the rods has a diameter of 0.1 cm). When you have finished collecting data, show it to your instructor to receive a stamp.

- 1. Move the sliders on the resistivity setup to their extreme ends (up the ramps)
- 2. Loosen the nuts on both ends
- 3. Insert one of the metal rods.
- 4. Tighten both of the nuts.
- 5. Move the left-hand slider to the 0 cm position, and the right-hand slider to the 5 cm position.
- 6. In the Pasco software, go to the Data tab, then click on Signal Generator.
- 7. Set the 850 Output to a DC voltage of 100 mV.
- 8. Click Record at the bottom of the screen. Wait a few seconds until the numbers stop changing and then click Stop.
- 9. Record the resistance (in units of m $\Omega$ ) in table 1.5.
- 10. Repeat with the right-hand slider at 10 cm, 15 cm, and 20 cm.
- 11. Repeat for each metal.

Length	Copper	Brass	Aluminum	Steel	Nichrome
5 cm					
10 cm					
15 cm					
20 cm					

Table 1.5: Data table for resistivity rods.

Instructor Stamp: \_\_\_\_

### 1.6 Resistance

Resistance (symbol: R) has to do with how much a circuit element inhibits the flow of electric current. The units of resistance are ohms ( $\Omega$ ). The way in which the resistors are configured (in series, where the current must flow through all resistors; in parallel, where current can flow through multiple paths; or a mixture of both) determines the overall resistance of the circuit.

**Circuit 4:** Gather three different resistors that have values between  $1 \text{ k}\Omega$  and  $9.9 \text{ k}\Omega$  (the third stripe color will be red). Use an ohmmeter to measure the resistance of each resistor, and record these values in table 1.6.

Resistor	Measured Value
1	
2	
3	

Table 1.6: Measured resistance values.

Then, measure the resistance of the resistor configurations shown in figures 1.10–1.12. Record your data in table 1.7. When you have finished recording your data, show it to your instructor to receive a stamp.



Figure 1.10: Circuit for resistance configuration 1 (left) and 2 (right).



Figure 1.11: Circuit for resistance configuration 3 (left) and 4 (right).



Figure 1.12: Circuit for resistance configuration 5 (left) and 6 (right).

Configuration	Resistance
1	
2	
3	
4	
5	
6	

Table 1.7: Table of resistance data.

Instructor Stamp: \_\_\_\_\_

#### 1.7 Power

Power (symbol: P) is the rate of energy loss within a circuit element. It is measured in units of watts (W). Power can be calculated using the equation P = IV. Within a circuit, all of the power absorbed by circuit elements is equal to all of the power supplied by sources.

Circuit 5: Ask your instructor what value of R to use in your circuit, and record the value below.

Build the circuit shown in figure 1.13, using the resistor value given to you by your instructor as R, and a potentiometer wired as a variable resistor as  $R_X$ .



Figure 1.13: Circuit diagram for the power circuit.

Dial the knob on the variable resistor  $(R_X)$  until  $V_X$  is zero. Record the measured value of  $V_X$  in table 1.8. Then, measure the value of I, which is the current flowing through the circuit. (Note: you cannot measure both the voltage and current simultaneously.) Record this in table 1.8.

Calculate  $V_X/I$  to find the value of the variable resistor. Calculate  $(V_X)(I)$  to find the power delivered to the variable resistor. Record both of these values in table 1.8.

Dial the potentiometer so that the voltage drop increases. Repeat until you have at least 10 data points from one extreme of the potentiometer to the other.

Type your data into the class Google drive folder (the link is available on Blackboard). Every group will share their recorded data here to ensure a wide range of values along the full range of the potentiometer. When you have finished recording your data, show it to your instructor to receive a stamp.

Vx (V)	I (mA)	P (mW)	Rx (k $\Omega$ )

 Table 1.8: Table of power measurements and calculations.

Instructor Stamp: \_\_\_\_\_

# Lab 1 Homework

Carefully read each question before answering. Show all work or justify your answers to receive credit. Attach a separate sheet of paper, if necessary, to show all work and calculations.

- 1. Based on the results you measured in lab Circuit 1, what general effect does the orientation of a voltage source have on measured output voltage? (Be specific with your answer: avoid saying something like "it switches signs.")
- 2. Based on the results you measured in lab Circuit 1, what general effect does the orientation of a current source have on the measured output voltage. (Again, be specific.)
- 3. For the next three graphs, you will be asked to plot current on the x-axis and voltage on the y-axis. What physical property does the slope of these graphs represent?
- 4. If current has units of mA, and voltage has units of V, what are the units of this physical property?
- 5. Use graphing software to create a graph of the current-voltage relationship for mystery box a. **Current** should be on the x-axis and voltage on the y-axis. Ensure that the data points are clearly shown. If the data is linear, generate a best-fit line and include the equation on your printout. If the data is not linear, do not include a fit line! No matter what, do not "connect-the-dots" between data points. Ensure that all axes are labeled with correct units. This graph is required to obtain points on the next two questions.
- 6. What element is inside of mystery box a? Justify your answer.
- 7. Is the value of the slope constant throughout the entire range of -5 V to 5 V? If so, what is the slope of the data? If not, does the slope increase or decrease as the current increases?

- 8. Use graphing software to create a graph of the current-voltage relationship for mystery box b. **Current should be on the x-axis and voltage on the y-axis.** Ensure that the data points are clearly shown. If the data is linear, generate a best-fit line and include the equation on your printout. If the data is not linear, do not include a fit line! No matter what, do not "connect-the-dots" between data points. Ensure that all axes are labeled with correct units. This graph is required to obtain points on the next two questions.
- 9. What element is inside of mystery box b? Justify your answer.

10. Is the value of the slope constant throughout the entire range of -5 V to 5 V? If so, what is the slope of the data? If not, does the slope increase or decrease as the current increases?

- 11. Use graphing software to create a graph of the current-voltage relationship for mystery box c. **Current should be on the x-axis and voltage on the y-axis.** Ensure that the data points are clearly shown. If the data is linear, generate a best-fit line and include the equation on your printout. If the data is not linear, do not include a fit line! No matter what, do not "connect-the-dots" between data points. Ensure that all axes are labeled with correct units. This graph is required to obtain points on the next two questions.
- 12. What element is inside of mystery box c? Justify your answer.

13. Is the value of the slope constant throughout the entire range of -5 V to 5 V? If so, what is the slope of the data? If not, does the slope increase or decrease as the current increases?

- 14. Use graphing software to plot all of the resistivity data that you collected in lab Circuit 3. All data should be on a single graph. The length should be on the x-axis and the resistance on the y-axis. Ensure that all axes are labeled with correct units. Include linear best-fit lines for all of the data, and include the equations on your plot. No matter what, do not "connect-the-dots" between data points. This graph is required to obtain points on the following resistivity calculation questions.
- 15. The slope of the best-fit equations is equal to the resistance divided by the length (in units of m $\Omega$  / cm). Multiply these slopes by the area of the metal rod (which you calculated in the pre-lab) to calculate resistivity using equation 1.1.

$$\rho = A\left(\frac{R}{l}\right) \tag{1.1}$$

(a) Copper (Cu)

(b) Brass

(c) Aluminum (Al)

(d) Steel

(e) Nichrome

16. Based on the results you measured in lab Circuit 4, qualitatively describe what happens to the overall resistance when two or more resistors are connected in series.

17. Based on the results you measured in lab Circuit 4, qualitatively describe what happens to the overall resistance when two or more resistors are connected in parallel.

18. Explain if all possible resistor configurations (not limited to those you've seen in this lab) are purely series or purely parallel combinations (or is there another alternative).

- 19. Use graphing software to create a graph of the resistance–power relationship that was measured in lab Circuit 5. Use the Google docs spreadsheet to include data from all students. Plot the load resistance on the x-axis and power on the y-axis. Ensure that the data points are clearly shown. The data will not be linear, so do not include a fit line. No matter what, do not "connect-the-dots" between data points. Ensure that all axes are labeled with correct units. This graph is required to obtain points on the next two questions.
- 20. Determine the approximate value of  $R_X$  (in units of  $k\Omega$ ) that leads to maximum power transfer. If your answer has a precision of hundredth's place or greater, justify the use of that precision. (In other words, explain exactly how you calculated, measured, or otherwise derived that value, and why your result is accurate enough to justify that amount of precision.) A range of values will not be accepted.

21. Compare the value of  $R_X$  that leads to maximum power transfer to the value of the other resistor in the circuit.

# Pre-Lab 2

Carefully read the entirety of Lab 2, then answer the following questions. Attach a separate sheet of paper, if necessary, to show all work and calculations.

1. Explain the difference between a short circuit and an open circuit. (You may want to let your answer to question 3 on this pre-lab help inform your answer to this question. Hint: are all shorts bad shorts? What are the properties of a short circuit **in general**?)

2. Why can short circuits be dangerous?

3. Are all short circuits dangerous? Why or why not?

- 4. When using 1/4 W resistors, what is the smallest single resistor value that can be placed in parallel with a constant voltage source of...
  - (a) ... 6 V?
  - (b) ... 8 V?
  - (c) ... 10 V?

(d) ... 12 V?

- 5. When using 1/4 W resistors, what is the largest single resistor value that can be placed in series with a constant current supply of...
  - (a)  $\dots$  4 mA?

(b) ... 6 mA?

(c) ... 8 mA?

(d) ... 10 mA?

6. Assume that you have access to all possible values of 1/4 W resistors. You need to use them to create an equivalent resistance of 50  $\Omega$  with a voltage source of 5 V. How can you accomplish this without burning out any resistors? Try to find a solution that uses as few resistors as possible.

7. Assume that you have access to all possible values of 1/4 W resistors. You need to use them to create an equivalent resistance of 4 k $\Omega$  with a current source of 10 mA. How can you accomplish this without burning out any resistors? Try to find a solution that uses as few resistors as possible.

# Lab 2: Ohm's Law and Kirchhoff's Circuit Laws

Ohm's law and Kirchhoff's circuit laws are two very powerful tools that can be used to analyze and design analog circuits. Ohm's law gives a relationship between voltage, current, and resistance. Kirchhoff's circuit laws can be used to determine the voltage and current at various parts in a circuit.

For lab resources and information, go to the following URL or scan the QR code. doctor-pasquale.com/circuit-analysis-lab-2



# 2.1 Series and Parallel Circuits

Circuit components can be wired together in different ways. Depending on how components are connected at nodes (a point where two or more circuit elements meet), they can be said to be either series or parallel. (Note: many circuits are not purely series or purely parallel but may contain elements of both configurations. Some circuits are neither. This section discusses the two specific sets of circuits that are purely series or purely parallel.)

### 2.1.1 Series Circuits

Circuit elements are said to be connected in series when they all exist within a single branch. There is only one path connecting each circuit component together, so electrons must travel through each and every one. In a circuit composed of a source and series resistors, current must flow through all of the resistors. Therefore, when resistors are connected in series, the equivalent resistance of the circuit goes up, as described by equation 2.1.

$$R_{EQ} = \sum_{i} R_i \tag{2.1}$$

### 2.1.2 Parallel Circuits

Circuit elements are said to be connected in parallel when they connect at both nodes (which causes each element to be within its own branch). This creates multiple branches for electrons to travel through. Therefore, when resistors are connected in parallel, the equivalent resistance of the circuit goes down, as described by equation 2.2.

$$\frac{1}{R_{EQ}} = \sum_{i} \frac{1}{R_i} \tag{2.2}$$

**Circuit 1:** With the resistors that are available to you, design a series circuit that has an equivalent resistance of 5,400  $\Omega$ . More than one resistor must be used. All resistors must be connected in series. Draw the circuit diagram. Build the circuit, then demonstrate its functionality to your instructor to receive a stamp.

Instructor Stamp: \_\_\_\_\_

**Circuit 2:** With the resistors that are available to you, design a series circuit that has an equivalent resistance of 195 k $\Omega$ . More than one resistor must be used. All resistors must be connected in series. Draw the circuit diagram. Build the circuit, then demonstrate its functionality to your instructor to receive a stamp.

Instructor Stamp: \_\_\_\_\_

**Circuit 3:** With the resistors that are available to you, design a parallel circuit that has an equivalent resistance of 250  $\Omega$ . More than one resistor must be used. All resistors must be connected in parallel. Draw the circuit diagram. Build the circuit, then demonstrate its functionality to your instructor to receive a stamp.

Instructor Stamp: \_\_\_\_\_

**Circuit 4:** With the resistors that are available to you, design a parallel circuit that has an equivalent resistance of 330  $\Omega$ . More than one resistor must be used. All resistors must be connected in parallel. Draw the circuit diagram. Build the circuit, then demonstrate its functionality to your instructor to receive a stamp.

Instructor Stamp: \_\_\_\_\_

#### 2.2 Ohm's Law

Ohm's describes the relationship between voltage and current in a resistor. This relationship is defined by equation 2.3.

$$V = IR \tag{2.3}$$

The power consumed by a resistor can be determined by solving the equation for power (P = IV) simultaneously with Ohm's law. Equations 2.4 and 2.5 are used to determine the power consumed by a resistor if only voltage or current are known along with the value of the resistor.

$$P = \frac{V^2}{R} \tag{2.4}$$

$$P = I^2 R \tag{2.5}$$

It is important to note that resistors are rated with a particular power level. Exceeding this amount of power in a resistor will cause it to melt. Most of the resistors used in this class are 1/4 W resistors. Circuits must therefore be designed not to exceed 1/4 W on any individual resistor!

#### 2.2.1 Short Circuits

Ideally, wires are highly conducting and have an effective resistance of 0  $\Omega$ . Because their resistance is 0, if you were to measure the voltage drop over a wire, it would be 0 V.

Short circuits can be dangerous. If a voltage source is directly connected at both nodes with a wire, as shown in figure 2.1, the effective resistance of the circuit is 0, causing the current  $I \to \infty$ . All of this current flows through the wire until it melts or a fuse blows in the power supply. For this reason it is imperative that the exposed contacts of red (wires that are reserved for connections to high potential) and black (wires that are reserved for connections to low potential) wires never touch each other! Furthermore, because the internal resistance of an ammeter is close to zero, this dangerous short circuit will occur if current is measured in parallel with a voltage source.



Figure 2.1: A short circuit with a low-resistance path, which will cause overheating.

Not all short circuits are inherently dangerous, however. It is possible to short out an individual circuit element while still maintaining a high enough resistance within a circuit to prevent any of the other elements from melting. An example circuit diagram of this scenario is shown in figure 2.2. Only a single resistor in figure 2.2 is shorted out; assuming that the two other resistors are high enough to withstand the current flowing through the circuit, everything will be fine. While this circuit might not cause catastrophic damage to the breadboard, power supply, and other circuit components, it still may lead to undesirable side effects (incorrect voltage and current values throughout the circuit) if the short circuit isn't part of the circuit design.



Figure 2.2: A short circuit with a resistance in the short circuit path, which may not cause overheating.

### 2.2.2 Open Circuits

An ideal insulator (for example, air) is a material that offers infinite resistance  $(R \to \infty \Omega)$  to current. In other words, electrons cannot flow through the material and current is zero (I = 0 A), even in the presence of a potential difference. The circuit diagram in figure 2.3 depicts a circuit that contains a branch with an open.



Figure 2.3: An circuit featuring a branch with an open.

**Circuit 5:** Use one or more resistors to generate a current of 1.5 mA with a voltage source of 10 V. Ensure that the power consumed by any individual resistor is less than 1/4 W. Draw the circuit diagram (include the placement of the ammeter in your diagram). Build the circuit, then demonstrate its functionality to your instructor to receive a stamp.

Instructor Stamp: \_

**Circuit 6:** Use one or more resistors to generate a current of 4.5 mA with a voltage source of 6 V. Ensure that the power consumed by any individual resistor is less than 1/4 W. Draw the circuit diagram (include the placement of the ammeter in your diagram). Build the circuit, then demonstrate its functionality to your instructor to receive a stamp.

Instructor Stamp: \_\_\_\_

### 2.3 Kirchhoff's Voltage Law (KVL) and the Voltage Divider Rule

KVL states that all voltage drops in a loop have to sum to zero (conservation of energy). A consequence of this is the voltage divider rule, which applies when all resistors are connected in series. The voltage divider rule is described by equation 2.6, where  $V_i$  is the voltage dropped over the  $i^{\text{th}}$  resistor,  $V_s$  is the source voltage,  $R_i$  is the resistance of the  $i^{\text{th}}$  resistor, and  $R_{EQ}$  is the equivalent series resistance of the circuit.

$$V_i = V_s \left(\frac{R_i}{R_{EQ}}\right) \tag{2.6}$$

**Circuit 7:** Connect 8 V to the red and black binding posts of the voltage divider circuit, shown in figure 2.4 (left). This circuit contains three unknown resistors, which are labeled in the diagram as  $R_1$ ,  $R_2$ , and  $R_3$ .

- Connect the voltmeter between nodes **a** and **b** to measure and record voltage  $V_{ab}$ .
- Connect the voltmeter between nodes **b** and **c** to measure and record voltage  $V_{bc}$ .
- Connect the voltmeter between nodes c and d to measure and record voltage  $V_{cd}$ .
- Verify that these three voltages add up to 8 V.



Figure 2.4: KVL circuit layout, without an ammeter (left) and with an ammeter (right).

Finally, connect an ammeter in series between your power supply and the circuit, shown in figure 2.4 (right), to measure the current that flows through all three resistors. Record the current below.

When you have finished recording and verifying your data, show the results to your instructor to receive a stamp.

Instructor Stamp: \_\_

**Circuit 8:** Design the circuit in figure 2.5 to give an output voltage equal to your group number divided by two. (Therefore, if you are in group 8, the output voltage should be 4 V.) Ensure that the power consumed by any one resistor will be less than one quarter watt.



Figure 2.5: Circuit diagram for the group number voltage divider circuit.

Record your resistor values (include units!) in table 2.1.

Resistor	Value
R1	
R2	
R3	

Table 2.1: Table of resistor values.

Build the circuit on your breadboard, and verify your results with your instructor to receive a stamp.

Instructor Stamp: \_\_\_\_\_

### 2.4 Kirchhoff's Current Law (KCL) and the Current Divider Rule

KCL states that the sum of all currents entering a node is equal to zero (conservation of charge). A consequence of this is the current divider rule, which applies when all resistors are connected in parallel. The current divider rule is described by equation 2.7, where  $I_i$  is the current flowing through the  $i^{\text{th}}$  resistor,  $I_s$  is the source current,  $R_i$  is the resistance of the  $i^{\text{th}}$  resistor, and  $R_{EQ}$  is the equivalent parallel resistance of the circuit.

$$I_i = I_s \left(\frac{R_{EQ}}{R_i}\right) \tag{2.7}$$

**Circuit 9:** Design the circuit in figure 2.6 to give an output current equal to your group number divided by two. (Therefore, if you are in group 6, the output current should be 3 mA.) Ensure that the power consumed by any one resistor will be less than one quarter watt.



Figure 2.6: Circuit diagram for the group number current divider circuit.

Record your resistor values (include units!) in table 2.2.

Resistor	Value
R1	
R2	
R3	

Table 2.2: Table of resistor values.

Build the circuit on your breadboard, and verify your results with your instructor to receive a stamp.

Instructor Stamp: \_\_\_

**Circuit 10:** Follow the instructions as given below. When you have finished measuring all of your data, verify your results with your instructor to receive a stamp. The circuit board diagram is shown in figure 2.7.



Figure 2.7: KCL circuit board layout.

- Connect the power supply to the appropriate terminals on the constant current source. Slowly and carefully dial the power supply to 15 V.
- If you would like, you may connect the output of the current source to the ammeter to verify that the current is actually 6 mA as you dial the potentiometer on the current source. Once you have finished this optional step, connect the current supply to the KCL board as shown below. (Note: the current source will not display any current reading until it is connected to a load or to the ammeter.)
- Connect the ammeter between the current source and resistor  $R_1$  (between binding posts **a** and **b**). Then, connect banana plugs between the power supply and resistor  $R_2$  (between binding posts **a** and **c**), and between the power supply and resistor  $R_3$  (between binding posts **a** and **d**). Record  $I_a$  below.
- Connect the ammeter between the current source and resistor  $R_2$  (between binding posts **a** and **c**). Then, connect banana plugs between the power supply and resistor  $R_1$  (between binding posts **a** and **b**), and between the power supply and resistor  $R_3$  (between binding posts **a** and **d**). Record  $I_b$  below.
- Connect the ammeter between the current source and resistor  $R_3$  (between binding posts **a** and **d**). Then, connect banana plugs between the power supply and resistor  $R_1$  (between binding posts **a** and **b**), and between the power supply and resistor  $R_2$  (between binding posts **a** and **c**). Record  $I_c$  below.
- Disconnect the power supply and current source. Then, connect banana plugs along the top rails (between a and b, between b and c, and between c and d). Connect the multimeter to measure resistance between the red and black binding posts (this is the equivalent resistance of the circuit). Record  $R_{EQ}$  below.

Instructor Stamp: \_\_\_

# Lab 2 Homework

Carefully read each question before answering. Show all work or justify your answers to receive credit. Attach a separate sheet of paper, if necessary, to show all work and calculations.

- 1. The following questions pertain to lab Circuit 7. You will also need to refer to equation 2.6 describing the voltage divider.
  - (a) Use experimental evidence from this lab to explain how this circuit verifies KVL.
  - (b) Use Ohm's Law to calculate the total resistance of the circuit.
  - (c) Use the voltage divider equation to determine the value of resistor  $R_1$ .
  - (d) Use the voltage divider equation to determine the value of resistor  $R_2$ .
  - (e) Use the voltage divider equation to determine the value of resistor  $R_3$ .
  - (f) Which resistor has the largest voltage drop? Is this the resistor with the highest or lowest resistance value?
  - (g) Which resistor has the smallest voltage drop? Is this the resistor with the highest or lowest resistance value?

- 2. The following questions pertain to lab Circuit 8.
  - (a) Calculate the total current flowing through all three resistors.

(b) Calculate the voltage drop over resistor  $R_1$ .

(c) Calculate the power consumed by resistor  $R_1$ .

(d) Calculate the voltage drop over resistor  $R_2$ .

(e) Calculate the power consumed by resistor  $R_2$ .

(f) Calculate the power consumed by resistor  $R_3$ . (You may not need to calculate the voltage drop if your circuit was correctly designed. Be sure to refer to your lab feedback to check that your output was adequate. If not, you will need to calculate the actual voltage drop over this resistor, or use the current flowing through the resistor to make this calculation.)

- 3. The following questions pertain to lab Circuit 9.
  - (a) Calculate the total voltage drop over all three resistors.

(b) Calculate the current flow through resistor  $R_1$ .

(c) Calculate the power consumed by resistor  $R_1$ .

(d) Calculate the current flow through resistor  $R_2$ .

(e) Calculate the power consumed by resistor  $R_2$ .

(f) Calculate the power consumed by resistor  $R_3$ . (You may not need to calculate the current if your circuit was correctly designed. Be sure to refer to your lab feedback to check that your output was adequate. If not, you will need to calculate the actual current through this resistor, or use the voltage through the resistor to make this calculation.)

- 4. The following questions pertain to lab Circuit 10. You will also need to refer to equation 2.7 describing the current divider.
  - (a) Use experimental evidence from this lab to explain how this circuit verifies KCL.

(b) Use the current divider equation to determine the value of resistor  $R_1$ .

(c) Use the current divider equation to determine the value of resistor  $R_2$ .

(d) Use the current divider equation to determine the value of resistor  $R_3$ .

(e) Which resistor has the largest current flow? Is this the resistor with the highest or lowest resistance value?

(f) Which resistor has the smallest current flow? Is this the resistor with the highest or lowest resistance value?

# Pre-Lab 3

Carefully read the entirety of Lab 3, then answer the following questions. Attach a separate sheet of paper, if necessary, to show all work and calculations.

- 1. Your instructor gives you a mystery box that has three nodes, as shown in figure 3.2. The following questions will help you to determine the process for figuring out what is inside the mystery box, which you will do in this lab. You measure the resistance between nodes a and b to be 500  $\Omega$ , between b and c to be 600  $\Omega$ , and between a and c to be 800  $\Omega$ .
  - (a) Calculate the wye resistances.

(b) Calculate the delta resistances.

- 2. Your instructor gives you a second mystery box that has three nodes, as shown in figure 3.2. You measure the resistance between nodes a and b to be  $1.2 \text{ k}\Omega$ , between b and c to be  $1.2 \text{ k}\Omega$ , and between a and c to be  $1.8 \text{ k}\Omega$ .
  - (a) Calculate the wye resistances.

(b) Calculate the delta resistances.

3. Carefully read Circuit 4. Select a battery voltage (between 3–4 V) to use as the low voltage limit, with a precision in the tenth's place.

4. Use the voltage divider rule to calculate the value of  $R_2$  on a 10 k $\Omega$  potentiometer that will be required to obtain an output voltage of 2.6 V given the threshold you selected in the previous question.
# Lab 3: Transformations and Measurement Circuits

Delta-wye and wye-delta transformations utilize circuit laws to convert resistor networks into simpler formations to more easily reduce circuits and calculate their properties. In addition, various measurement circuits will be build that can characterize temperature, voltage, and resistance properties.

For lab resources and information, go to the following URL or scan the QR code. doctor-pasquale.com/circuit-analysis-lab-3

## 3.1 Delta-Wye and Wye-Delta Transformations

Delta and wye refer to the configuration of resistors in a circuit, shown in figure 3.1. Measuring any circuit property (voltage, current, or resistance) between any two identical nodes on each circuit gives identical values. Because of this, if you are given a black box with three terminals and three resistors inside, it would be impossible for you to know whether the configuration is delta or wye without opening the box.



Figure 3.1: Delta circuit configuration (left) and wye circuit configuration (right).

The delta-wye transformation converts a circuit from a delta configuration to a wye configuration, using equations 3.1-3.3.

$$R_a = \frac{R_1 R_3}{R_1 + R_2 + R_3} \tag{3.1}$$

$$R_b = \frac{R_1 R_2}{R_1 + R_2 + R_3} \tag{3.2}$$

$$R_c = \frac{R_2 R_3}{R_1 + R_2 + R_3} \tag{3.3}$$

The wye-delta transformation converts a circuit from a wye configuration to a delta configuration. This is simply the reverse of a delta-wye transform. The equations describing the wye-delta transformation are given in equations 3.4–3.6.

$$R_{1} = \frac{R_{a}R_{b} + R_{b}R_{c} + R_{a}R_{c}}{R_{c}}$$
(3.4)

$$R_{2} = \frac{R_{a}R_{b} + R_{b}R_{c} + R_{a}R_{c}}{R_{a}}$$
(3.5)

$$R_{3} = \frac{R_{a}R_{b} + R_{b}R_{c} + R_{a}R_{c}}{R_{b}}$$
(3.6)



Two mystery boxes have been built with three terminals, as shown in figure 3.2. Use these to complete the following two circuits.



Figure 3.2: Mystery box circuit configuration.

**Circuit 1:** Obtain delta-wye **mystery box 1** from your instructor. Take as many measurements as you need to determine what is inside of the box. Draw and label an equivalent delta circuit and an equivalent wye circuit. When these drawings are complete, show them to your instructor to receive a stamp.

Instructor Stamp: \_\_

**Circuit 2:** Obtain delta-wye **mystery box 2** from your instructor. Take as many measurements as you need to determine what is inside of the box. Draw and label an equivalent delta circuit and an equivalent wye circuit. When these drawings are complete, show them to your instructor to receive a stamp.

Instructor Stamp: \_\_\_\_\_

### 3.2 Thermistor

A thermistor is a resistor whose resistance is a function of temperature. If the relationship between the thermistor resistance and temperature is known, any device that can measure the resistance can be used to determine the temperature. The circuit symbol for a thermistor is shown in figure 3.3.

Figure 3.3: Circuit symbol of a thermistor.

The thermistor that will be used in this lab is a negative temperature coefficient (NTC) thermistor, as the resistance decreases as temperature increases. The relationship between temperature and resistance is quantified by equation 3.7, where T is the ambient temperature in Kelvin,  $T_0$  is the reference temperature in Kelvin,  $\beta$  is the thermistor constant, R is the resistance of the thermistor, and  $R_0$  is the reference resistance.

$$\frac{1}{T} = \frac{1}{T_0} + \frac{1}{\beta} \ln\left(\frac{R}{R_0}\right) \tag{3.7}$$

Note that the thermistor is temperature compensated, which means that your measurements will not be affected by any power consumed by the thermistor itself.

**Circuit 3:** Connect the thermistor as shown in figure 3.4. In the lab homework, you will be asked to calculate the ambient temperature. The thermistor used in the circuit has parameters given in table 3.1.



Figure 3.4: Thermistor voltage divider circuit.

Parameter	Value
β	4038
Т0	25 °C
R0	10 k $\Omega$

Table 3.1: Table of thermistor parameters.

Measure the output voltage and the resistance of the 14 k $\Omega$  resistor and record both values in table 3.2.

Parameter	Value
Vout	
R14k	

Table 3.2: Table of measured values.

When you have completed this circuit, demonstrate your results to your instructor to receive a stamp.

Instructor Stamp: \_\_\_\_

### 3.3 Light-Emitting Diodes

Light-emitting diodes (LEDs) are passive circuit components that light up if a sufficient amount of current flows through the device from anode to cathode. The circuit symbol is shown in figure 3.5.



Figure 3.5: Circuit symbol of a light-emitting diode (LED)

Current can only flow in one direction through an LED. Schematically, current flows in the same direction of the triangle in the circuit symbol (shown in figure 3.5). This means that high potential must be present on the anode (left side of the circuit diagram) and low potential must be present on the cathode (right side of the circuit diagram) for the LED to light up.

There are two ways to distinguish the anode from cathode in a physical LED, depicted in figure 3.6. The anode has a longer leg and is connected to the epoxy cap in a section where the cap has a rounded edge. The cathode has a shorter leg and is connected to the epoxy cap in a section where the cap has a flat edge.



**Figure 3.6:** Depiction of an actual LED. The anode corresponds to the long leg connected to the round end of the epoxy cap. The cathode corresponds to the short leg connected to the flat end of the epoxy cap.

### 3.4 Adjustable Shunt Regulator

An adjustable shunt regulator is a rather complicated device that can act with a simple function: above a certain voltage, it will act like an open circuit, and below that voltage, it will act like a short circuit. In this way, when the device is acting as a short circuit, it will act as a shunt (which is a device that causes current to be diverted through itself, rather than go somewhere else). The device is called adjustable because the threshold voltage can be set by the circuit designer. While the specifics of the shunt regulator are outside the scope of this lab (and this class), the circuit symbol is shown in figure 3.7.



Figure 3.7: Circuit symbol for an adjustable shunt regulator.

The shunt regulator works as follows.

- If  $V_{REF} > V_F$ , the device acts as a shunt (short circuit)
- If  $V_{REF} < V_F$ , the device acts as an open

The adjustable shunt regulator that will be used in this lab is the TL431, with a pinout diagram shown in figure 3.8.



Figure 3.8: Pinout diagram for the TL431 adjustable shunt regulator.

**Circuit 4:** This circuit will use a voltage divider to set the threshold on an adjustable shunt regulator to create a low battery indicator circuit. A potentiometer will be used as a voltage divider. The fixed resistance of the potentiometer will be applied across a battery, and the position of the wiper will be selected specifically to produce a particular voltage  $(V_{OUT})$  under the condition of a low battery voltage  $(V_{BATT})$ , as shown in figure 3.9.





As long as the battery remains above a low voltage threshold, an LED will be shorted out and will remain unlit. Once the battery voltage dips below the threshold, the short will become an open, causing the LED to light up. The threshold for this particular circuit (shown in figure 3.10), will be 2.6 V. Therefore, the voltage divider equation for the potentiometer is described by equation 3.8, where  $R_2$  is the resistance between the wiper and ground, and  $R_1 + R_2$  is the total resistance of the potentiometer.

$$2.6 \text{ V} = V_{LOW BATT} \left(\frac{R_2}{R_1 + R_2}\right) \tag{3.8}$$





In the pre-lab, you selected a low battery voltage between 3–4 V, and an  $R_2$  value required to obtain a wiper voltage of 2.6 V when that voltage is present on the battery. Tune a potentiometer to obtain  $R_2$  equal to the value you calculated. Then, build the circuit as shown in figure 3.10. Use a DC power supply to act as the battery. Start with the power supply at 5 V. The LED should remain unlit. Then, slowly dial the voltage down. You should see that once the power supply dips below your low voltage threshold, that the LED turns on. When the circuit is working correctly, demonstrate it to your instructor to receive a stamp.

#### Instructor Stamp: \_\_

#### 3.5 Wheatstone Bridge

A Wheatstone bridge is an electric circuit that can be used to measure the resistance of an unknown material. In the circuit diagram shown in figure 3.11,  $R_X$  is an unknown resistor and  $R_2$  is a variable resistor. The bridge is balanced when  $V_{db} = 0$  V. In this case,  $R_X$  can be calculated based on the other resistor values, given by equation 3.9.

$$R_X = \frac{R_2 R_3}{R_1}$$
(3.9)



Figure 3.11: Circuit diagram of a Wheatstone bridge.

The Wheatstone bridge may seem antiquated when precision multimeters are available. However, there are benefits to the Wheatstone bridge. First, it is not necessary to know the value of the input voltage to measure the value of  $R_X$ . In addition, the input voltage can fluctuate and not affect the measurements. In addition, because the voltmeter measures a value of 0, the voltmeter does not need to be calibrated. These days, the Wheatstone bridge is not used to measure resistances, but to measure very small fluctuations in resistance (for example, due to temperature) that an ohmmeter is not capable of measuring.

**Circuit 5:** Wire up a Wheatstone bridge circuit with  $V_S$  no greater than 4 V. Use  $R_1 = 100 \Omega$  and  $R_3 = 560 \Omega$ . Measure and record these values to make your calculations as accurate as possible.

Dial the variable resistor until the voltmeter reads as close to 0 V as possible. To measure the value of the variable resistor, you must physically disconnect the potentiometer from the other resistors and the power supply. Otherwise, the presence of other resistors or the voltage source will change the measurement. Use the measured value to calculate the value of each mystery resistor.

When you have calculated each of the mystery resistor values, record them in table 3.3 and show the values to your instructor to receive a stamp. (Note: be sure that you record the value of the mystery resistor, and not the value of the variable resistor!)

Resistor	Value
red	
yellow	
blue	

Table 3.3: Mystery resistor values.

Instructor Stamp: \_\_\_\_

# Lab 3 Homework

Carefully read each question before answering. Show all work or justify your answers to receive credit. Attach a separate sheet of paper, if necessary, to show all work and calculations.

1. Consider the mystery boxes in Circuit 1 and Circuit 2. Without opening the box, could you tell if the circuit is a delta configuration or a wye configuration? Why or why not?

2. Consider the thermistor in Circuit 3. Calculate the resistance of the thermistor.

3. Using equation 3.7, calculate the ambient temperature in °C. (Be careful, the equation requires you to use units of Kelvin for temperature.)

- 4. Consider the battery level indicator in Circuit 4.
  - (a) Would it be possible for the battery level circuit to turn on the LED when the battery voltage dips below 2 V?

(b) If your answer to the previous question was yes, how could you do this? If your answer to the previous question was no, why not?

5. Consult the inventory of parts to determine the most likely value of the red mystery resistor.

6. Consult the inventory of parts to determine the most likely value of the yellow mystery resistor.

7. Consult the inventory of parts to determine the most likely value of the blue mystery resistor.

# Pre-Lab 4

Carefully read the entirety of Lab 4, then answer the following questions. Attach a separate sheet of paper, if necessary, to show all work and calculations.

- 1. In Circuit 1, you will be building a circuit to test the superposition theorem. You are allowed to choose the values of voltage, current, and resistance that you would like to use (within limits).
  - (a) Choose a value to use for the voltage source. Keep it between 3 V and 8 V. (The precision of this value should be no more than tenth's place.)
  - (b) Choose a value to use for the current source. Keep it between 5 mA and 10 mA. (The precision of this value should be no more than tenth's place.)
  - (c) Choose two **different** resistor values to use. Consult the inventory of parts to determine what resistor values are available. (Do not use series and parallel combinations of resistors.) Verify that the resistors do not exceed 1/4 W of power consumption with either one or both sources activated. (You are welcome to use a simulation tool such as CircuitJS to test the component values to ensure that the power rating is satisfied.)
  - (d) Re-draw your circuit diagram to indicate how you will build it with only the voltage source connected and the current source deactivated. You will refer to this in lab to help you build the circuit. Include the polarity/direction of each measured quantity.

(e) Re-draw your circuit diagram to indicate how you will build it with only the current source connected and the voltage source deactivated. You will refer to this in lab to help you build the circuit. Include the polarity/direction of each measured quantity.

2. What precaution should you take with the LM317 voltage regulator component? Why?

3. Which signal should you connect the BNC cable to on the function generator?

4. In lab Circuit 2, you are asked to create a square wave with  $V_{MIN} = 1$  V,  $V_{MAX} = 4$  V, and a frequency of 200 Hz. Plot this waveform using the graph below, then answer the following questions.



(a) What is the amplitude of the waveform?

(b) What is the peak-to-peak voltage of the waveform?

(c) What is the DC offset of the waveform?

5. In lab Circuit 3, you are asked to create a triangular wave with  $V_{MIN} = 0$  V,  $V_{MAX} = 5$  V, and a frequency of 250 Hz. Plot this waveform using the graph below, then answer the following questions.



- (a) What is the amplitude of the waveform?
- (b) What is the peak-to-peak voltage of the waveform?
- (c) What is the DC offset of the waveform?
- 6. In lab Circuit 4, you are asked to create a sinusoidal wave with  $V_{MIN} = -5$  V,  $V_{MAX} = 5$  V, and a frequency of 50 Hz. Plot this waveform using the graph below, then answer the following questions.



- (a) What is the amplitude of the waveform?
- (b) What is the peak-to-peak voltage of the waveform?
- (c) What is the DC offset of the waveform?

- 7. The following questions refer to lab Circuit 5. Read it carefully before answering these questions.
  - (a) Derive a mathematical expression for the desired output waveform, which is shown in figure 4.4. (Note that the argument of a sinusoid should be unit-less, and the assumed units of the variable t are seconds.)

(b) The DC voltage source will generate the DC component (that is, the DC offset) of the output waveform. Using superposition with the AC source deactivated, derive an equation that relates resistors  $R_1$  and  $R_2$ .

(c) The AC voltage source will generate the AC component of the output waveform. Using superposition with the DC source deactivated, derive an equation that relates resistors  $R_1$  and  $R_2$ .

(d) Use these two equations to calculate values for resistors  $R_1$  and  $R_2$ .

# Lab 4: Superposition

In this lab, you will experimentally verify the concept of superposition. Then you will utilize superposition to determine voltages and currents in a circuit. Superposition can be used to simplify measurements and calculations of current and voltage values within a linear circuit.

For lab resources and information, go to the following URL or scan the QR code. doctor-pasquale.com/circuit-analysis-lab-4



## 4.1 Superposition

The purpose of the first section of this lab is to prove the validity of the superposition principle by performing an experimental verification. The superposition principle states that all voltages and currents within a linear circuit are equal to the sum of the responses caused by each independent source acting alone, with all other independent sources deactivated.

A voltage source is deactivated by replacing the source with a circuit element that drops zero volts: a wire (short circuit). A current source is deactivated by replacing the source with a circuit element that allows no current to flow: an open.

## 4.2 Voltage Regulator

In this lab, you will use a voltage regulator to create a constant voltage source that is not affected by the presence of multiple independent sources in your circuits. The LM317 voltage regulator is an adjustable voltage regulator that you will use as a voltage source in Circuit 1 and Circuits 6 and 7 in this lab. The package of the LM317 is known as a TO-220, and the pinout diagram is shown in Figure 4.1. Note that the heatsink (metal component) of the package may get hot, so you are cautioned not to touch it while the circuit is in operation!



Figure 4.1: Pinout diagram for LM317 in the TO-220 package.

The circuit diagram in Figure 4.2 demonstrates how to connect the LM317 with other components to obtain an adjustable constant voltage source.



Figure 4.2: Circuit diagram for adjustable voltage source.

**Circuit 1:** In the pre-lab, you chose voltage, current, and resistance values for the circuit below (taking care to ensure that the power consumed by any resistor with one or both sources activated will be less than one quarter watt). Write those values down on the circuit diagram in figure 4.3. The constant voltage will be sourced by the adjustable voltage source shown in Figure 4.2. You will use the DC power supply (dialed to no higher than 20 V) to act as the voltage input for both the constant current source and the adjustable voltage regulator.



Figure 4.3: Circuit diagram for lab Circuit 1.

Measure each property with only the voltage source connected; with only the current source connected; and with both supplies connected. Take care to respect the polarity of each measurement! Record your data in table 4.1.

	Voltage Only	Current Only	Both
V1 (V)			
l1 (mA)			
V2 (V)			
l2 (mA)			

 Table 4.1: Superposition voltage and current measurements.

When you have finished collecting your data, demonstrate it to your instructor to receive a stamp.

## 4.3 Time-Varying Signals

While DC signals have been used in all labs up until now, this lab will introduce AC signals, which have voltage values that change with respect to time. Three types of AC signals (sinusoidal, triangular, and square waves) will be used in this lab.

### 4.3.1 Function Generator

The function generator is used to generate time-varying voltage signals. It is capable of creating sinusoidal waves, square waves, and triangular waves with frequencies from the Hz to MHz range. For information about how to use the function generator, refer to the YouTube video.

The most important attributes of any time-varying signal can be generated by changing the wave **shape**, **frequency**, **output level**, and **DC offset**.

- Shape The shape of the wave can either be sinusoidal, square, or triangular. Press the button corresponding to each of these shapes and see how the output changes on the oscilloscope. The corresponding signal will be made available on the BNC connector labeled output.
- **Frequency** The frequency can be changed by pressing the range buttons on the function generator to set the range of frequencies, with the number appearing above the button representing (approximately) the maximum frequency available at that range. The frequency can be tuned using the coarse and fine knobs.
- **Output level** The output level dial on the function generator changes the amplitude of the wave. This should be set to achieve the desired peak-to-peak value *before* changing the DC offset.
- **DC offset** The DC offset is activated by pressing the corresponding button, and changes the distance between the waveform's equilibrium position and 0 V. If there is no DC offset, the waveform will be centered vertically at 0 V. The offset knob is then used to set the actual value of the DC offset.

#### 4.3.2 Oscilloscope

The oscilloscope is the device used to measure time-varying voltages. For information about how to use the oscilloscope, refer to the YouTube video. You will be using the oscilloscope frequently throughout the semester, so take any notes or pictures of your setups that are necessary to help you in future labs.

**Circuit 2:** Use the function generator to create a square wave with  $V_{MIN} = 1$  V,  $V_{MAX} = 4$  V, and a frequency of 200 Hz. Demonstrate the waveform on the oscilloscope to your instructor to receive a stamp.

#### Instructor Stamp: \_\_\_\_\_

**Circuit 3:** Use the function generator to create a triangular wave with  $V_{MIN} = 0$  V,  $V_{MAX} = 5$  V, and a frequency of 250 Hz. Demonstrate the waveform on the oscilloscope to your instructor to receive a stamp.

Instructor Stamp: \_\_\_\_

**Circuit 4:** Use the function generator to create a sinusoidal wave with  $V_{MIN} = -5$  V,  $V_{MAX} = 5$  V, and a frequency of 50 Hz. Demonstrate the waveform on the oscilloscope to your instructor to receive a stamp. Keep this signal on your function generator for use in your next circuit!

Instructor Stamp: \_\_\_\_

**Circuit 5:** This circuit uses superposition between the DC source and AC source to create an output wave that both has a sinusoidal component (from the function generator) and a DC component (from the DC power supply). Because the sinusoidal wave has no offset, the DC offset of the output waveform must come from the DC power supply. This output wave is given in figure 4.4.



Figure 4.4: Desired output waveform for superposition Circuit 5.

In the pre-lab, you should have derived values for both resistors to obtain the desired output waveform. Record both resistor values (with units) below.

Using those resistor values, build the circuit shown in figure 4.5 using a DC power supply and function generator as sources.



Figure 4.5: Circuit diagram for superposition Circuit 5.

Measure the output voltage on an oscilloscope. When the output voltage matches that given in figure 4.4, demonstrate it to your instructor to receive a stamp.

Instructor Stamp: \_

## 4.4 Superposition in Linear and Nonlinear Circuits

The last section of this lab will test superposition one last time using mystery boxes. One of the mystery boxes will contain non-linear circuit elements, and the other mystery box will contain only linear circuit elements. You should be able to differentiate between them by seeing if superposition holds up or not.

Build the following circuit, inserting the mystery box with the labeled nodes as shown in figure 4.6. The 6 V will be sourced by the adjustable voltage source shown in Figure 4.2.



Figure 4.6: Schematic of superposition mystery box setup.

You will use the DC power supply (dialed no higher than 20 V) to act as the voltage input for both the constant current source and the adjustable voltage regulator.

**Circuit 6:** Connect **mystery box a** to your circuit. Measure each property with only the voltage source connected; with only the current source connected; and with both supplies connected. Record the data in table 4.2. Take care to respect the polarity of each measurement! When you have finished collecting your data, demonstrate it to your instructor to receive a stamp.

	Voltage Only	Current Only	Both
V1 (V)			
V2 (V)			

Table 4.2: Mystery box a data table.

### Instructor Stamp: \_\_

**Circuit 7:** Connect **mystery box b** to your circuit. Measure each property with only the voltage source connected; with only the current source connected; and with both supplies connected. Record the data in table 4.3. Take care to respect the polarity of each measurement! When you have finished collecting your data, demonstrate it to your instructor to receive a stamp.

	Voltage Only	Current Only	Both
V1 (V)			
V2 (V)			

Table 4.3: Mystery box b data table.

Instructor Stamp: \_\_\_\_

# Lab 4 Homework

Carefully read each question before answering. Show all work or justify your answers to receive credit. Attach a separate sheet of paper, if necessary, to show all work and calculations.

- 1. The following questions refer to lab Circuit 1.
  - (a) Does voltage obey the superposition theorem? Use numerical evidence from circuit 1 to justify your answer.

(b) Does current obey the superposition theorem? Use numerical evidence from circuit 1 to justify your answer.

(c) Calculate the power consumed by each resistor if only the voltage source is activated and the current source is deactivated.

(d) Calculate the power consumed by each resistor if only the current source is activated and the voltage source is deactivated.

(e) Calculate the power consumed by each resistor if both sources are activated.

(f) Does power obey the superposition theorem? Justify your answer.

2. Using the data you gathered in lab, are the contents of mystery box a linear or non-linear? How do you know?

3. Using the data you gathered in lab, are the contents of mystery box b linear or non-linear? How do you know?

# Pre-Lab 5

Carefully read the entirety of Lab 5, then answer the following questions. Attach a separate sheet of paper, if necessary, to show all work and calculations.

1. Read lab Circuit 1. Convert the circuit shown in lab to one that has a single independent source (either a voltage source or a current source) and two resistors, one of which is the 18 k $\Omega$  load resistor. Draw a circuit diagram of your completed circuit. You will use this in lab to verify the property of source transformation.

2. In lab Circuit 4, you will have to determine the Thévenin resistance for three mystery circuits. How will you measure  $R_{TH}$  in lab? Why?

3. Choose 5 different values between 0–255. Then determine what the analog output voltage would be for these values given an 8-bit digital to analog converter (DAC). Assume that  $V_{HIGH} = 5$  V, and  $V_{LOW} = 0$  V. Record these in table 5.6.

# Lab 5: Source Transformation and Equivalent Circuits

In this lab, the concept of source transformation will be used to simplify circuits so that they only include a single voltage or current source. Then, dependent source circuits will be explored to determine their properties. Thévenin and Norton equivalencies will be used to simplify complicated circuits. You will measure the Thévenin properties of mystery circuits so that you can identify them in the homework. Finally, you will build a practical example of a circuit that uses voltage division: a digital to analog converter.



## 5.1 Source Transformation

Source transformation is a circuit analysis tool that can be used to change a current source to voltage source or vice versa. The method applies for both independent and dependent sources. It can be shown that the two circuits in figure 5.1 have equivalent values for V measured over the load and I going into the load, regardless of the load circuit.



Figure 5.1: The circuit on the left is equivalent to the circuit on the right.

Source transformation can be used to simplify a circuit by reducing the number of independent sources in a circuit. It can also be used to change the source type in situations where a current source cannot be obtained, for example. **Circuit 1:** The purpose of this circuit is to verify that source transformation is valid. Start by building the circuit in figure 5.2, which contains both a voltage source and a current source. (Because you will be using the DC power source as the voltage source, you will use the dual supply to power the current source, as you have done previously.)



Figure 5.2: Source transformation circuit, before transforming to a single source.

Measure the voltage V and current I and record in table 5.1.

Property	Value
V	
I	

#### Table 5.1: Circuit measurements before transforming to a single source.

In the pre-lab, you determined how to convert the circuit to one that uses only one independent source. Draw the circuit diagram below. (Your circuit diagram should represent the circuit you built: the values of the resistors that were available and should respect the precision of the current or voltage source.)

Build the circuit. Then, measure the voltage V and current I. Record this data in table 5.2. When you have finished collecting data, demonstrate your results to your instructor to receive a stamp.

Property	Value
V	
I	

Table 5.2: Circuit measurements after transforming to a single source.

Instructor Stamp: \_\_\_\_

## 5.2 Dependent Sources

Dependent sources deliver current or voltage proportional to other circuit characteristics. There are four types of dependent sources: voltage-controlled voltage source (VCVS), voltage-controlled current source (VCCS), current-controlled voltage source (CCVS), and current-controlled current source (CCCS).

**Circuit 2:** The purpose of this circuit is to determine the proportionality constant of a voltage-controlled voltage source (VCVS). The model of the circuit you will build is shown in figure 5.3, where  $V_d = kV_x$ . You will vary the value of  $R_x$  to empirically determine the value of the proportionality constant k.



Figure 5.3: Circuit model of the VCVS circuit.

The VCVS circuit has four nodes. Connect it to resistors and the constant-current source as shown in figure 5.4. Use the dual power supply to power the constant-current source, as the DC power supply will be used for the VCVS box.



Figure 5.4: Circuit diagram showing connections between the circuit and VCVS.

Vary the value of  $R_x$  and measure the corresponding values of  $V_x$  and  $V_d$ . Record each of the values in table 5.3. You will be asked to calculate the proportionality constant of the VCVS in the lab homework.

<b>Rx (</b> Ω)	Vx	Vd
1000		
1500		
2000		
2500		

Table 5.3: Voltage measurements as a function of Rx.

When you have finished collecting data, demonstrate your results to your instructor to receive a stamp.

Instructor Stamp: \_

**Circuit 3:** The purpose of this circuit is to determine the proportionality constant of a current-controlled voltage source (CCVS). The model of the circuit you will build is shown in figure 5.5, where  $V_d = r_m I_x$ . You will vary the value of  $R_x$  to empirically determine the value of the proportionality constant  $r_m$ .



Figure 5.5: Circuit model of the CCVS circuit.

The CCVS circuit has four nodes. Connect it to resistors and the constant-current source as shown in figure 5.6. Use the dual power supply to power the constant-current source, as the DC power supply will be used for the CCVS box.



Figure 5.6: Circuit diagram showing connections between the circuit and CCVS.

Vary the value of  $R_x$  and measure the corresponding values of  $I_x$  and  $V_d$ . Record each of the values in table 5.4. You will be asked to calculate the proportionality constant of the CCVS in the lab homework.

<b>Rx (</b> Ω)	lx	Vd
50		
100		
200		
250		

**Table 5.4:** Voltage and current measurements as a function of Rx.

When you have finished collecting data, demonstrate your results to your instructor to receive a stamp.

Instructor Stamp: \_\_

## 5.3 Thévenin and Norton Equivalent Circuits

Thévenin and Norton's theorems state that any linear circuit can be represented by an equivalent circuit composed of a single voltage source in series with a resistor (Thévenin) or a single current source in parallel with a resistor (Norton). These equivalencies can be used to simplify circuits composed of any number of linear circuit elements (resistors, capacitors, inductors, op-amps, independent and dependent current and voltage sources).

## 5.3.1 Thévenin Equivalent Circuit

A Thévenin equivalent circuit, shown in figure 5.7, consists of a voltage source in series with a resistor.  $V_{TH}$ , the value of the voltage source, is equal to the open-circuit voltage ( $V_{OC}$ ) as calculated or measured across terminals **a** and **b** of the linear circuit.



Figure 5.7: Thévenin equivalent circuit.

The value of the Thévenin resistance  $(R_{TH})$  can be determined in various ways.

- $R_{TH}$  can be **directly measured** if all independent sources within the linear circuit can be and are deactivated. If all independent sources have been deactivated,  $R_{TH}$  is simply equal to the resistance as measured across terminals **a** and **b** of the linear circuit.
- $R_{TH}$  can be **indirectly measured** by measuring the short-circuit current  $(I_{SC})$  that flows between terminals **a** and **b** when they are shorted together.  $R_{TH}$  is then equal to  $V_{TH}/I_{SC}$ .

### 5.3.2 Norton Equivalent Circuit

A Norton equivalent circuit, shown in figure 5.8, consists of a voltage source in parallel with a resistor.  $I_N$ , the value of the current source, is equal to the short-circuit current  $(I_{SC})$  as calculated or measured through a short between terminals **a** and **b** of the linear circuit.



Figure 5.8: Norton equivalent circuit.

The value of the Norton resistance  $(R_N)$  can be determined in various ways.

- $R_N$  can be **directly measured** if all independent sources within the linear circuit can be and are deactivated. If all independent sources have been deactivated,  $R_N$  is simply equal to the resistance as measured across terminals **a** and **b** of the linear circuit.
- $R_N$  can be **indirectly measured** by measuring the open-circuit voltage  $(V_{OC})$  between terminals **a** and **b**.  $R_N$  is then equal to  $V_{OC}/I_N$ .

Circuit 4: There are three Thévenin mystery boxes. They all have four terminals, A, B, C, and D. Connect each one in turn to a 4 V power supply using the following circuit diagram.



Measure the properties  $V_{OC}$  and  $I_{SC}$  for each circuit and record the data in table 5.5. Then, either directly or indirectly measure  $R_{TH}$ . Assume that there may be independent sources inside of the box that cannot be deactivated! When you have finished, show your results to the instructor to receive a stamp.

Box	Voc (V)	lsc (mA)	Rth (k $\Omega$ )
а			
b			
с			

Table 5.5: Mystery box data table.

Instructor Stamp: \_\_\_\_

## 5.4 Digital to Analog Converter (DAC)

A digital to analog converter (DAC) is a device that converts a digital (binary) value to a correspondingly weighted analog voltage. For example, an 8-bit digital number ranges from values 0 - 255 in base ten, or from  $0000\ 0000 - 1111\ 1111$  in binary. A computer or microcontroller can use this digital value to accomplish some type of functionality using digital hardware and software.

A digital value can be transformed to a corresponding analog voltage given by equation 5.1, where  $V_{HIGH}$  is the voltage corresponding to a logic HIGH signal,  $V_{LOW}$  is the voltage corresponding to a logic LOW signal, and n is equal to the digital resolution of the system given in number of bits.

$$V_{ANALOG} = (V_{HIGH} - V_{LOW}) \times \frac{\text{value}}{2^n}$$
(5.1)

One type of DAC is known as an R - 2R ladder, shown in figure 5.9. It can be built out of resistors of identical value. This resistor ladder is a very practical application of a very simple circuit theory: the voltage divider. Each digital input bit is weighted by the fraction of resistance that exists between that bit and ground compared to the total resistance between the analog output and ground.



**Figure 5.9:** An R - 2R ladder digital to analog converter.

**Circuit 5:** Build an R - 2R ladder as shown in figure 5.9. Choose a value for R such that there are plenty of resistors available. Use one of the DAC printed circuit boards to enter and connect the digital values (the MSB and LSBs are labeled). You will need to connect a ground wire using an alligator clip to the heat sink on the voltage regulator.

Choose 10 values between 0–255 and record them in table 5.6. You should already have 5 digital values and their expected analog values from the pre-lab. Add to these the 5 digital values (assuming that they are all unique) that your lab partner chose and include their expected values in the table as well. Measure the analog output voltage with a voltmeter and record it as well. When you've finished recording data, show it to your instructor to receive a stamp.

Digital Value	Expected Value (V)	Vout (∨)	% Error

**Table 5.6:** Digital to analog converter data table.

Instructor Stamp: \_\_

# Lab 5 Homework

Carefully read each question before answering. Show all work or justify your answers to receive credit. Attach a separate sheet of paper, if necessary, to show all work and calculations.

1. Consider the VCVS circuit from Circuit 2. Calculate the proportionality constant (k) of the VCVS by taking the average of  $V_d/V_x$  for each value of  $R_x$  that you used. (As always: include units in your answer as needed.)

2. Consider the CCVS circuit from Circuit 3. Calculate the proportionality constant  $(r_m)$  of the CCVS by taking the average of  $V_d/I_x$  for each value of  $R_x$  that you used. (As always: include units in your answer as needed.)

3. Calculate the average error of the DAC that you built in Circuit 5. If it is greater than 10%, comment on the possible sources of error. (Human error is not a source of error.)

4. Consider the circuit shown in figure 5.10. Calculate the Thévenin equivalent voltage and resistance with respect to nodes a and b. When you have completed this, identify which of the mystery circuits that it is.



Figure 5.10: One of the Thévenin mystery box circuits.

5. Consider the circuit shown in figure 5.11. Calculate the Thévenin equivalent voltage and resistance with respect to nodes a and b. When you have completed this, identify which of the mystery circuits that it is.



Figure 5.11: One of the Thévenin mystery box circuits.

6. Consider the circuit shown in figure 5.12. Calculate the Thévenin equivalent voltage and resistance with respect to nodes a and b. When you have completed this, identify which of the mystery circuits that it is.



Figure 5.12: One of the Thévenin mystery box circuits.
# Pre-Lab 6

Carefully read the entirety of Lab 6, then answer the following questions. Attach a separate sheet of paper, if necessary, to show all work and calculations.

1. What is meant by an active or passive circuit element?

2. What types of circuit elements are active?

3. What types of circuit elements are passive?

- 4. Assume that the inverting amplifier built in Circuit 2 will saturate at output levels greater than  $\pm 10$  V. Given that the maximum value of your source voltage will be 2 V, and the minimum value of your potentiometer resistance will be 0  $\Omega$ , determine values for the following resistors such that the output will not saturate.
  - (a) Resistor  $R_1$  in Circuit 2.

(b) Resistor  $R_2$  in Circuit 3.

(c) Resistor  $R_3$  in Circuit 4.

5. You are building Circuit 5 or 6 in lab and notice that your output voltage never goes above zero volts. What is the most likely cause of this issue?

6. This is not actually a question but a piece of advice. You are going to be working with DIP chips in this class, which is very different from other circuits you have built up until this point. You may want to consider using the space below to draw pictures of how to connect your circuits based on the pins of the DIP chip. Or, you may want to use TinkerCAD to play around with the op-amps and build one or two circuits online before coming to class. You can use your TinkerCAD layout to help you out when you are in class.

# Lab 6: Operational Amplifiers

In this lab, we will make practical use of operational amplifiers, both in their saturation mode as comparators, and with negative feedback as closed-loop, differential-input amplifiers.

For lab resources and information, go to the following URL or scan the QR code. doctor-pasquale.com/circuit-analysis-lab-6



### 6.1 Operational Amplifiers

Operational amplifiers, or op-amps, are differential amplifiers that create an output voltage that is proportional to the difference between two input voltages, described by equation 6.1.

$$V_{OUT} = A(V_P - V_N) \tag{6.1}$$

The schematic of an op-amp is shown in figure 6.1. The gain of the amplifier, denoted as A in equation 6.1, can be determined by reading the device datasheet.



Figure 6.1: Differential amplifier schematic.

An op-amp is known as an active circuit element because it requires connections to an external power supply in order to function properly. (These connections are labeled  $V^+$  and  $V^-$  in figure 6.1.) Resistors, capacitors, inductors, and diodes are passive circuit elements due to the fact that they do not require a power supply in order to function.

The power supply connected to the device limits the range of output of the op-amp. Voltage cannot be amplified indefinitely. If the circuit's voltage gain is very high, the output voltage will become saturated at the points when  $V_{OUT} \ge V^+$  or when  $V_{OUT} \le V^-$ . In fact, the output voltage will never quite achieve the same levels as the power supply voltages (i.e. the maximum voltage output will be somewhat less than  $V^+$ and the minimum voltage output will be somewhat greater than  $V^-$ ).

Negative feedback is used to tune an amplifier gain and prevent the output voltage from saturating all the time due to large A values.

#### 6.1.1 741 General Purpose Op-Amp Integrated Circuit

The 741 op-amp integrated circuit is a single op-amp package that can be operated with a dual-supply power source. The pinout diagram is provided in figure 6.2. The chip must be placed across the trench in the breadboard to electrically isolate each of the pins, which has a unique function.



Figure 6.2: Pinout diagram of the 741 general purpose op-amp integrated circuit chip.

Circuit 1: Use the function generator to create a sinusoidal wave that has a frequency of 1 kHz,  $V_{MIN} = 0$  V, and  $V_{MAX} = 2$  V. Demonstrate the waveform to your instructor to receive a stamp. You will use this waveform in Circuits 2–4.

#### Instructor Stamp: \_\_\_\_

**Circuit 2:** Build an inverting amplifier, shown in figure 6.3 with the 741 integrated circuit using the value of  $R_1$  that you derived in the pre-lab. Record this value below.

Use a dual power supply to supply at least  $\pm 12$  V to the op-amp. Show both the input and output waveforms on the oscilloscope, with no voltage offset and using the same voltage scale per division. Note what happens as you turn the potentiometer from one extreme to the other. You may want to record your observations, as you will be asked questions about the behavior of this circuit in the lab homework. Demonstrate your results to your instructor when you have completed this circuit. **Do not take this circuit apart when you are done; you will build on to it in the next circuit.** 



Figure 6.3: Inverting op-amp circuit diagram.

Instructor Stamp: \_\_\_\_

**Circuit 3:** Using the same conditions as the previous circuit, add another inverting amplifier to the output of the one you built in the previous circuit, as shown in figure 6.4. You should have derived the value of  $R_2$  in the pre-lab. Record this value below.

Show both the input and output waveforms on the oscilloscope, with no voltage offset and using the same voltage scale per division. Note what happens as you turn the potentiometer from one extreme to the other. You may want to record your observations, as you will be asked questions about the behavior of this circuit in the lab homework. Demonstrate your results to your instructor when you have completed this circuit.



Figure 6.4: Cascaded op-amp circuit diagram.

#### Instructor Stamp: \_\_\_\_

**Circuit 4:** Using the same conditions as the previous circuit, build a non-inverting amplifier, as depicted in figure 6.5. You should have derived the value of  $R_3$  in the pre-lab. Record this value below.

Show both the input and output waveforms on the oscilloscope, with no voltage offset and using the same voltage scale per division. Note what happens as you turn the potentiometer from one extreme to the other. You may want to record your observations, as you will be asked questions about the behavior of this circuit in the lab homework. Demonstrate your results to your instructor when you have completed this circuit.



Figure 6.5: Non-inverting op-amp circuit diagram.

Instructor Stamp: \_\_

### 6.2 Comparators

A comparator is an op-amp built specifically to work with saturated output voltages. Any time the noninverting input is greater than the inverting input, the output will be equal to  $V^+$ . Whenever the inverting input is greater than the non-inverting input, the output will be equal to  $V^-$ . This acts as a circuit element capable of comparing two voltages, which is why the device is called a comparator.

### 6.2.1 339 Differential Comparator Integrated Circuit

The 339 comparator integrated circuit is a quad package (i.e. it contains four comparators) that can be operated with a single-supply power source (i.e. power and ground, rather than  $V^+$  and  $V^-$ ). The pinout diagram for this chip is provided in figure 6.6.





It is important to note that all outputs of the 339 chips that are to be used must be connected via a pull-up resistor. These pull-up resistors are included in all schematics in this lab. If the pull-up resistors are not included, the output signals of your circuits will never be greater than zero.

Circuit 5: Generate a triangular wave that has a frequency of 1 kHz,  $V_{MIN} = 0$  V, and  $V_{MAX} = 5$  V.

Use the 339 comparator to generate a pulse-width modulation signal, using the schematic shown in figure 6.7. The non-inverting input should be connected to the triangle wave. The inverting input should come from a soft pot, shown as a generic potentiometer in the circuit diagram. The output will be connected to a light-emitting diode (LED).



Figure 6.7: Pulse-width modulation circuit diagram.

The soft pot should be connected as shown in figure 6.8.



Figure 6.8: Soft pot connection diagram.

The circuit will be functioning properly if by swiping your finger along the soft pot, the brightness of the LED changes. A gif of this functionality is available for you to look at to see if your circuit acts correctly or not. It is also valuable to view your input and output waves on the oscilloscope to see what is happening to the electrical signals due to your circuit. Demonstrate your results to your instructor when you have completed this circuit.

Instructor Stamp: \_

**Circuit 6:** Build a flash-type analog to digital converter (ADC) using the 339 comparator using the circuit diagram given in figure 6.9. The input signal should come from a potentiometer connected between 0 and 5 V. The output of each comparator should go to an LED with a 1 k $\Omega$  current-limiting resistor on the output. Demonstrate your results to your instructor when you have completed this circuit.



Figure 6.9: Flash-type analog to digital converter circuit diagram.

Instructor Stamp: \_\_

## Lab 6 Homework

Carefully read each question before answering. Show all work or justify your answers to receive credit. Attach a separate sheet of paper, if necessary, to show all work and calculations.

1. Use the voltage divider rule to calculate at what input voltage each op-amp in the 3-bit ADC shown in figure 6.10 will saturate. Ensure that each of your calculations is clearly labeled as  $V_7$ ,  $V_6$ , etc.



Figure 6.10: 3-bit analog to digital converter.

2. Consider the inverting amplifier that you built in lab Circuit 2. Calculate the minimum and maximum possible values of the absolute value of the voltage gain  $(|V_{OUT}/V_{IN}|)$  given the values of the resistors that you used.

3. Qualitatively describe the behavior of the inverting amplifier. (Things to include in your answer: Did the output ever saturate? Was the output always inverted, sometimes inverted, or never inverted? Was the output always amplified, always attenuated, or was there the possibility for both situations to occur?)

4. If the gain of the amplifier went from < 1 (attenuating) to > 1 (amplifying), calculate the value of the potentiometer that would lead to that transition.

5. Consider the cascaded inverting amplifier that you built in lab Circuit 3. Calculate the minimum and maximum possible values of the absolute value of the voltage gain given the values of the resistors that you used.

6. Qualitatively describe the behavior of the cascaded inverting amplifier. (Things to include in your answer: Did the output ever saturate? Was the output always inverted, sometimes inverted, or never inverted? Was the output always amplified, always attenuated, or was there the possibility for both situations to occur?)

7. If the gain of the amplifier went from < 1 (attenuating) to > 1 (amplifying), calculate the value of the potentiometer that would lead to that transition.

8. Consider the non-inverting amplifier that you built in lab Circuit 4. Calculate the minimum and maximum possible values of the absolute value of the voltage gain given the values of the resistors that you used.

9. Qualitatively describe the behavior of the non-inverting amplifier. (Things to include in your answer: Did the output ever saturate? Was the output always inverted, sometimes inverted, or never inverted? Was the output always amplified, always attenuated, or was there the possibility for both situations to occur?)

10. If the gain of the amplifier went from < 1 (attenuating) to > 1 (amplifying), calculate the value of the potentiometer that would lead to that transition.

## Pre-Lab 7

Carefully read the entirety of Lab 7, then answer the following questions. Attach a separate sheet of paper, if necessary, to show all work and calculations.

1. Given the circuit shown in figure 7.1, derive an equation for  $v_{OUT}(t)$  in terms of R, C, and  $v_{IN}(t)$ .



Figure 7.1: Lab 7 pre-lab op-amp circuit 1.

2. Given the circuit shown in figure 7.2, derive an equation for  $v_{OUT}(t)$  in terms of R, C, and  $v_{IN}(t)$ .



Figure 7.2: Lab 7 pre-lab op-amp circuit 2.

## Lab 7: Capacitors

This lab will focus on a new passive circuit element: the capacitor. Capacitors are capable of storing energy and open the door to an exploration of alternating-current circuit applications.

For lab resources and information, go to the following URL or scan the QR code. doctor-pasquale.com/circuit-analysis-lab-7



Capacitors are passive circuit elements that are capable of storing and releasing energy in the form of an electric field. This capacity to store energy is known as capacitance, which has a symbol of C and units of farads (F). This property physically arises from two parallel conductors held some distance apart from each other with an insulating material (known as a dielectric) in between them. When a high potential is applied to one of the conductors, and a low potential applied to the other, an electric field is generated. The two types of capacitors that will be used predominantly in this course are ceramic capacitors and aluminum electrolytic capacitors.

### 7.1.1 Capacitors in Series & Parallel

As described and derived in the Circuit Analysis textbook, connecting capacitors in series causes the equivalent capacitance of a circuit to decrease, following equation 7.1.

$$\frac{1}{C_{EQ}} = \sum_{i} \frac{1}{C_i} \tag{7.1}$$

When capacitors are connected in parallel, the result is that the equivalent capacitance of a circuit increases, following equation 7.2.

$$C_{EQ} = \sum_{i} C_i \tag{7.2}$$

#### 7.1.2 Voltage-Current Relationship

Capacitors have a relationship between voltage and current that is much different from that of a resistor. The current passing through a capacitor is given by equation 7.3, where C is the capacitance, v(t) is the voltage, and t is time.

$$i(t) = C \frac{dv(t)}{dt},\tag{7.3}$$



**Circuit 1:** These circuits will get you familiar with connecting capacitors in series and parallel combinations, and also familiarize you with the LRC meter that is used to measure capacitance. Measuring capacitance requires that the capacitors are discharged. Electrolytic capacitors must be measured with the positive lead of the LRC meter connected to the positive leads of the capacitors.

Obtain the **mystery ceramic capacitor** (labeled  $C_X$  in figure 7.3) from your instructor, and connect it as shown in figure 7.3. (Remember: ceramic capacitors are non-polar so it doesn't matter which orientation it is connected in.) Then, measure and record the equivalent capacitance of the circuit in table 7.1.



Figure 7.3: Circuit diagram for the mystery ceramic capacitor.

Obtain the **mystery electrolytic capacitor** (labeled  $C_X$  in figure 7.4) from your instructor, and connect it as shown in figure 7.4. (Remember: electrolytic capacitors are polar so be sure to connect the anode – which is labeled on the mystery box – in the correct orientation.) Then, measure and record the equivalent capacitance of the circuit in table 7.1.



Figure 7.4: Circuit diagram for the mystery electrolytic capacitor.

Capacitor	Equivalent Capacitance
Ceramic	
Electrolytic	

Table 7.1: Equivalent capacitance measurements.

When you have finished collecting data, show your results to your instructor to receive a stamp.

Instructor Stamp: \_\_\_\_

**Circuit 2:** Use a capacitance meter to measure the capacitance of the four capacitor mystery boxes, and record the data in table 7.2. You will use this data to answer some of the lab homework questions. When you have finished collecting data, show your results to your instructor to receive a stamp.

Box	C (nF)
A	
В	
С	
D	

Table 7.2: Capacitor mystery box measurements.

#### Instructor Stamp: \_\_\_\_\_

**Circuit 3:** The circuit shown in figure 7.5 is an op-amp differentiator. The circuit also contains a **low-pass** filter at the output in order to filter out spikes in the output waveform.



Figure 7.5: Op-amp differentiator circuit.

Build the circuit, using the function generator to create the sinusoidal waveform as labeled on the circuit diagram as the input wave. Adjust the value of the potentiometer and see what happens to the result. Adjust the value of the frequency (no more than 50–60 Hz) and see what happens to the result. Then, change your input waveform to a triangle wave and observe the outcome. Record your observations to help you answer the lab homework. Demonstrate your results to your instructor to receive a stamp.

Instructor Stamp: \_\_\_\_

**Circuit 4:** The circuit shown in figure 7.6 is an op-amp integrator. The circuit also contains a high value  $(100 \text{ k}\Omega)$  resistor in parallel with the feedback capacitor in order to reduce integration offset.



Figure 7.6: Op-amp integrator circuit.

Build the circuit, using the input wave as labeled on the circuit diagram. Adjust the value of the potentiometer and see what happens to the result. Adjust the value of the frequency (no more than 50–60 Hz) and see what happens to the result. Then, change your input waveform to a square wave and observe the outcome. Record your observations to help you answer the lab homework. Demonstrate your results to your instructor to receive a stamp.

Instructor Stamp: \_\_\_\_

**Circuit 5:** Build the circuit shown in figure 7.7, using a transformer as the input signal. The circuit is known as a full-wave rectifier. It has a frequency of 60 Hz. **Do not view the input signal on the oscilloscope**, as this will cause problems with grounding! You will, however, want to view the output on the oscilloscope. View the output first without an output capacitor. Then, use 4.7 nF, 47 nF, and 470 nF capacitors on the output. Record your observations to help you answer the lab homework. Demonstrate your results to your instructor to receive a stamp.



Figure 7.7: Full-wave rectifier circuit.

Instructor Stamp: \_\_

# Lab 7 Homework

Carefully read each question before answering. Show all work or justify your answers to receive credit. Attach a separate sheet of paper, if necessary, to show all work and calculations.

1. Qualitatively describe what happens to the overall capacitance of a circuit when two or more capacitors are connected in series.

2. Qualitatively describe what happens to the overall capacitance of a circuit when two or more capacitors are connected in parallel.

3. Consider the mystery **ceramic** capacitor from lab Circuit 1. Calculate the value of the capacitor. Note that **one** capacitor was included in the mystery box (not a series or parallel combination). (Consult the inventory of parts to determine the available capacitors that could have been used to perform a "smell test" on your answer. The ceramic capacitors have a tolerance of 20%. Errors larger than that should prompt you to double-check your math.)

4. Consider the mystery **electrolytic** capacitor from lab Circuit 1. Calculate the value of the capacitor. Note that **one** capacitor was included in the mystery box (not a series or parallel combination). (Consult the inventory of parts to determine the available capacitors that could have been used to perform a "smell test" on your answer. The electrolytic capacitors have a tolerance of 20%. Errors larger than that should prompt you to double-check your math.) 5. The following four circuits resemble the inner circuitry of the capacitor mystery boxes. All capacitors have a value of 1 nF. Calculate the equivalent capacitance of the following circuits, and then identify the corresponding mystery box.









- 6. The following questions pertain to lab Circuit 3 (the op-amp differentiator circuit).
  - (a) Describe the shape of the output waveform when your input waveform was a sinusoid. How were the two signals similar? How were they different?

(b) Describe the shape of the output waveform when your input waveform was a triangle wave. Why would this shape be expected on the output?

(c) When the slope of the triangle wave was positive, was the output positive or negative? What aspect of the circuit causes this to be so?

(d) What happened to  $|V_{OUT,MAX}|$  as the frequency of the input wave increased? Mathematically, why is this so?

(e) What happened to  $|V_{OUT,MAX}|$  as the resistance of the variable resistor increased? Mathematically, why is this so?

- 7. The following questions pertain to lab Circuit 4 (the op-amp integrator circuit).
  - (a) Describe the shape of the output waveform when your input waveform was a sinusoid. How were the two signals similar? How were they different?

(b) Describe the shape of the output waveform when your input waveform was a square wave. Why would this shape be expected on the output?

(c) When the value of the square wave was positive, was the output increasing or decreasing? What aspect of the circuit causes this to be so?

(d) What happened to  $|V_{OUT,MAX}|$  as the frequency of the input wave increased? Mathematically, why is this so?

(e) What happened to  $|V_{OUT,MAX}|$  as the resistance of the variable resistor increased? Mathematically, why is this so?

- 8. The following questions pertain to lab Circuit 5 (the full-wave rectifier circuit).
  - (a) Explain the purpose of this circuit (without the output capacitor).

(b) What is the purpose of the output capacitor?

(c) What effect does increasing the output capacitance have on the output of the circuit?

(d) The transformer that you used for this circuit plugs into a wall outlet and outputs a sinusoidal wave with a frequency of 60 Hz. Explain why the frequency is 60 Hz.

# Pre-Lab 8

Carefully read the entirety of Lab 8, then answer the following questions. Attach a separate sheet of paper, if necessary, to show all work and calculations.

1. Read lab Circuit 2. You will be asked to build an RC circuit with a transient response given in equation 8.3. Calculate the values of  $R_1$ ,  $R_2$ , and C to achieve this response. Keep your capacitor value at or below 1  $\mu$ F. (It is strongly recommend that you select a single ceramic capacitor value from the inventory of parts, and then determine the values of  $R_1$  and  $R_2$  based on that.) Note: there are multiple correct answers to this question!

2. Read lab Circuit 4. You will be asked to build an RC circuit with a transient response given in equation 8.5. Calculate the values of  $R_1$ ,  $R_2$ , and C to achieve this response. Keep your capacitor value at or below 1  $\mu$ F. (It is strongly recommend that you select a single ceramic capacitor value from the inventory of parts, and then determine the values of  $R_1$  and  $R_2$  based on that.) Note: there are multiple correct answers to this question!

## Lab 8: RC Circuits

In this lab, we will learn about the transient response of RC circuits. Because capacitors can store energy, they are capable of causing a discharging or charging effect that occurs due to changes in voltage.

For lab resources and information, go to the following URL or scan the QR code. doctor-pasquale.com/circuit-analysis-lab-8



A capacitor is capable of storing energy. For this reason, the voltage drop over the capacitor will change in response to a change in other circuit conditions (e.g. voltage). This change is known as a transient response. When voltage is removed from the circuit, the capacitor will undergo a discharging response, defined by equation 8.1, where v(0) is the initial voltage drop over the capacitor, R is the equivalent resistance seen by the capacitor, and C is the equivalent capacitance.

$$v(t) = \left(v(0^+)e^{-\frac{t}{RC}}\right) \ u(t) + v(0^-) \ u(-t)$$
(8.1)

The amount of time it takes for the voltage to decay to 36.79% of its original value is known as the RC time constant, which is defined in equation 8.2.

$$\tau = RC \tag{8.2}$$

A discharging transient response can be explored by using a square wave input. A square wave that oscillates between 0 V and  $V_S$  will have rising edges (from 0 V to  $V_S$ ) and falling edges (from  $V_S$  to 0 V). The falling edge can be used to reproduce the discharging transient response.

The frequency of the square wave must be carefully selected to enable the RC circuit to fully charge or fully discharge every half cycle of the wave. (In other words: the voltage should completely discharge or completely charge before the square wave changes its value.) Figure 8.1 (left) shows a fully charging and discharging response, while figure 8.1 (right) shows a circuit that is unable to fully charge and discharge between half cycles.



**Figure 8.1:** An RC circuit that is able to fully charge and discharge every cycle (left), and is unable to fully charge and discharge every cycle (right).

In order to obtain accurate measurements, trigger the oscilloscope on a falling edge for discharging circuits and on a rising edge for charging circuits. In the trigger menu, select the correct edge response as well as "single" which will cause the oscilloscope to generate a snapshot of a single transient response. Zoom in by using the horizontal scale dial as much as possible to get the best resolution and therefore the best results in your measurements.



**Circuit 1:** Build the circuit shown in figure 8.2. Use one of the three mystery capacitors as the capacitance. (You will repeat this for all three mystery capacitors.)



Figure 8.2: Lab 8 circuit 1 diagram.

Use a square wave with any suitable frequency that allows the capacitor to fully charge and discharge between cycles. Use an ohmmeter to measure the actual resistance value of the 100 k $\Omega$  resistor and record the value below.

Connect the oscilloscope across the capacitor, as shown in figure 8.2, to measure v(t). Set the triggering to falling edge. Use the cursors to find  $\tau$ . One cursor will be placed at v(0) and the other at  $v(\tau)$ . (Note: you will have to calculate this value before you can place the cursor there! Use equation 8.1 to do this calculation.) The time between these two values is equal to  $\tau$ . Record the data in table 8.1 for each of the three mystery capacitors.

С	v(0) (V)	ν(τ) (V)	au (ms)
red			
yellow			
blue			

Table 8.1: Mystery capacitance data table.

When you have completed recording your data, demonstrate it to your instructor to receive a stamp.

Instructor Stamp: \_\_\_\_

**Circuit 2:** In the pre-lab, you determined the resistance and capacitance values necessary to build an RC circuit with a transient response given by equation 8.3.

$$v(t) = 4 \text{ V } e^{-100t} u(t) + 4 \text{ V } u(-t)$$
(8.3)

Record the values of  $R_1$ ,  $R_2$ , and C (with units) in table 8.2. (There is no need to measure these components.)

Parameter	Value
R1	
R2	
С	

 Table 8.2:
 Component values in circuit 2.

Build the circuit. shown in figure 8.3 and display the output on the oscilloscope. Use a square wave with any suitable frequency that allows the capacitor to fully charge and discharge between cycles. Set the triggering to falling edge. Place one of the cursors at v(0) and the other at  $v(\tau)$  (which you can calculate using equation 8.3). When you have finished, demonstrate the output waveform (with cursors placed at the correct voltages) to your instructor to receive a stamp.



Figure 8.3: Lab 8 circuit 2 diagram.

Instructor Stamp: \_\_\_\_

**Circuit 3:** Build the circuit shown in figure 8.4, being careful to to respect the polarity of the electrolytic capacitor. **The order of components matters!** (I.E., do not swap the resistor and the capacitor.) Use a square wave with any suitable frequency that allows the capacitor to fully charge and discharge between cycles. Set the triggering to falling edge.



Figure 8.4: Lab 8 circuit 3 diagram.

Measure the actual value of the 2.2  $\mu$ F capacitor and record it in table 8.3. In addition, measure and record the value of the 2 k $\Omega$  resistor (which is used to ensure that the capacitor does not charge and discharge too quickly for the oscilloscope to accurately measure). For the unknown resistor (labeled  $R_X$  in the circuit diagram), you will use one of the mystery resistors. Record the resistor number in table 8.3. You will measure the RC time constant and use it to determine the value of the resistor. Use the cursors on the oscilloscope at v(0) and  $v(\tau)$  to measure the time constant  $\tau$ . Record the value of the time constant in table 8.3.

Parameter	Value
2.2 $\mu$ F capacitor	
2 k $\Omega$ resistor	
resistor number	
time constant	

Table 8.3: Component values in circuit 3.

Instructor Stamp: \_\_

### 8.2 RC Circuit Charging Transient Response

The charging transient response of an RC circuit refers to how the output voltage changes in response to an abrupt increase in voltage. This charging response (assuming no initial voltage drop over the capacitor) is defined by equation 8.4, where  $v(\infty)$  is the steady-state voltage over the capacitor, R is the equivalent resistance seen by the capacitor, and C is the equivalent capacitance.

$$v(t) = v(\infty) \left[ 1 - e^{-\frac{t}{RC}} \right] u(t)$$
(8.4)

The amount of time it takes for the voltage to reach 63.21% of its final voltage is equal to the time constant  $\tau$ , and is characterized by equation 8.2.

**Circuit 4:** In the pre-lab, you determined the resistance and capacitance values necessary to build an RC circuit with a charging transient response given by equation 8.5.

$$v(t) = 3.5 \text{ V}(1 - e^{-250t}) u(t)$$
(8.5)

Record the values of  $R_1$ ,  $R_2$ , and C (with units!) in table 8.4. (There is no need to measure them.)

Parameter	Value
R1	
R2	
С	

Table 8.4: Component values in circuit 4.

Build the circuit, shown in figure 8.5 and display the output on the oscilloscope. Use a square wave with any suitable frequency that allows the capacitor to fully charge and discharge between cycles. Set the triggering to rising edge. Place one of the cursors at v(0) and the other at  $v(\tau)$  (which you can calculate using equation 8.7). When you have finished, demonstrate the output waveform (with cursors placed at the correct voltages) to your instructor to receive a stamp.



Figure 8.5: Lab 8 circuit 4 diagram.

Instructor Stamp: \_\_\_\_

#### 8.3 RC Circuit General Transient Response

In general, an RC circuit may have an initial value v(0) and a final value  $v(\infty)$  that are not both zero (not a fully discharging circuit, nor a charging from zero circuit). This is called the general RC transient response and is the most general form of the transient. (In other words, this analysis and equation can be used all the time, rather than in specific situations as with the discharging or charging cases.)

The voltage dropped over the capacitor is defined by equation 8.6, where v(0) is the initial voltage drop over the capacitor,  $v(\infty)$  is the steady-state voltage drop over the capacitor, R is the equivalent resistance seen by the capacitor, and C is the equivalent capacitance.

$$v(t) = \left[v(\infty) + (v(0^+) - v(\infty))e^{-\frac{t}{RC}}\right] u(t) + v(0^-) u(-t)$$
(8.6)

In the general case,  $\tau$  (which is still defined by equation 8.2) describes the amount of time it takes the voltage to reach a value defined by equation 8.7.

$$v(\tau) = 0.6321v(\infty) + 0.3679v(0^+).$$
(8.7)

**Circuit 5:** There are two mystery RC circuits enclosed in boxes. They all have four terminals, **A**, **B**, **C**, and **D**, as shown in figure 8.6. You will measure the **charging** response of each one. Use a square wave with any suitable frequency that allows the capacitor to fully charge and discharge between cycles. Set the triggering to rising edge.



Figure 8.6: Mystery RC circuit connection diagram.

For each mystery circuit, measure  $v(0^+)$ ,  $v(\infty)$ ,  $v(\tau)$  (which you will have to calculate using equation 8.7), and  $\tau$ . Record these values in table 8.5. When you have finished, demonstrate your results to your instructor to receive a stamp.

Box	v(0+) (V)	v(∞) (V)	ν(τ) (V)	au (ms)
а				
b				

Table 8.5: Component values in circuit 5.

Instructor Stamp: \_\_

# Lab 8 Homework

Carefully read each question before answering. Show all work or justify your answers to receive credit. Attach a separate sheet of paper, if necessary, to show all work and calculations.

- 1. The following questions pertain to lab Circuit 1. You will also need to refer to equation 8.2. Use your measured component values for all calculations. For each question, consult the inventory of parts to determine the available capacitors that could have been used to perform a "smell test" on your answer. The ceramic capacitors have a tolerance of 20%. Errors larger than that should prompt you to double-check your math.
  - (a) Calculate the value of the red mystery capacitor.

(b) Calculate the value of the yellow mystery capacitor.

(c) Calculate the value of the blue mystery capacitor.

2. Calculate the value of the mystery resistor in lab Circuit 3. (Carefully consider how to compensate for the 2 k $\Omega$  resistor in your calculations.) Use measured component values and refer to equation 8.2. Consult the inventory of parts to determine the available resistors that could have been used to perform a "smell test" on your answer. The resistors have a tolerance of 5%. Errors larger than that should prompt you to double-check your math.

3. The circuit shown in figure 8.7 corresponds to one of the mystery RC circuits from Circuit 5.



Figure 8.7: One of the mystery RC circuits from Circuit 5.

(a) Calculate  $v(0^+)$  of this circuit.

(b) Calcualte  $v(\infty)$  of this circuit.

(c) Calculate  $\tau$  of this circuit.

(d) Which of the mystery boxes does this circuit correspond to?

4. The circuit shown in figure 8.8 corresponds to one of the mystery RC circuits from Circuit 5.



Figure 8.8: One of the mystery RC circuits from Circuit 5.

(a) Calculate  $v(0^+)$  of this circuit.

(b) Calcualte  $v(\infty)$  of this circuit.

(c) Calculate  $\tau$  of this circuit.

(d) Which of the mystery boxes does this circuit correspond to?
## Pre-Lab 9

Carefully read the entirety of Lab 9, then answer the following questions. Attach a separate sheet of paper, if necessary, to show all work and calculations.

- 1. Read lab Circuit 1. You will be asked to build an RL circuit with a time constant of  $\approx 500 \ \mu$ s. Use equation 9.5 to assist you in your calculations.
  - (a) Calculate the value of L required to obtain this response.

(b) Draw a circuit diagram of how you will obtain this inductance using choke inductors that are 100 mH and 50 mH.

2. In the lab homework, you will be asked to calculate an equation for lab Circuit 1 based on your measured parameters. Derive an equation (in symbolic form) for i(t) based on v(0),  $\tau$ , and any measured resistor values.

- 3. Read lab Circuit 2 and Circuit 3.
  - (a) Will the presence of an iron core in lab Circuit 3 increase or decrease the inductance of the solenoid?

(b) When the iron core is present in the solenoid, what should happen (qualitatively) to the value of  $\tau$ ?

(c) Derive an equation (in symbolic form) for  $\tau$  based on L,  $R_L$ , and R.

- 4. Read lab Circuit 4. You will be asked to build an RL circuit with a time constant of  $\approx 1$  ms. Use equation 9.5 to assist you in your calculations.
  - (a) Calculate the value of L required to obtain this response.

(b) Draw a circuit diagram of how you will obtain this inductance using choke inductors that are 100 mH and 50 mH.

(c) Draw a sketch of the expected voltage output when the switch closes. (The time constant does not need to be perfect, but draw the expected initial value of the voltage and the expected final value of the voltage, and either an increasing or decreasing exponential, depending on the circuit.)

5. In the lab homework, you will be asked to calculate an equation for lab Circuit 4 based on your measured parameters. Derive an equation (in symbolic form) for i(t) based on v(0),  $v(\infty)$ ,  $\tau$ , and any measured resistor values.

# Lab 9: Inductors and RL Circuits

In this lab, we will learn about inductors, which in addition to resistors and capacitors are important passive electric components.

For lab resources and information, go to the following URL or scan the QR code. doctor-pasquale.com/circuit-analysis-lab-9



## 9.1 Inductors

Inductors are coils of wire that induce a magnetic field. While a capacitor can store and release energy in the form of an electric field, an inductor can store and release energy in the form of a magnetic field. This ability to store magnetic energy is known as inductance, which has a symbol of L and units of henrys (H).

### 9.1.1 Inductors in Series & Parallel

As described and derived in the Circuit Analysis textbook, connecting inductors in series causes the equivalent inductance of a circuit to increase, following equation 9.1.

$$L_{EQ} = \sum_{i} L_i \tag{9.1}$$

When inductors are connected in parallel, the result is that the equivalent inductance of a circuit decreases, following equation 9.2.

$$\frac{1}{L_{EQ}} = \sum_{i} \frac{1}{L_i} \tag{9.2}$$

Note that these relationships are the same as the series and parallel resistance relationships! As with resistors, it is possible that an inductive circuit may consist of a combination of series and parallel elements.

#### 9.1.2 Voltage-Current Relationship

Inductors have a relationship between voltage and current that is similar to that of a capacitor, and greatly different from that of a resistor. Calculus is required to obtain a full understanding of the I-V properties. The voltage drop over an inductor is defined by equation 9.3.

$$v(t) = L \frac{di(t)}{dt} \tag{9.3}$$

#### 9.1.3 Parasitic Effects

Because an inductor is formed from many turns of wire, and wire is made of an imperfect conductor, inductors contain a parasitic resistance, which cannot always be ignored in analyzing and designing circuits. The model of a real inductor therefore consists of an inductor in series with a resistor. Parasitic resistance can be measured with an ohmmeter.

### 9.2 RL Circuit Discharging Transient Response

If an inductor has a stored magnetic field, and is connected in parallel with a resistor, current will flow through the circuit until it decreases to zero. The discharging response is defined by equation 9.4, where i(0) is the initial current flow through the inductor, and  $\tau$  is the RL time constant defined by equation 9.5.

$$i(t) = i(0^{+}) e^{-t/\tau} u(t) + i(0^{-}) u(-t)$$
(9.4)

The RL time constant,  $\tau$ , is defined by equation 9.5, where R is the equivalent resistance seen by the inductor, and L is the equivalent inductance. The RL time constant defines the amount of time it takes for the current has decayed to 36.79% of its original value.

$$\tau = \frac{L}{R} \tag{9.5}$$

### 9.3 Using a Toggle Switch

A toggle switch will be used in a few of the circuits in this lab. A toggle switch has three terminals. Therefore, one of the toggle switch connections (labeled A in figures 9.1 and 9.2) will not be connected to any part of the circuit.

#### 9.3.1 Closing Switch

To create a closing switch, start with the toggle switch between terminals A and B. (A will be disconnected from the circuit.) When the switch is moved, it will complete the connection between terminals B and C, closing the switch. This is shown in figure 9.1.



**Figure 9.1:** Closing a toggle switch, as described by a circuit diagram (left) and the actual toggle switch (right).

#### 9.3.2 Opening Switch

To create an opening switch, start with the toggle switch between terminals B and C. (A will be disconnected from the circuit.) When the switch is moved, it will disconnect the two terminals from each other, opening the switch. This is shown in figure 9.2.



**Figure 9.2:** Opening a toggle switch, as described by a circuit diagram (left) and the actual toggle switch (right).

**Circuit 1:** In the pre-lab, you determined the inductor values necessary to build an RL circuit with a time constant equal to  $\approx 500 \ \mu$ s. Measure the actual value of the inductance and record it in table 9.1 as  $L_{REAL}$ .

Use an ohmmeter to measure the parasitic resistance of the inductor and record it in table 9.1 as  $R_L$ . (The parasitic resistance of the inductor is not explicitly shown on the circuit diagram!) The value of resistor  $R_1$  below should be equal to 400  $\Omega - R_L$ . Record the **measured** value of this resistor in table 9.1. Measure the value of the 50  $\Omega$  resistor, and record it in table 9.1, to assist in homework calculations.

Parameter	Value
Lreal	
RL	
R1	
R50	

Table 9.1:Circuit 1 data table.

Build the circuit, as shown in figure 9.3, and display the output on the oscilloscope. (A resistor is used to measure voltage because there are no current probes for the oscilloscope. Instead of measuring current directly, we are measuring it indirectly.)



Figure 9.3: Lab 9 circuit 1 diagram.

Record output values  $v(0^+)$ ,  $v(\tau)$ ,  $v(\infty)$ , and  $\tau$  in table 9.2. Use equation 9.4 to calculate  $v(\tau)$ .

Parameter	Value
v(0+)	
v( au)	
$v(\infty)$	
au	

Table 9.2: Circuit 1 output values.

Use a toggle switch and set the oscilloscope to trigger on a single falling edge. Place one cursor at v(0) and the other at  $v(\tau)$ . You should see that the time between these two cursors is approximately 500  $\mu$ s. If not, check your circuit diagram and double check your component values with your instructor before proceeding. When you have achieved the correct output response, demonstrate to your instructor to receive a stamp.

## 9.4 RL Circuit Charging Transient Response

The charging transient response of an RL circuit refers to how the output changes in response to an abrupt increase in voltage or current. The equation that describes the charging transient response of an RL circuit is defined by equation 9.6 (assuming no initial current flow through the inductor), where  $i(\infty)$  is the steady-state current flowing through the inductor, and  $\tau$  is defined by equation 9.5. In the case of a charging RL circuit,  $\tau$  defines the amount of time it takes for the current to reach 63.21% of its final value.

$$i(t) = i(\infty) \left[ 1 - e^{-t/\tau} \right] u(t)$$
(9.6)

**Circuit 2:** Use the function generator to create a square wave with a frequency of approximately 10 Hz as shown in figure 9.4. Because current cannot be measured directly on the oscilloscope, the voltage over a resistor will be measured instead. Ohm's law can be used to calculate the current through the inductor.



Figure 9.4: Lab 9 circuit 2 diagram. Use the very large solenoid as the inductor.

Build the circuit using a 1 k $\Omega$  resistor (over which you will measure the output voltage) and one of the very large solenoids. Measure the actual value of the resistor using an ohmmeter. Measure the parasitic resistance of the solenoid and record both values in table 9.3.

Parameter	Value
R1k	
RL	

 Table 9.3:
 Circuit 2 data table.

Use equation 9.6 to calculate the value of  $v(\tau)$ . Trigger the oscilloscope on a rising edge and place a cursor at v(0) and at  $v(\tau)$ . The time difference between these two cursors is equal to the time constant  $\tau$ . Record all of these values in table 9.4.

Parameter	Value
v( au)	
$v(\infty)$	
τ	

Table 9.4: Circuit 2 output values.

When you have finished recording data, demonstrate your results to your instructor to receive a stamp.

**Circuit 3:** Repeat the above experiment, but this time with **one** iron core inserted entirely into the solenoid. Otherwise: the circuit diagram remains exactly the same! Use equation 9.6 to calculate the value of  $v(\tau)$ . Trigger the oscilloscope on a rising edge and place a cursor at v(0) and at  $v(\tau)$ . The time difference between these two cursors is equal to the time constant  $\tau$ . Record these values in table 9.5.

Parameter	Value
v( au)	
$v(\infty)$	
au	

**Table 9.5:** Circuit 3 output values.

When you have finished recording data, demonstrate your results to your instructor to receive a stamp.

Instructor Stamp: \_\_\_\_

## 9.5 RL Circuit General Transient Response

In general, an RL circuit may have an initial value i(0) and a final value  $i(\infty)$  that are not both zero (not a fully discharging circuit, nor a charging from zero circuit). This is called the general RL transient response and is the most general form of the transient. (In other words, this analysis and equation can be used all the time, rather than in specific situations as with the discharging or charging cases.) The current flowing through the inductor is defined by equation 9.7, where i(0) is the initial current flow through the inductor,  $i(\infty)$  is the steady-state current flow through the inductor, and  $\tau$  is defined by equation 9.5.

$$i(t) = \left[i(\infty) + (i(0^+) - i(\infty))e^{-t/\tau}\right] u(t) + i(0^-) u(-t)$$
(9.7)

In the general case,  $\tau$  describes the amount of time it takes the current to reach a value given by equation 9.8.

$$i(\tau) = 0.6321i(\infty) + 0.3679i(0^+).$$
(9.8)

Circuit 4: In the pre-lab, you determined the inductor values necessary to build an RL circuit with a time constant equal to 1 ms. Measure the actual value of your inductor, and record it in table 9.6 as  $L_{REAL}$ .

Use an ohmmeter to measure the resistance of the inductor and record it as  $R_L$ . (The parasitic resistance of the inductor is not explicitly shown on the circuit diagram!) The value of resistor  $R_1$  below should be equal to 100  $\Omega - R_L$ . Record the **measured** value of this resistor in table 9.6. To assist with your calculations in the lab homework, you will also measure the actual values of the 47  $\Omega$  and 18  $\Omega$  resistors.

Parameter	Value
Lreal	
RL	
R1	
R47	
R18	

Table 9.6: Circuit 4 data table.

Build the circuit shown in figure 9.5 and display the output on the oscilloscope.



Figure 9.5: Lab 9 circuit 4 diagram.

Record  $v(0^+)$ ,  $v(\infty)$ ,  $v(\tau)$ , and  $\tau$  in table 9.7. Use equation 9.8 to calculate  $v(\tau)$ . When the circuit has the correct time constant (1 ms), demonstrate the output waveform to your instructor to receive a stamp.

Parameter	Value
v(0+)	
v( au)	
$v(\infty)$	
τ	

Table 9.7: Circuit 4 output values.

# Lab 9 Homework

Carefully read each question before answering. Show all work or justify your answers to receive credit. Attach a separate sheet of paper, if necessary, to show all work and calculations.

1. Based on your measurements in Circuit 1, and the symbolic equation you derived in question 2 of the pre-lab, derive an equation for i(t) for your circuit. Use a calculator to format your equation in the form given in equation 9.4, do not leave terms in fractional form. (Assuming you used the oscilloscope correctly in lab, you can use your measured value of  $\tau$  from table 9.2. Otherwise, you can calculate it from your passive component values.)

- 2. The following questions pertain to lab Circuits 2 and 3. You will also need to refer to equation 9.5, as well as the equation that you derived in pre-lab question 3 (c).
  - (a) Calculate the value of the inductance of the solenoid without the iron core. Be sure to account for the parasitic resistance in your calculation!

(b) Calculate the value of the inductance of the solenoid with the iron core. Be sure to account for the parasitic resistance in your calculation!

3. Based on your measurements in Circuit 4, and the symbolic equation you derived in question 5 of the pre-lab, derive an equation for i(t) for your circuit. Use a calculator to format your equation in the form given in equation 9.7, do not leave terms in fractional form. (Assuming you used the oscilloscope correctly in lab, you can use your measured value of  $\tau$  from table 9.7. Otherwise, you can calculate it from your passive component values.)

4. Use your measurements of inductance and parasitic resistance in Circuits 1 and 4 to answer the following questions based on figure 9.6.



Figure 9.6: Lab 9 homework circuit diagram.

(a) How is  $\tau$  affected by the parasitic resistance of the inductor? Does this cause the circuit to discharge faster or slower than an "ideal" circuit?

(b) Let's say you need to modify the circuit in figure 9.6 so that its time constant is as close to ideal as possible. You cannot change the solenoid, and you cannot change the resistor R that's in parallel with it. What modification(s) can you make to the circuit that reduce(s) the effect of the parasitic resistance to get the time constant as close to ideal as possible?

## Pre-Lab 10

Carefully read the entirety of Lab 10, then answer the following questions. Attach a separate sheet of paper, if necessary, to show all work and calculations.

1. Is figure 10.1 a first order or second order circuit? How do you know?



Figure 10.1: RC circuit diagram.

2. Is figure 10.2 a first order or second order circuit? How do you know?



Figure 10.2: RC circuit diagram.

3. Is figure 10.3 a first order or second order circuit? How do you know?



Figure 10.3: RLC circuit diagram.

4. Is figure 10.4 a first order or second order circuit? How do you know?



Figure 10.4: RLC circuit diagram.

5. Calculate the necessary resistance value for the series RLC circuit shown in figure 10.5 to obtain a critically damped output. Assume that each inductor has 50  $\Omega$  of parasitic resistance.



Figure 10.5: RLC circuit diagram.

6. Calculate the necessary resistance value for the parallel RLC circuit shown in figure 10.6 to obtain a critically damped output. Assume that each inductor has 50  $\Omega$  of parasitic resistance.



Figure 10.6: RLC circuit diagram.

## Lab 10: Second Order Circuits

In this lab, we will learn about second order circuits, which are composed of two energy storage devices. These devices have very different properties than RL and RC circuits.

For lab resources and information, go to the following URL or scan the QR code. doctor-pasquale.com/circuit-analysis-lab-10



## 10.1 Second Order Circuits

Second order circuits are so-called because they consist of two energy storage devices (two capacitors, two inductors, or one capacitor and one inductor).

There are three possible transient responses (i.e. the voltage abruptly changes from one level to another) possible for a second order circuit. An **overdamped** response indicates that the circuit takes a long time to respond to change in the input voltage (or current). A **critically damped** circuit has the quickest possible approach from the initial to the final value that does not oscillate, overshoot, or undershoot the final value. Finally, a **underdamped** circuit responds so quickly to changes in the input that the output oscillates around the final voltage value.

The nature of the circuit (whether it is overdamped, critically damped, or underdamped) has to do with the solution to the characteristic equation of the differential equation that is obtained upon calculating the output voltage (or current). These conditions will be explored for RLC series circuits and RLC parallel circuits in this lab.

## 10.2 Series RLC Circuit

A series RLC circuit is comprised of a resistor, capacitor, and inductor all in series with each other. The second order differential equation defining a series RLC circuit is provided in equation 10.1, where v(t) defines the voltage dropped over the capacitor, R is the equivalent resistance of the circuit, L is the equivalent inductance, and C is the equivalent capacitance.

$$\frac{d^2 v(t)}{dt^2} + \frac{R}{L} \frac{dv(t)}{dt} + \frac{1}{LC} v(t) = 0$$
(10.1)

The damping coefficient ( $\alpha$ ) and resonant frequency ( $\omega_0$ ) of a series RLC circuit are given by equations 10.2 and 10.3.

$$\alpha = \frac{R}{2L} \tag{10.2}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \tag{10.3}$$

These parameters can be used to determine if the circuit is overdamped, critically damped, or underdamped. When  $\alpha > \omega_0$ , the circuit is overdamped. When  $\alpha = \omega_0$ , it is critically damped. Otherwise, the circuit is underdamped. **Circuit 1:** Measure the inductance of the very large solenoid and the capacitance of a 50 nF ceramic capacitor (you will likely need to use series or parallel combinations to generate this capacitance value). Record these values in table 10.1. Use an ohmmeter to measure the parasitic resistance of the inductor and record it as  $R_L$ . The value of resistor  $R_1$  below should be equal to 2 k $\Omega$ . Record the **measured** value of this resistor in table 10.1.

Parameter	Value
С	
L	
RL	
R1	

Table 10.1: Circuit 1 data table.

Build a series RLC circuit (defined by figure 10.7) using the components that you just measured. Use an input wave that is a square wave with a voltage between 0 V and 5 V and a frequency of approximately 10 Hz (any frequency that allows the transient to fully discharge to the steady-state is acceptable). Configure the oscilloscope to trigger on a falling edge. Zoom in so that the output signal can easily be viewed on the oscilloscope screen.



Figure 10.7: Series RLC circuit diagram.

Record both the input and output waveforms on the oscilloscope and demonstrate them to your instructor to receive a stamp. If there are any oscillations in the output signal that cannot be explained by the input signal, measure the frequency of the oscillations and record the value below.

**Circuit 2:** Keeping the circuit from above otherwise the same, replace resistor  $R_1$  with a 100 k $\Omega$  resistor. Record the measured value of this resistor as  $R_2$  in table 10.2.

Parameter	Value
R2	

Table 10.2: Circuit 2 data table.

Record both the input and output waveforms on the oscilloscope and demonstrate them to your instructor to receive a stamp. If there are any oscillations in the output signal that cannot be explained by the input signal, measure the frequency of the oscillations and record the value below.

Instructor Stamp: \_\_\_\_\_

**Circuit 3:** Calculate the value of resistance that must be used to cause  $\alpha = \omega_0$  (be sure that you account for any parasitic resistances in your calculation!) and record this value in table 10.3 as  $R_{IDEAL}$ . Use a potentiometer to achieve this resistance. Dial the potentiometer until you obtain a critically damped response. Once this has been achieved, record the measured value of your potentiometer as  $R_3$ .

Parameter	Value
Rideal	
R3	

Table 10.3: Circuit 3 data table.

Record both the input and output waveforms on the oscilloscope and demonstrate them to your instructor to receive a stamp. If there are any oscillations in the output signal that cannot be explained by the input signal, measure the frequency of the oscillations and record the value below.

### 10.3 RLC Parallel Circuit

An RLC parallel circuit is comprised of a resistor, capacitor, and inductor all in parallel with each other. A parallel RLC circuit is defined by the second order differential equation shown in equation 10.4, where i(t) is the current flowing through the inductor, R is the resistance, L is the inductance, and C is the capacitance.

$$\frac{d^2 i(t)}{dt^2} + \frac{1}{RC} \frac{di(t)}{dt} + \frac{1}{LC} i(t) = 0$$
(10.4)

The damping coefficient ( $\alpha$ ) and resonant frequency ( $\omega_0$ ) of a parallel RLC circuit are defined by equations 10.5 and 10.6.

$$\alpha = \frac{1}{2RC} \tag{10.5}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \tag{10.6}$$

These parameters can be used to determine if the circuit is overdamped, critically damped, or underdamped. When  $\alpha > \omega_0$ , the circuit is overdamped. When  $\alpha = \omega_0$ , it is critically damped. Otherwise, the circuit is underdamped.

#### 10.3.1 A Parallel RLC Circuit with Parasitic Resistance

A parallel RLC circuit, taking into account the parasitic resistance of the inductor, has a different differential equation. (In a series RLC circuit, the parasitic resistance can easily be added to the resistor because the two components are in series; that is not the case in the parallel circuit.) The circuit diagram in figure 10.8 shows the parasitic resistance of the inductor (labeled as  $R_L$ ) to make it very clear.



Figure 10.8: Parallel RLC circuit with parasitic resistance.

The differential equation for current is defined by equation 10.7.

$$\frac{d^2i(t)}{dt^2} + \left(\frac{1}{RC} + \frac{R_L}{L}\right)\frac{di(t)}{dt} + \left(\frac{1}{LC} + \frac{R_L}{RLC}\right)i(t) = 0$$
(10.7)

The damping coefficient ( $\alpha$ ) and resonant frequency ( $\omega_0$ ) of a parallel RLC circuit with parasitic resistance are given by equations 10.8 and 10.9.

$$\alpha = \frac{1}{2} \left( \frac{1}{RC} + \frac{R_L}{L} \right) \tag{10.8}$$

$$\omega_0 = \sqrt{\frac{R_L}{RLC} + \frac{1}{LC}} \tag{10.9}$$

To obtain a critically damped circuit, the value of the resistor R must be as defined in equation 10.10

$$R = \frac{LR_L - \frac{2L^{3/2}}{\sqrt{C}}}{CR_L^2 - 4L} \tag{10.10}$$

### 10.3.2 Building a Parallel RLC Circuit in Lab

Because we don't have an AC current source, and because we are unable to directly measure current with the oscilloscope, a few changes will be made to the circuit to make it suitable for lab measurements. A small load resistor, labeled  $R_{LOAD}$  in the circuit diagram shown in figure 10.9, is used to convert the current i(t) into voltage using Ohm's Law. In addition, source transformation has been used to turn the current source (in parallel with resistor R) into a voltage source (in series with resistor R). Keep in mind that the voltage value will need to be changed if the value of the resistor R is changed to obtain different damping characteristics! Note that the parasitic resistance of the inductor is not explicitly shown on this circuit diagram!



Figure 10.9: Modified parallel RLC circuit to be built in lab.

This load resistance can be added to the parasitic resistance of the inductor  $(R_L + R_{LOAD})$  in all calculations to obtain appropriately critically damped circuits.

**Circuit 4:** Continue to use the very large solenoid as your inductor. Use a capacitor of between 400 nF and 450 nF. (Combine the available ceramic capacitors in series or parallel, and measure and record your actual capacitance value in table 10.4.)

Finally, measure the two resistances used in the circuit.  $R_4$  is the 100  $\Omega$  resistor in series with the voltage source (that would be in parallel with a current source if this were a normal parallel RLC circuit.) The load resistor should have a resistance of 100  $\Omega$ . Record both values in table 10.4.

Parameter	Value
С	
R4	
Rload	

Table 10.4: Circuit 4 data table.

Build a parallel RLC circuit (defined by figure 10.10) using the components that you just measured. Use an input wave that is a square wave with a voltage between 0 V and 10 V and a frequency of approximately 10 Hz (any frequency that allows the transient to fully discharge to the steady-state is acceptable). Configure the oscilloscope to trigger on a falling edge. Zoom in so that the output signal can easily be viewed on the oscilloscope screen.



Figure 10.10: Parallel RLC circuit diagram.

Record both the input and output waveforms on the oscilloscope and demonstrate them to your instructor to receive a stamp. If there are any oscillations in the output signal that cannot be explained by the input signal, measure the frequency of the oscillations and record the value below.

**Circuit 5:** Keeping the circuit from above otherwise the same, replace the 100  $\Omega$  resistor (the resistor in series with the source, **not** the load resistor) with a 10 k $\Omega$  resistor. Measure and record the actual value of this resistor in table 10.5 as  $R_5$ .

Parameter	Value
R5	

 Table 10.5:
 Circuit 5 data table.

Record both the input and output waveforms on the oscilloscope and demonstrate them to your instructor to receive a stamp. If there are any oscillations in the output signal that cannot be explained by the input signal, measure the frequency of the oscillations and record the value below.

Instructor Stamp: \_\_\_\_\_

**Circuit 6:** Calculate the value of resistance that must be used to cause  $\alpha = \omega_0$  (be sure that you account for any parasitic resistances and load resistances in your calculation!) and record this value in table 10.6 as  $R_{IDEAL}$ . You will use a potentiometer to achieve this resistance. Dial the potentiometer until you obtain a critically damped response. Once this has been achieved, record the measured value of your potentiometer in table 10.6 as  $R_6$ .

Parameter	Value
Rideal	
R6	

Table 10.6: Circuit 6 data table.

Record both the input and output waveforms on the oscilloscope and demonstrate them to your instructor to receive a stamp. If there are any oscillations in the output signal that cannot be explained by the input signal, measure the frequency of the oscillations and record the value below.

## Lab 10 Homework

Carefully read each question before answering. Show all work or justify your answers to receive credit. Attach a separate sheet of paper, if necessary, to show all work and calculations.

- 1. Use measured component values and account for any and all parasitic component effects in the following questions. Show all calculations for all parameter values ( $\omega_0$ ,  $\alpha$ , roots, initial conditions, etc.) to receive credit for your work.
  - (a) Derive a non-differential question for v(t) in Circuit 1.

(b) Derive a non-differential question for v(t) in Circuit 2.

(c) Derive a non-differential question for v(t) in Circuit 3. (You can assume that  $\alpha = \omega_0$  for the purposes of writing out the equation.)

2. Which of the circuits (1 or 2) was overdamped? How do you know?

3. Which of the circuits (1 or 2) was underdamped? How do you know?

4. For the **underdamped** circuit, **meaningfully** compare your calculated oscillation frequency ( $\beta$ ) to your recorded oscillation frequency. (Ensure that units are equivalent before comparing. Meaningful comparisons do not include qualitative words such as "close", "good", "excellent", or other similar words.)

- 5. For the **critically damped** circuit that you built...
  - (a) ...**meaningfully** compare your calculated damping parameter ( $\alpha$ ) to your calculated resonant frequency ( $\omega_0$ ).

(b) If your damping parameter and resonant frequency differ by any amount, would you need to increase or decrease the potentiometer to achieve a more critically damped response? Justify your answer.

- 6. Use measured component values and account for any and all parasitic component effects in the following questions. Show **all** calculations for **all** parameter values ( $\omega_0$ ,  $\alpha$ , roots, initial conditions, etc.) to receive credit for your work.
  - (a) Derive a non-differential question for i(t) in Circuit 4.

(b) Derive a non-differential question for i(t) in Circuit 5.

(c) Derive a non-differential question for i(t) in Circuit 6. (You can assume that  $\alpha = \omega_0$  for the purposes of writing out the equation.)

7. Which of the circuits (4 or 5) was overdamped? How do you know?

8. Which of the circuits (4 or 5) was underdamped? How do you know?

9. For the **underdamped** circuit, **meaningfully** compare your calculated oscillation frequency ( $\beta$ ) to your recorded oscillation frequency. (Ensure that units are equivalent before comparing.)

- 10. For the **critically damped** circuit that you built...
  - (a) ...**meaningfully** compare your calculated damping parameter ( $\alpha$ ) to your calculated resonant frequency ( $\omega_0$ ).

(b) If your damping parameter and resonant frequency differ by any amount, would you need to increase or decrease the potentiometer to achieve a more critically damped response? Justify your answer.

## Pre-Lab 11

Carefully read the entirety of Lab 11, then answer the following questions. Attach a separate sheet of paper, if necessary, to show all work and calculations.

1. What type of filter is shown in figure 11.1? How do you know?



Figure 11.1: Filter circuit diagram.

2. What type of filter is shown in figure 11.2? How do you know?



Figure 11.2: Filter circuit diagram.

3. What type of filter is shown in figure 11.3? How do you know?



Figure 11.3: Filter circuit diagram.

4. What type of filter is shown in figure 11.4? How do you know?



Figure 11.4: Filter circuit diagram.

5. In lab Circuit 1, you will design a first-order low-pass RC filter that has a cutoff frequency of 5 kHz. Use only a single available resistor and single available ceramic capacitor value to obtain a cutoff frequency within 10% of the desired frequency. Use resistor and capacitor values available in the inventory of parts.

6. In lab Circuit 3, you will design a first-order high-pass RC filter that has a cutoff frequency of 30 kHz. Use only a single available resistor and single available ceramic capacitor value to obtain a cutoff frequency within 10% of the desired frequency. Use resistor and capacitor values available in the inventory of parts. The capacitor needs to be 10 nF or smaller to keep resistances from being too small.

- 7. In lab Circuit 5, you will design a band-pass RC filter that consists of two stages.
  - (a) The first stage is a high-pass filter with a cutoff frequency of 500 Hz. Use only a single available resistor and single available ceramic capacitor value to obtain a cutoff frequency within 10% of the desired frequency. Use resistor and capacitor values available in the inventory of parts.

(b) The second stage is a high-pass filter with a cutoff frequency of 1,000 Hz. Use only a single available resistor and single available ceramic capacitor value to obtain a cutoff frequency within 10% of the desired frequency. Use resistor and capacitor values available in the inventory of parts.

## Lab 11: Filters

In this lab, we will design filters, which are used to remove portions of signals from an electronic waveform.

For lab resources and information, go to the following URL or scan the QR code. doctor-pasquale.com/circuit-analysis-lab-11

## 11.1 Low-Pass Filters

A low-pass filter is used to allow low frequencies to pass from the input to the output, while blocking higher frequencies. As the input voltage frequency changes, the magnitude of the output voltage will change. This can be characterized by calculating  $v_{out}/v_{in}$  at varying frequencies and looking at the response. This normalized output is shown in figure 11.5 (note the use of the log-log plot).



Figure 11.5: Normalized response of a low-pass filter.

## 11.1.1 First Order Low-Pass Filters

A first order low-pass filter uses a single energy-storage element (capacitor or inductor) to block high frequencies from reaching the output. The frequency at which this output blockage occurs is known as the cutoff frequency and is related to the time constant of the circuit, defined in equation 11.1.

$$\omega_c = \frac{1}{\tau} \tag{11.1}$$

## 11.1.2 Second Order Low-Pass Filters

Second order low-pass filters contain two energy storage devices. A second-order circuit increases the rate at which the voltage drops off after the cutoff frequency. Each individual stage has the effect of increasing the slope by 20 dB/decade (that is, 20 dB per power of ten increase in the frequency). The cutoff frequency of a second order low-pass filter is described by equation 11.2

$$\omega_c = \sqrt{a_0} \tag{11.2}$$

There is no limit to the number of stages that can be used in a circuit to achieve higher-order filters. Filters that use inductors are generally not used due to the non-trivial parasitic resistance present in most inductors. In RC filters, ceramic capacitors are used due to their bipolar operation. However, they are limited as to their maximum capacitance value. **Circuit 1:** Design a first-order low-pass RC filter that has a cutoff frequency of 5 kHz. In the pre-lab, you should have calculated the ideal component values to use to achieve this cutoff frequency. Measure the actual resistance and capacitance of these components and record them in table 11.1 as  $R_1$  and  $C_1$ .

Parameter	Value
R1	
C1	

Table 11.1: Circuit 1 data table.

Use the circuit diagram in figure 11.6 to build the low-pass RC filter. **Note:** in this and all circuits in this lab, you will be using a purely AC input signal. That means there should be no DC offset component to the input voltage.



Figure 11.6: First order low-pass RC filter.

Display both  $V_{IN}$  and  $V_{OUT}$  simultaneously on the oscilloscope. Ensure that the amplitude of  $V_{IN}$  remains at 5 V – if it does not, be sure to record the value of  $V_{IN}$  at each frequency where it is no longer equal to 5 V. Vary the frequency of the input wave to measure the output amplitude at each frequency. Record these values in table 11.2. Demonstrate your results to your instructor to receive a stamp.

f (Hz)	Vout (V)	f (Hz)	Vout (V)	f (Hz)	Vout (V)
50		2k		7k	
100		3k		10k	
200		4k		20k	
500		5k		50k	
1k		6k		100k	

 Table 11.2:
 Circuit 1 output voltage values.

**Circuit 2:** Add a second identical RC stage to the filter that you built in Circuit 1 to achieve a second-order filter. Measure the actual resistance and capacitance of the new components, and record these in table 11.3 as  $R_2$  and  $C_2$ .

Parameter	Value
R2	
C2	

**Table 11.3:** Circuit 2 data table.

Use the circuit diagram in figure 11.7 to build the second order low-pass RC filter.



Figure 11.7: Second order low-pass RC filter.

Display both  $V_{IN}$  and  $V_{OUT}$  simultaneously on the oscilloscope. Ensure that the amplitude of  $V_{IN}$  remains at 5 V – if it does not, be sure to record the value of  $V_{IN}$  at each frequency where it is no longer equal to 5 V. Vary the frequency of the input wave to measure the output amplitude at each frequency. Record these values in table 11.4. Demonstrate your results to your instructor to receive a stamp.

f (Hz)	Vout (V)	f (Hz)	Vout (V)	f (Hz)	Vout (V)
50		2k		7k	
100		3k		10k	
200		4k		20k	
500		5k		50k	
1k		6k		100k	

 Table 11.4:
 Circuit 2 output voltage values.

## 11.2 High-Pass Filters

A high-pass filter is used to allow high frequencies to pass from the input to the output, while blocking lower frequencies. This is essentially the opposite of a low-pass filter. Just as with low-pass filters, a high-pass filter is characterized by calculating  $v_{out}/v_{in}$  at varying frequencies and looking at the response. This normalized output is shown in figure 11.8 (note the use of the log-log plot).



Figure 11.8: Normalized response of a high-pass filter.

## 11.2.1 First Order High-Pass Filters

A first order high-pass filter has a single energy storage element. First order filters always have a cutoff frequency given in equation 11.1, regardless of the type of filter.

## 11.2.2 Second-Order High-Pass Filters

There are many ways to realize a second order high-pass filter. As we saw with the low-pass filter, a secondorder high-pass filter can be achieved using two cascading first-order filters. Another option is to use an RLC filter. All second order filters have a cutoff frequency given in equation 11.2. The limitations of cascaded filters is the same for high-pass as for low-pass filters. Parasitic resistance in inductors and relatively low ceramic capacitor values constrain the limits of the filter response. **Circuit 3:** Design a first-order high-pass RC filter that has a cutoff frequency of 30 kHz. In the pre-lab, you should have calculated the ideal component values to use to achieve this cutoff frequency. Measure the actual resistance and capacitance of these components and record them in table 11.5 as  $R_3$  and  $C_3$ . For this circuit to function properly,  $C_3$  must be no greater than 10 nF.

Parameter	Value
R3	
C3	

**Table 11.5:** Circuit 3 data table.

Use the circuit diagram in figure 11.9 to build the high-pass RC filter.



Figure 11.9: First order high-pass RC filter.

Display both  $V_{IN}$  and  $V_{OUT}$  simultaneously on the oscilloscope. Ensure that the amplitude of  $V_{IN}$  remains at 5 V – if it does not, be sure to record the value of  $V_{IN}$  at each frequency where it is no longer equal to 5 V. Vary the frequency of the input wave to measure the output amplitude at each frequency. Record these values in table 11.6. Demonstrate your results to your instructor to receive a stamp.

f (Hz)	Vout (V)	f (Hz)	Vout (V)	f (Hz)	Vout (V)
1k		24k		34k	
2k		26k		50k	
5k		28k		100k	
10k		30k		200k	
20k		32k		500k	

Table 11.6:	Circuit	3	output	voltage	values.
-------------	---------	---	--------	---------	---------

**Circuit 4:** Add a second identical RC stage to the filter that you built in Circuit 3 to achieve a second-order filter. Measure the actual resistance and capacitance of the new components, and record these values in table 11.7 as  $R_4$  and  $C_4$ .

Parameter	Value
R4	
C4	

 Table 11.7:
 Circuit 4 data table.

Use the circuit diagram in figure 11.10 to build the second order high-pass RC filter.



Figure 11.10: Second order high-pass RC filter.

Display both  $V_{IN}$  and  $V_{OUT}$  simultaneously on the oscilloscope. Ensure that the amplitude of  $V_{IN}$  remains at 5 V – if it does not, be sure to record the value of  $V_{IN}$  at each frequency where it is no longer equal to 5 V. Vary the frequency of the input wave to measure the output amplitude at each frequency. Record these values in table 11.8. Demonstrate your results to your instructor to receive a stamp.

f (Hz)	Vout (V)	f (Hz)	Vout (V)	f (Hz)	Vout (V)
1k		24k		34k	
2k		26k		50k	
5k		28k		100k	
10k		30k		200k	
20k		32k		500k	

Table 11.8: Circuit 4 output voltage values.
#### 11.3 Band-Pass Filters

A band-pass filter allows only a particular band of frequencies to reach the output. All other inputs are blocked from the output altogether. An example output response of a band-pass filter is shown in figure 11.11.



Figure 11.11: Normalized response of a band-pass filter.

A band-pass filter must be at least a second order circuit. The center frequency of a second order bandpass filter is given in equation 11.2. It is called the center frequency because it defines the center of the pass-band, which is the band of frequencies that are sent through to the output. Two cutoff frequencies define the limits of the pass-band and are defined in equations 11.3 and 11.4.

$$\omega_{c1} = -\alpha + \sqrt{\alpha^2 + \omega_0^2} \tag{11.3}$$

$$\omega_{c2} = \alpha + \sqrt{\alpha^2 + \omega_0^2} \tag{11.4}$$

The range of frequencies that are allowed to pass through to the output is known as the bandwidth, which is described by equation 11.5.

$$\beta = \omega_{c2} - \omega_{c1} \tag{11.5}$$

The quality-factor defines the "narrowness" of the filter response. A high quality-filter means that very few frequencies will be passed to the output. Bandwidth and quality-factor are therefore inversely related. The quality-factor of a second order band-pass filter is given by

$$Q = \frac{\sqrt{a_0}}{a_1}.\tag{11.6}$$

The design of a band-pass filter will depend on the final application of the device. If only one frequency needs to be passed through to the output, then it is important to design a filter with a high quality factor. If many frequencies need to be passed through to the output, then a lower quality factor will suffice.

#### 11.4 Band-Stop Filters

A band-stop filter only blocks a particular band of frequencies to reach the output. All other inputs are passed to the output. An example output response of a band-stop filter is shown in figure 11.12.



Figure 11.12: Normalized response of a band-stop filter.

As with band-pass filters, band-stop filters must be at least second order. They can be characterized by equations 11.2–11.6.

**Circuit 5:** In the pre-lab, you should have calculated the ideal component values to use to achieve this cutoff frequency. Measure the actual resistance and capacitance of these components and record them in table 11.9 as  $R_1$ ,  $C_1$ ,  $R_2$ , and  $C_2$ .

Parameter	Value
R1	
C1	
R2	
C2	

Table 11.9: Circuit 5 data table.

The band-pass RC filter consists of two stages. The first stage is a high-pass filter with a cutoff frequency of 500 Hz, and the second stage is a low-pass filter with a cutoff frequency of 1,000 Hz, as shown in figure 11.13.



Figure 11.13: Second order band-pass RC filter.

Display both  $V_{IN}$  and  $V_{OUT}$  simultaneously on the oscilloscope. Ensure that the amplitude of  $V_{IN}$  remains at 5 V – if it does not, be sure to record the value of  $V_{IN}$  at each frequency where it is no longer equal to 5 V. Vary the frequency of the input wave to measure the output amplitude at each frequency. Record these values in table 11.10. Demonstrate your results to your instructor to receive a stamp.

f (Hz)	Vout (V)	f (Hz)	Vout (V)	f (Hz)	Vout (V)
5		200		5k	
10		500		10k	
20		700		20k	
100		1k		50k	
150		2k		100k	

Table 11.10: Circuit 5 output voltage values.

**Circuit 6:** Build the twin-notch band-stop filter shown in figure 11.14. There is no need to measure the component values.



Figure 11.14: Twin-notch band-stop filter.

Display both  $V_{IN}$  and  $V_{OUT}$  simultaneously on the oscilloscope. Ensure that the amplitude of  $V_{IN}$  remains at 5 V – if it does not, be sure to record the value of  $V_{IN}$  at each frequency where it is no longer equal to 5 V. Vary the frequency of the input wave to measure the output amplitude at each frequency. Record these values in table 11.11. Demonstrate your results to your instructor to receive a stamp.

f (Hz)	Vout (V)	f (Hz)	Vout (V)	f (Hz)	Vout (V)
10		500		5k	
20		1k		10k	
50		2k		20k	
100		3k		50k	
200		4k		100k	

 Table 11.11: Circuit 6 output voltage values.

## Lab 11 Homework

Carefully read each question before answering. Show all work or justify your answers to receive credit. Attach a separate sheet of paper, if necessary, to show all work and calculations.

- 1. For the data that you took in both Circuit 1 and Circuit 2, graph  $v_o/v_i$  as a function of frequency, using a log-log plot. Put both sets of data on the same graph. Ensure that all axes and data are labeled with correct units.
- 2. What effect would a third identical stage added to the output of Circuit 2 have on the output response?

- 3. For the data that you took in both Circuit 3 and Circuit 4, graph  $v_o/v_i$  as a function of frequency, using a log-log plot. Put both sets of data on the same graph. Ensure that all axes and data are labeled with correct units.
- 4. What effect would a third identical stage added to the output of Circuit 4 have on the output response?

5. What are at least two reasons that inductors are not generally used in filter design?

6. Derive the differential equation (in terms of v(t)) for the circuit that you built in Circuit 5 in **symbolic** form.

7. Using the diff-eq you just derived, calculate the cutoff frequency of the circuit. Convert to  $f_c$  in units of Hz.

8. Using the diff-eq you just derived, calculate the quality factor of the circuit.

- 9. For the data that you took in both Circuit 5 and Circuit 6, graph  $v_o/v_i$  as a function of frequency, using a log-log plot. Put both sets of data on the same graph. Ensure that all axes and data are labeled with correct units.
- 10. By qualitatively analyzing the graph, which circuit has the larger quality factor? How do you know?

11. Using the graph, estimate the cutoff frequency of the band-stop filter from lab Circuit 6.

# Pre-Lab 12

Carefully read the entirety of Lab 12, then answer the following questions. Attach a separate sheet of paper, if necessary, to show all work and calculations.

1. What channel on the oscilloscope will you use for input voltage? Why?

2. If your oscilloscope measures a phase angle of  $42^{\circ}$ , what is the actual phase angle?

3. If your oscilloscope measures a phase angle of 289°, what is the actual phase angle?

4. What setting on the oscilloscope will you use to remove noise from the output signal?

5. Carefully read lab Circuit 3. You will be asked to design two different circuits that achieve an output phase angle of  $45^{\circ}$  using a capacitance of 470 nF. Select two different resistor values from the inventory of parts (use values between 100  $\Omega$  and 1 M $\Omega$ ) and calculate the frequencies (ensure the values are between 50 Hz and 500 kHz) required to obtain an output phase angle of  $45^{\circ}$ .

6. Carefully read lab Circuit 6. You will be asked to design two different circuits that achieve an output phase angle of  $-20^{\circ}$  using an inductance of 200 mH. Assume that the inductor will have a parasitic resistance of approximately 160  $\Omega$ . Select two different resistor values from the inventory of parts (use values between 100  $\Omega$  and 1 M $\Omega$ ) and calculate the frequencies (ensure the values are between 50 Hz and 500 kHz) required to obtain an output phase angle of  $-20^{\circ}$ .

## Lab 12: Phasors and Impedance

In this lab, we will explore the concepts of phasors and impedances, which can be used to simplify circuit elements in AC circuits.

For lab resources and information, go to the following URL or scan the QR code. doctor-pasquale.com/circuit-analysis-lab-12



### 12.1 Phasors

All elements of a circuit can be phasor-transformed. The word phasor is short for phase vector. Phasors give us information about the amplitude and phase of a waveform, and follow the same arithmetical properties as vectors. Voltage and current sources get transformed into phasors by noting the amplitude and phase of the source. Circuit elements get transformed into impedances.

By performing a phasor-transform on a circuit, circuit laws such as KCL, KVL, voltage and current dividers, source transformation, etc., still hold and are relatively simple to perform.

A complex number z can be transformed into a phasor by following the rules of Cartesian to polar coordinate transformation.

### 12.2 Impedance

Impedance (which has units of Ohms) is a useful concept that allows us to use all of the circuit laws that we learned earlier in the semester without having to use calculus to determine voltages and currents in elements such as inductors and capacitors. Impedances use the complex number system to express the resistance (R) and reactance (X) of circuit components, as described in equation 12.1.

$$\mathbf{Z} = R + jX \tag{12.1}$$

The impedance of a resistor is real, and does not have any dependence on frequency. The impedances of capacitors and inductors, however, are completely imaginary, and do depend on the frequency of the source. The impedance of a capacitor is defined by equation 12.2 and the impedance of an inductor is defined by equation 12.3.

$$\mathbf{Z}_{\mathbf{C}} = \frac{-j}{\omega C} \tag{12.2}$$

$$\mathbf{Z}_{\mathbf{L}} = j\omega L \tag{12.3}$$

### 12.3 Oscilloscope Setup

In this lab, you will need to measure the phase angle between input and output voltages. It is necessary to be consistent with all of your measurements in order to obtain correct phase angles. For this reason, always measure input voltage on channel 1, and always measure output voltage on channel 2. Phase measurements should be set to  $\phi = \text{CH1-CH2}$ . Finally, you will need to apply the following corrections to the phase angles to obtain correct values.

if 
$$0^{\circ} \le |\phi| < 180^{\circ} \dots \phi_{REAL} = -\phi$$
 (12.4)

if 
$$180^{\circ} \le |\phi| < 360^{\circ} \dots \phi_{REAL} = 360 - \phi$$
 (12.5)

It is possible that the output voltage may have some high-frequency noise fluctuations that cause the oscilloscope to have a hard time settling on a value for the phase angle. It is therefore recommended that

you use low-pass filtering on the oscilloscope itself. This can be accessed under the channel 2 menu (under the violet voltage dial) then by going to filter. Use a low-pass filter with a cutoff frequency of approximately 35 kHz, or until the signal is cleaned up sufficiently.

**Circuit 1:** Build the circuit shown in figure 12.1.



Figure 12.1: RC circuit diagram.

Use an RLC meter to measure the capacitance and record the value below.

You will use multiple resistor values to determine the effect of load resistance on output voltage and phase angle. Use the resistor values given, and measure and record each of the actual values in the  $R_{REAL}$  column of table 12.1. Measure the amplitude of the output voltage and phase angles using the oscilloscope. When you have finished collecting data, demonstrate your results to your instructor to receive a stamp.

<b>R</b> (Ω)	Rreal ( $\Omega$ )	Vout (V)	φ <b>(</b> ° <b>)</b>
220			
470			
820			
2.2k			
4.7k			
6.8k			
10k			

Table 12.1: Circuit 1 output data.

Instructor Stamp: \_\_\_\_

150

**Circuit 2:** Return to the 220  $\Omega$  resistor. Measure the output voltage amplitude as well as the phase difference between the input and output signals for at least 10 different frequencies between 50 – 600 Hz. Fill out the data in table 12.2. You will use this data in the lab homework to explore the relationship between frequency and impedance in a capacitor. When you have finished collecting data, demonstrate your results to your instructor to receive a stamp.

f (Hz)	Vout (V)	φ <b>(</b> ° <b>)</b>	f (Hz)	Vout (V)	φ <b>(</b> ° <b>)</b>
50					
			600		

Table 12.2: Circuit 2 output data.

#### Instructor Stamp: \_\_\_\_\_

**Circuit 3:** Now that you understand the relationship between load resistance and frequency on the output properties of this circuit, design two circuits to obtain an output phase angle of 45°. Both circuits must have the capacitor value of 470 nF. You should have calculated these values in the pre-lab. Measure the actual resistances of both resistors and record them in table 12.3. After building the circuits, record the frequencies of operation in table 12.3.

Circuit	Resistance ( $\Omega$ )	Frequency (Hz)
1		
2		

1

Table 12.3: Circuit 3 data table.

When you have achieved the correct output response, demonstrate your results to your instructor to receive a stamp.

Circuit 1:

Circuit 2:

Instructor Stamp: \_\_\_\_\_

Circuit 4: Build the circuit shown in figure 12.2.



Figure 12.2: RL circuit diagram.

Use a calibrated RLC meter to measure the inductance and record the value below. Measure and record the parasitic resistance of the inductor using an ohmmeter.

You will use multiple resistor values to determine the effect of load resistance on output voltage and phase angle. Use the resistor values given, and measure and record each of the actual values in the  $R_{REAL}$  column of table 12.4. Measure the amplitude of the output voltage and phase angles using the oscilloscope. When you have finished collecting data, demonstrate your results to your instructor to receive a stamp.

<b>R (</b> Ω)	Rreal ( $\Omega$ )	Vout (V)	φ <b>(</b> ° <b>)</b>
10			
18			
24			
47			
100			
220			
330			

Table 12.4: Circuit 4 output data.

**Circuit 5:** Return to the 10  $\Omega$  resistor. Measure the output voltage amplitude as well as the phase difference between the input and output signals for at least 10 different frequencies between 50 – 600 Hz. Fill out the data in table 12.5. You will use this data in the lab homework to explore the relationship between frequency and impedance in an inductor. When you have finished collecting data, demonstrate your results to your instructor to receive a stamp.

f (Hz)	Vout (V)	φ <b>(</b> ° <b>)</b>	f (Hz)	Vout (V)	φ (°)
50					
			600		

Table 12.5:Circuit 5 output data.

#### Instructor Stamp: \_\_\_\_

**Circuit 6:** Now that you understand the relationship between load resistance and frequency on the output properties of this circuit, design two circuits to obtain an output phase angle of  $-20^{\circ}$ . Both circuits must have an inductor value of 200 mH. You should have calculated these values in the pre-lab and recorded the appropriate frequencies and resistance values below. Measure and record the parasitic resistance of the inductor.

Measure the actual resistances of both resistors and record them in table 12.6. After building the circuit, record the frequencies of operation in table 12.3. Keep in mind that you may need to adjust the resistance values due to the parasitic resistance of the inductor!

Circuit	Resistance ( $\Omega$ )	Frequency (Hz)
1		
2		

Table 12.6: Circuit 6 data table.

When you have achieved the correct output response, demonstrate your results to your instructor to receive a stamp.

Circuit 1:

Instructor Stamp: \_\_\_\_\_

Circuit 2:

Circuit 7: There are three mystery impedance boxes which contain unknown circuit elements. They all have four terminals, A, B, C, and D. Connect each one in turn to an AC power supply, as shown in figure 12.3.



Figure 12.3: Mystery impedance box circuit diagram.

Then, measure the amplitude of  $v_o$  as well as the phase difference between  $v_i$  and  $v_o$  at three different frequencies and record in table 12.7. When you have finished recording data, show your results to the instructor to receive a stamp.

	Box A		Box B		Box C	
f (kHz)	Vout (V)	φ <b>(</b> ° <b>)</b>	Vout (V)	φ (°)	Vout (V)	φ <b>(</b> ° <b>)</b>
1						
5						
10						

Table 12.7: Circuit 7 data table.

## Lab 12 Homework

Carefully read each question before answering. Show all work or justify your answers to receive credit. Attach a separate sheet of paper, if necessary, to show all work and calculations.

- 1. Refer to lab Circuits 1–3.
  - (a) What type of filter is this type of circuit? How do you know?

(b) How does the output voltage change with respect to the load resistance (all other things being equal)?

(c) Explain the result of the previous question in terms of the cutoff frequency of the filter. (In other words, are you raising or lowering the cutoff frequency with increasing values of R, and does that allow more or less of the input to pass through to the output with f = 100 Hz?)

(d) Calculate the total impedance of the circuit in Cartesian form, using the values that you measured and recorded in lab, using the 220  $\Omega$  resistor and a frequency of 100 Hz.

(e) Calculate the total impedance of the circuit **in Cartesian form**, using the values that you measured and recorded in lab, using the 4.7 k $\Omega$  resistor and a frequency of 100 Hz.

- 2. Refer to lab Circuits 4–6.
  - (a) What type of filter is this type of circuit? How do you know?

(b) How does the output voltage change with respect to the load resistance (all other things being equal)?

(c) Explain the result of the previous question in terms of the cutoff frequency of the filter. (In other words, are you raising or lowering the cutoff frequency with increasing values of R, and does that allow more or less of the input to pass through to the output with f = 100 Hz?)

(d) Calculate the total impedance of the circuit in Cartesian form, using the values that you measured and recorded in lab, using the 10  $\Omega$  resistor and a frequency of 100 Hz. Include the parasitic resistance of the inductor in all of your calculations!

(e) Calculate the total impedance of the circuit **in Cartesian form**, using the values that you measured and recorded in lab, using the 100  $\Omega$  resistor and a frequency of 100 Hz. Include the parasitic resistance of the inductor in all of your calculations!

3. How does the impedance of a capacitor change with respect to frequency? How does your data support this?

4. How does the impedance of an inductor change with respect to frequency? How does your data support this?

5. How does the impedance of a resistor change with respect to frequency? How does your data support this? (Hint: how would the data in Tales 12.2 and 12.5 change if the impedance of a resistor increased / decreased / remained constant with frequency?)

6. Consider the circuit shown in figure 12.4.



Figure 12.4: One of the mystery impedance box circuit diagrams.

(a) Calculate the phasor of  $\mathbf{v_o}$  in polar form at 1 kHz.

(b) Calculate the phasor of  $\mathbf{v_o}$  in polar form at 5 kHz.

(c) Calculate the phasor of  $\mathbf{v_o}$  in polar form at 10 kHz.

(d) Which mystery impedance box does this circuit correspond to?

7. Consider the circuit shown in figure 12.5.



Figure 12.5: One of the mystery impedance box circuit diagrams.

(a) Calculate the phasor of  $\mathbf{v_o}$  in polar form at 1 kHz.

(b) Calculate the phasor of  $\mathbf{v_o}$  in polar form at 5 kHz.

(c) Calculate the phasor of  $\mathbf{v_o}$  in polar form at 10 kHz.

(d) Which mystery impedance box does this circuit correspond to?

8. Consider the circuit shown in figure 12.5.



Figure 12.6: One of the mystery impedance box circuit diagrams.

(a) Calculate the phasor of  $\mathbf{v_o}$  in polar form at 1 kHz.

(b) Calculate the phasor of  $\mathbf{v_o}$  in polar form at 5 kHz.

(c) Calculate the phasor of  $\mathbf{v_o}$  in polar form at 10 kHz.

(d) Which mystery impedance box does this circuit correspond to?

# Pre-Lab 13

Carefully read the entirety of Lab 13, then answer the following questions. Attach a separate sheet of paper, if necessary, to show all work and calculations.

1. What is the purpose of the 100  $\Omega$  resistor used in lab Circuits 1 and 5?

2. What is a pull-down resistor? In which lab questions will you use them?

3. Explain the difference between a regular diode and a Zener diode. Use your own words, don't just copy what was written in this lab!

## Lab 13: Diodes

In this lab, we will take a little break from in-class material to explore a very important electronic component: the diode. The diode is a passive, nonlinear circuit element that is used in many electronics applications.

For lab resources and information, go to the following URL or scan the QR code. doctor-pasquale.com/circuit-analysis-lab-13



### 13.1 The Diode

The diode is a semiconductor device composed of a P-N junction. This means that silicon has been doped with acceptor atoms in one area (named the P-type area due to an excess of "holes," or the absence of electrons) and adjacent to that area it has been doped with donor atoms (named the N-type area due to an excess of electrons). When a forward voltage is applied to the P-N junction (positive potential to the P-type area, and negative potential to the N-type area), an electric field is created and current will flow. When a reverse voltage is applied (positive potential to the N-type area, and negative potential to the P-type area), current is blocked. At this time, it is not too important to understand the semiconductor physics behind this device, what is important is that the result of this P-N junction is that the diode acts as a one-way valve for current, only allowing it to flow when voltage is applied in a particular polarity.

Another characteristic of note about the diode is that it does require a particular voltage level before current will flow through the device. This is called the turn-on voltage and is required to generate enough of an electric field to allow electrons to flow from the N-type area to the P-type area.

### 13.1.1 Connecting a Diode

The circuit symbol for a diode is shown in figure 13.1. (Note the similarity to the symbol for a light-emitting diode in figure 3.5.)

anode —— cathode

Figure 13.1: Circuit symbol of a diode.

Current will flow only in the direction that the arrow points, from anode to cathode. An actual diode will generally have a stripe printed on it; that stripe corresponds to the cathode of the diode. An example is shown in figure 13.2.

anode ——— cathode



**Circuit 1:** Measure the current-voltage characteristics of a diode. First, connect the power supply to supply a positive voltage, as shown in figure 13.3 (left). Then reverse the power supply connection (without changing any of the other components; note the polarity on the ammeter and voltmeter in the circuit diagram!) to supply a negative voltage, as shown in figure 13.3 (right). You will record the voltage drop over the diode as well as the current flow through the diode under several different supply voltage values.



Figure 13.3: Diode circuit with positive supply voltage (left) and negative supply voltage (right).

Record your data in table 13.1. When you have finished collecting data, show it to your instructor to receive a stamp.

Vs (V)	Vdiode (V)	I (mA)	Vs (V)	Vdiode (V)	I (mA)	Vs (V)	Vdiode (V)	I (mA)
-5			-1			3		
_4			0			4		
-3			1			5		
-2			2					

 Table 13.1:
 Data table for diode circuit 1.

#### Instructor Stamp: \_\_\_\_

**Circuit 2:** Use the function generator to create a triangular wave with a frequency of 100 Hz,  $V_{MIN} = -5$  V, and  $V_{MAX} = 5$  V. Use this as the input wave to the circuit shown in figure 13.4, known as a waveform clipper.



Figure 13.4: Waveform clipper circuit.

Record your observations as you change the value of variable resistors  $R_1$  and  $R_2$ . Demonstrate your results to your instructor to receive a stamp.

### 13.1.2 Diode-Resistor Logic (DRL)

Diode-resistor logic (DRL) is the construction of Boolean logic functions using resistors and diodes. DRL was used in the beginning of computing due to the small size of diodes over vacuum tubes. However, they were eventually replaced by transistor logic due to the faster speed and lower power consumption of transistor-based logic devices. In addition, inverting operations are not possible using diodes.

Boolean logic uses values of ON and OFF, HIGH and LOW, or 1 and 0, corresponding to voltage levels (usually 5 V and 0 V). Two basic Boolean logic functions are AND and OR. These logic functions can be described using truth tables, which give output values for all possible input values. An AND gate is a logic gate that has an output of ON only when all inputs are ON, as described in table 13.2.

Α	В	Output
OFF	OFF	OFF
OFF	ON	OFF
ON	OFF	OFF
ON	ON	ON

Table 13.2: AND gate truth table.

An OR gate is a logic gate that has an output of ON when at least one input is ON, as described in table 13.3.

Α	В	Output	
OFF	OFF	OFF	
OFF	ON	ON	
ON	OFF	ON	
ON	ON	ON	

**Table 13.3:** OR gate truth table.

Circuit 3: Build the circuit shown in figure 13.5.



Figure 13.5: Diode-resistor logic circuit diagram.

Pushbuttons A and B correspond to the inputs. The 10 k $\Omega$  resistors are known as pull-down resistors and allow connections to both 5 V and 0 V while avoiding short circuits. The 1 M $\Omega$  resistor is part of the diode-resistor logic. When current flows through the resistor, the output is 5 V. Otherwise the output is 0 V. The 220  $\Omega$  resistor limits the current through the output LED. Fill out the truth table (table 13.4) corresponding to how the circuit functions based on the values of A and B. Demonstrate your results to your instructor to receive a stamp.

Α	В	Output

Table 13.4: Truth table for circuit 3.

**Circuit 4:** Build the circuit shown in figure 13.6.



Figure 13.6: Diode-resistor logic circuit diagram.

Pushbuttons A and B correspond to the inputs. The 100  $\Omega$  resistors are pull-down resistors. The 1 k $\Omega$  resistor is part of the diode-resistor logic. The 220  $\Omega$  resistor limits the current through the output LED. Fill out a truth table (table 13.5) corresponding to how the circuit functions based on the values of A and B. Demonstrate your results to your instructor to receive a stamp.

Α	В	Output

 Table 13.5:
 Truth table for circuit 4.

### 13.2 The Zener Diode

A Zener diode acts very similarly to a regular diode in the forward-bias (positive voltage) mode. However, the Zener diode is built to operate in the reverse-bias mode as well. At a particular value of voltage (known as the Zener voltage), the Zener diode will allow current to flow. The voltage will be regulated at that particular value as long as the output voltage is higher than the Zener voltage.

Zener diodes operate based on the principle of quantum tunneling. At high enough reverse voltages, electrons will tunnel through the P-N junction and cause current to flow. They are generally used to prevent overvoltages from occurring in circuits. The circuit symbol for a Zener diode is shown in figure 13.7. An actual Zener diode has a stripe corresponding to the cathode, the same as in figure 13.2.



Figure 13.7: Circuit symbol of a Zener diode.

**Circuit 5:** Measure the current-voltage characteristics of a Zener diode. First, connect the power supply to supply a positive voltage, as shown in figure 13.8 (left). Then reverse the power supply connection (without changing any of the other components) to supply a negative voltage, as shown in figure 13.8 (right). You will record the voltage drop over and current flow through the Zener diode under several different supply voltage values. Record your data in table 13.6. When you have finished collecting data, show it to your instructor to receive a stamp.



Figure 13.8: Zener diode circuit with positive supply voltage (left) and negative supply voltage (right).

Vs (V)	Vzener (V)	I (mA)	Vs (V)	Vzener (V)	l (mA)	Vs (V)	Vzener (V)	I (mA)
-9.0			-3.0			0.5		
-8.0			-2.5			1.0		
-7.0			-2.0			1.5		
-6.0			-1.5			2		
-5.0			-1.0			3		
-4.0			-0.5			4		
-3.5			0			5		

Table 13.6: Data table for diode circuit 5.

**Circuit 6:** Use the function generator to create a sinusoidal wave with a frequency of approximately 100 Hz and an amplitude of approximately 2 V. Use this as the input wave to the circuit shown in figure 13.9, known as a voltage regulator.



Figure 13.9: Voltage regulator circuit.

Vary the amplitude from approximately 2 V to approximately 10 V and record your observations about the output below.

Now replace the Zener diode with a normal diode, as shown in figure 13.10. This is a half-wave rectifier circuit.



Figure 13.10: Half-wave rectifier circuit.

Vary the amplitude from approximately 2 V to approximately 10 V and record your observations about the output below.

Demonstrate your results to your instructor to receive a stamp.

## Lab 13 Homework

Carefully read each question before answering. Show all work or justify your answers to receive credit. Attach a separate sheet of paper, if necessary, to show all work and calculations.

- 1. Use graphing software to create a graph of the current-voltage relationship for the diode that you measured in lab Circuit 1. Current should be on the x-axis and voltage on the y-axis. Ensure that the data points are clearly shown. The data will not be linear, so do not include a fit line. Ensure that all axes are labeled with correct units.
- 2. The following questions pertain to lab Circuit 2 (the waveform clipper circuit).
  - (a) Explain the purpose of this circuit.

(b) What is the purpose of resistor  $R_1$ ?

(c) What is the purpose of resistor  $R_2$ ?

3. What type of logic gate did you build in lab Circuit 3? How do you know?

4. What type of logic gate did you build in lab Circuit 4? How do you know?

- 5. Use graphing software to create a graph of the current-voltage relationship for the Zener diode that you measured in lab Circuit 5. Current should be on the x-axis and voltage on the y-axis. Ensure that the data points are clearly shown. The data will not be linear, so do not include a fit line. Ensure that all axes are labeled with correct units.
- 6. What is the advantage of using a regular diode over using a Zener diode?

7. What is the advantage of using a Zener diode over using a regular diode?

8. You are building a circuit where the output voltage can swing between 0 and 5 V, but cannot be larger than 5 V. Would you use a normal diode or a Zener diode to prevent excess voltage from traveling to the load? Justify your answer using the observations you made in lab Circuit 6.

# Pre-Lab 14

Carefully read the entirety of Lab 14, then answer the following questions. Attach a separate sheet of paper, if necessary, to show all work and calculations.

- 1. The following questions pertain to lab Circuit 1.
  - (a) Calculate the impedance of the load. Assume that each inductor has a parasitic resistance of 50  $\Omega.$

(b) Calculate  $V_a$  and express the value **in polar form**. ( $V_a$  is depicted in figure 14.1.) You will use this calculation to get an idea of what you should expect to see on the oscilloscope when you build the circuit in lab.

(c) Calculate the RMS value of the load current  $(I_{LOAD,RMS})$ .

(d) Use equation 14.2 to calculate the complex power consumed by the load.

(e) Calculate the value of the capacitor that should be placed in parallel with the load in order to increase the power factor of the circuit to one. (The placement of the compensating capacitor is depicted schematically in figure 14.2.)

- 2. The following questions pertain to lab Circuit 3. (Note: the only difference between Circuits 1 and 3 is the frequency of operation.)
  - (a) Calculate the impedance of the load. Assume that each inductor has a parasitic resistance of 50  $\Omega.$

(b) Calculate  $V_a$  and express the value **in polar form**. ( $V_a$  is depicted in figure 14.1.) You will use this calculation to get an idea of what you should expect to see on the oscilloscope when you build the circuit in lab.

(c) Calculate the RMS value of the load current ( $I_{LOAD,RMS}$ ).

(d) Use equation 14.2 to calculate the complex power consumed by the load.

(e) Calculate the value of the capacitor that should be placed in parallel with the load in order to increase the power factor of the circuit to one. (The placement of the compensating capacitor is depicted schematically in figure 14.2.)

- 3. The following questions pertain to lab Circuit 7.
  - (a) Calculate the Thévenin impedance  $(\mathbf{Z}_{TH})$  of the circuit shown in figure 14.6. (Assume that the parasitic resistance of the inductor is 50  $\Omega$ .)

(b) Calculate the load impedance  $(\mathbf{Z}_{\mathbf{L}})$  of the circuit that would lead to maximum power transferred to the load.

(c) What component values will need to be placed on the load to obtain the maximum transfer of power? Use the inventory of parts to determine what values are available. You may use series and parallel combinations to get as close as possible to the desired values.

(d) Based on the component values you chose above, calculate the average power that will be consumed by the load.
# Lab 14: AC Power

In this lab, we will explore the concept of complex power, which can occur when the voltage and current through a load are not in phase.

For lab resources and information, go to the following URL or scan the QR code. doctor-pasquale.com/circuit-analysis-lab-14



## 14.1 Complex Power

Complex power refers to both the real and imaginary components of power. This occurs due to a phase difference between current and voltage in AC circuits with complex impedances. Because impedance is frequency dependent, the real and imaginary components of AC power are also frequency dependent.

The components of AC power include the quantities described in table 14.1. Each quantity can be distinguished by its symbol and also by the units. While all units of AC power are technically the same, the units are differentiated to make each type of power more easily identifiable.

Quantity	Symbol	Units
Average power	Р	watts (W)
Reactive power	Q	volt-amp reactive (VAR)
Complex power	S	volt-amp (VA)
Apparent power	$ \mathbf{S} $	volt-amp (VA)
Power factor	pf	(none)

### Table 14.1: Components of AC power.

The equation used to calculate the power consumed by the load of a complex circuit element is defined by equation 14.1. From this value, all other quantities related to power can be derived.

$$\mathbf{S} = \mathbf{V}_{\mathbf{LOAD}, \mathbf{RMS}} \mathbf{I}_{\mathbf{LOAD}, \mathbf{RMS}}^* \tag{14.1}$$

In the first four circuits you will build in this lab, it is not possible measure both the load voltage and a voltage that is directly proportional to the current. Instead, you will measure only a voltage value that is proportional to current by measuring the voltage dropped over a small resistor in series with the load. Using Ohm's law, it will be possible to calculate the load current. Because the load voltage is not known, equation 14.1 can be rewritten in terms of current and impedance. This relationship is defined by equation 14.2.

$$\mathbf{S} = |\mathbf{I}_{\mathbf{LOAD},\mathbf{RMS}}|^2 \mathbf{Z}_{\mathbf{LOAD}}$$
(14.2)

## 14.2 Power Factor Correction

The power factor of a load relates to the ratio of average power to reactive power, as defined in equation 14.3.

$$pf = \frac{P}{|\mathbf{S}|} \tag{14.3}$$

When the power factor of a load is one, the current and voltage are in phase with each other and all of the power consumed by the load is real. Real power is power that is capable of doing useful work. Reactive power is simply power that oscillates back and forth from load to source. Reactive power is therefore not useful, but still costs money when electric power is billed by a power utility.

For this reason, it is generally desirable to increase the power factor of a load as much as possible. Circuits containing an inductive load (which exists in most consumer appliances that are powered by a motor) can have the reactance cancelled out or reduced by a capacitor placed in parallel. A circuit containing a capacitive load can have the reactance reduced by an inductor placed in parallel.

#### 14.2.1 Correcting an Inductive Load

To correct an inductive load and obtain a power factor of one, equation 14.4 will be used to determine the value of the compensating capacitor to be placed in parallel with the load, where  $Q_i$  is the initial reactive power consumed by the load of the uncompensated circuit,  $\omega$  is the angular frequency of the circuit, and  $|\mathbf{V}_{LOAD,RMS}|$  is the magnitude of the RMS value of the load voltage.

$$C = \frac{Q_i}{|\mathbf{V}_{\mathbf{LOAD},\mathbf{RMS}}|^2\omega} \tag{14.4}$$

In the first four circuits, you will not be able to measure the load voltage. For this reason, you will use Ohm's law to determine another way to calculate the value of the compensating capacitor, described in equation 14.5, where the load current and load impedance are used instead of load voltage.

$$C = \frac{Q_i}{|\mathbf{I}_{\mathbf{LOAD},\mathbf{RMS}}|^2 |\mathbf{Z}_{\mathbf{LOAD}}|^2 \omega}$$
(14.5)

#### 14.2.2 Correcting a Capacitive Load

To correct a capacitive load and obtain a power factor of one, equation 14.6 will be used to determine the value of the compensating inductor to be placed in parallel with the load, where  $Q_i$  is the initial reactive power consumed by the load of the uncompensated circuit,  $\omega$  is the angular frequency of the circuit, and  $|\mathbf{V}_{LOAD,RMS}|$  is the magnitude of the RMS value of the load voltage.

$$L = \frac{|\mathbf{V}_{\mathbf{LOAD},\mathbf{RMS}}|^2}{-Q_i\omega} \tag{14.6}$$

You will use this equation in circuits 5 and 6, where you will know the value of the load voltage. Therefore, there is no need to use Ohm's law to calculate this in terms of current and impedance.

**Circuit 1:** Collect components for the circuit shown in figure 14.1. Measure and record all component values (including the parasitic resistance of the inductor), and record these values in table 14.2.

Parameter	Value	Parameter	Value
L		R150	
RL		R10	

**Table 14.2:** Circuit 1 data table.

Build the circuit shown in figure 14.1. You will use the oscilloscope to monitor the input voltage  $(V_s)$  and ensure that its magnitude and frequency are correct. Because current cannot be directly measured using the oscilloscope, a 10  $\Omega$  resistor will be used to indirectly measure it. You will measure the voltage dropped over the 10  $\Omega$  resistor (labeled as  $V_a$  in figure 14.1) and use Ohm's law in the lab homework to calculate the current through the load. Because the oscilloscope and function generator both contain an Earth ground connection, the voltage dropped over the load cannot be measured in this circuit. (Fortunately, it does not need to be.)



**Figure 14.1:** Inductive load circuit diagram. The load is indicated with dashed lines. The placement of oscillo-scope probes is shown.

Using the oscilloscope, measure the amplitude of  $V_a$  and **convert to an RMS value**. Then use the cursors to measure the phase angle between  $V_a$  and  $V_s$ . ( $V_s$  and  $V_{load}$  will have nearly identical phase angles, so this is a reasonable measurement to take to determine the phase difference between load voltage and load current.) Record these values in table 14.3.

Parameter	Magnitude (RMS)	Phase
Va		

Table 14.3: Circuit 1 output data table.

When you have finished gathering data, show it to your instructor to receive a stamp.

**Circuit 2:** In the pre-lab, you should have calculated the value of the capacitor that would have to be placed in parallel with the load, as shown in figure 14.2, to compensate for the phase angle generated by the inductor. You may need to recalculate this value if assumptions made in the pre-lab about the parasitic resistance of the inductor are incorrect. Connect capacitors in series and parallel as needed to create this value. Wire the capacitor in parallel with the load, as shown in figure 14.2.



**Figure 14.2:** Inductive load circuit diagram with capacitive load correction. The load is indicated with dashed lines. The placement of oscilloscope probes is shown.

Record the amplitude and phase of  $V_a$  (convert the amplitude to an RMS value). Record this data in table 14.4. You may need to slightly modify your capacitance value in order to obtain a phase angle that is as close to 0 as possible. In table 14.4, record the final measured value of the capacitance that you use as  $C_{final}$ .

Parameter	Value
Va (RMS)	
Phase	
Cfinal	

Table 14.4: Circuit 2 output data table.

When you have finished gathering data, show it to your instructor to receive a stamp.

Circuit 3: Repeat the Circuit 1 activity (no compensating capacitance should be placed in parallel with the load) using a frequency of 1500 Hz. Measure the amplitude and phase of  $V_a$  (convert the amplitude to an RMS value) and record this data in table 14.5.

Parameter	Magnitude (RMS)	Phase
Va		

 Table 14.5:
 Circuit 3 output data table.

When you have completed recording this data, demonstrate it to your instructor to receive a stamp.

Instructor Stamp: \_\_\_\_

**Circuit 4:** In the pre-lab, you should have calculated the value of the capacitor that would have to be placed in parallel with the load, as shown in figure 14.2, to compensate for the phase angle generated by the inductor. You may need to recalculate this value if assumptions made in the pre-lab about the parasitic resistance of the inductor are incorrect. Connect capacitors in series and parallel as needed to create this value. Wire the capacitor in parallel with the load, as shown in figure 14.2.

Record the amplitude and phase of  $V_a$  (convert the amplitude to an RMS value). Record this data in table 14.6. You may need to slightly modify your capacitance value in order to obtain a phase angle that is as close to 0 as possible. In table 14.6, record the final measured value of the capacitance that you use as  $C_{final}$ .

Parameter	Value
Va (RMS)	
Phase	
Cfinal	

T.

Table 14.6: Circuit 4 output data table.

When you have finished gathering data, show it to your instructor to receive a stamp.

## 14.3 Mystery Complex Circuits

There are two mystery circuits that contain unknown complex loads, consisting of a resistance in series with either an inductance or a capacitance. Each was connected to current and voltage probes as shown in figure 14.3. (Note: the Vernier current and voltage probes are capable of measuring both current and voltage directly. They are not as good at measuring AC circuits as an oscilloscope, however, and will only be used for the next two circuits in this lab.)



**Figure 14.3:** Circuit diagram for complex mystery boxes, including both components necessary to create a complex load. The placement of the current and voltage probes is indicated.

At the conclusion of the previous lab, you determined the component values that were used in each mystery box and recorded them in tables 14.7 and 14.9.

For the two following circuits, you will determine the value of capacitance or inductance to place in parallel with the load to compensate for the reactive power and increase the power factor to one. This compensation circuit will be connected as shown in figure 14.4.



**Figure 14.4:** Circuit diagram for complex mystery boxes, including both components necessary to create a complex load. The placement of the compensating circuitry required to increase the power factor is shown. The placement of the current and voltage probes is indicated.

#### 14.3.1 Vernier Current and Voltage Probes

Because the frequency of operation is low enough, the Vernier current and voltage probes can be used to directly measure both load current and load voltage simultaneously in the following two circuits. Using the LabQuest Mini box, a sample rate of 10,000 samples/second can be obtained, which will be adequate for the following two circuits that operate at 500 Hz. Following is the procedure for using the Vernier probes and LoggerPro software.

- 1. Connect the voltage probe to channel 1 and the current probe to channel 2.
- 2. Open LoggerPro.

- 3. With nothing connected to the probes, zero both of them.
- 4. Build the circuit (figures 14.3 and 14.4) and connect the probes as shown in the circuit diagrams.
- 5. In LoggerPro, go to Experiment > Data Collection.
  - (a) Duration: enter 0.02 seconds
  - (b) Sampling rate: enter 10,000 samples/second
  - (c) Click on OK
- 6. Collect data.
- 7. Ensure that the voltage has the same value as the original circuit, otherwise the compensation values will be incorrect.
- 8. Autoscale the current data so that you can see it.
- 9. Use the curve fitting function and fit both voltage and current to  $A\cos(Bt+C) + D$ . (Note that the units of C are radians.)

**Circuit 5:** Record the measured values of voltage and current for mystery box A (**convert to RMS values**), as well as the calculated impedance value in table 14.7.

Parameter	Magnitude	Phase
V (VRMS)		
I (ARMS)		
Ζ (Ω)		

 Table 14.7:
 Mystery box A measurements prior to load compensation.

Calculate the actual component values used for the resistance and capacitance/inductance in the original mystery box, and record them below. (You will probably want to refer to the inventory of parts to confirm the components that were used.)

Build the circuit using the component values you determined and the schematic given in figure 14.3. You may want to use the Vernier current and voltage probes to verify that your resistance and capacitance/inductance calculations were correct before continuing.

Determine the circuit element(s) that must be placed in parallel with the load (as shown in figure 14.4) in order to increase the power factor of the load to one. Record the value of this circuit element below. (Be clear if it is a capacitor or inductor.)

What value of circuit element will you actually use in your circuit? If using series and parallel combinations, draw the circuit diagram of component connections. Each individual component should be a part that exists in the inventory of parts.

Build that circuit. Use the Vernier current and voltage probes to measure the new values of voltage and current (convert to RMS values). Record these values in table 14.8. Verify that the phase difference is as close to  $0^{\circ}$  as possible.

Parameter	Magnitude	Phase
V (VRMS)		
I (ARMS)		

 Table 14.8:
 Mystery box A measurements after load compensation.

When you have completed this circuit, demonstrate your results to your instructor to receive a stamp.

**Circuit 6:** Record the measured values of voltage and current for mystery box B (**convert to RMS values**), as well as the calculated impedance value in table 14.9.

Parameter	Magnitude	Phase
V (VRMS)		
I (ARMS)		
Ζ (Ω)		

Table 14.9: Mystery box B measurements prior to load compensation.

Calculate the actual component values used for the resistance and capacitance/inductance in the original mystery box, and record them below. (You will probably want to refer to the inventory of parts to confirm the components that were used.)

Build the circuit using the component values you determined and the schematic given in figure 14.3. You may want to use the Vernier current and voltage probes to verify that your resistance and capacitance/inductance calculations were correct before continuing.

Determine the circuit element(s) that must be placed in parallel with the load (as shown in figure 14.4) in order to increase the power factor of the load to one. Record the value of this circuit element below. (Be clear if it is a capacitor or inductor.)

What value of circuit element will you actually use in your circuit? If using series and parallel combinations, draw the circuit diagram of component connections. Each individual component should be a part that exists in the inventory of parts.

Build that circuit. Use the Vernier current and voltage probes to measure the new values of voltage and current (convert to RMS values). Record these values in table 14.10. Verify that the phase difference is as close to  $0^{\circ}$  as possible.

Parameter	Magnitude	Phase
V (VRMS)		
I (ARMS)		

 Table 14.10:
 Mystery box B measurements after load compensation.

When you have completed this circuit, demonstrate your results to your instructor to receive a stamp.

## 14.4 Maximum Power Transfer

Any linear complex circuit can be modeled as a Thévenin equivalent circuit, with a Thévenin equivalent voltage  $(V_{TH})$  and Thévenin equivalent impedance  $(Z_{TH})$ , as shown in figure 14.5.



Figure 14.5: Thévenin equivalent circuit for a linear complex circuit.

Maximum power will be transferred to any load placed between nodes a and b in the circuit when the load impedance is equal to the complex conjugate of the Thévenin equivalent impedance, as defined in equation 14.7.

$$\mathbf{Z}_{\mathbf{LOAD}} = \mathbf{Z}_{\mathbf{TH}}^* \tag{14.7}$$

The value of the maximum average power consumed by the load when equation 14.7 is satisfied is defined by equation 14.8.

$$P_{LOAD,MAX} = \frac{|\mathbf{V}_{\mathbf{TH},\mathbf{RMS}}|^2}{4R_{TH}}$$
(14.8)

**Circuit 7:** Using the circuit diagram in figure 14.6 as your guide, determine what load circuit must be placed in the circuit to lead to maximum power transferred by the load. **Be sure that you consider the parasitic resistance of the inductor in these values!** Measure the component values that you choose and record these values in table 14.11.

Parameter	Value
Rload	
Xload	

Table 14.11: Circuit 4 component data table.



**Figure 14.6:** Complex circuit diagram for maximum power transfer to the load. The placement of oscilloscope probes is shown.

Build the circuit (shown in figure 14.6) with the load components that you measured. Use the oscilloscope to measure the voltage dropped over the load (depicted as  $V_{load}$ ), as well as  $V_a$ , which is proportional to the current through the load. Convert these values to RMS and record them in table 14.12. Use the cursors to determine the phase difference between the two voltages and record that value in table 14.12 as well.

Parameter	Value
Vload (RMS)	
Va (RMS)	
Phase	

T.

 Table 14.12:
 Circuit 7 output data table.

When you have completed this circuit, demonstrate your results to your instructor to receive a stamp.

# Lab 14 Homework

Carefully read each question before answering. Show all work or justify your answers to receive credit. Attach a separate sheet of paper, if necessary, to show all work and calculations.

- 1. Use component values that were measured and recorded in table 14.2 to answer the following questions about lab Circuit 1. Use the output voltage data that you collected in table 14.3.
  - (a) Use Ohm's law to calculate the complex value of the current flowing through the load using RMS values.

(b) Calculate the impedance of the load.

(c) Use equation 14.2 to calculate the complex power consumed by the load. Express the value in Cartesian form so that it is in terms of P + jQ.

(d) Calculate the power factor.

- 2. Use component values that were measured and recorded in table 14.2 to answer the following questions about lab Circuit 2. Use the output voltage data that you collected in table 14.4.
  - (a) Use Ohm's law to calculate the complex value of the current flowing through the load (which now contains the compensating capacitor) using RMS values.

(b) Calculate the impedance of the load (which now contains the compensating capacitor).

(c) Use equation 14.2 to calculate the complex power consumed by the load (which now contains the compensating capacitor). Express the value in Cartesian form so that it is in terms of P + jQ.

(d) Calculate the power factor. (As this circuit contained a compensating capacitor, this value should be close to one. If it is not, the most likely issue is a math error in part a or b of this question.)

- 3. Use component values that were measured and recorded in table 14.2 to answer the following questions about lab Circuit 3. Use the output voltage data that you collected in table 14.5.
  - (a) Use Ohm's law to calculate the complex value of the current flowing through the load using RMS values.

(b) Calculate the impedance of the load.

(c) Use equation 14.2 to calculate the complex power consumed by the load. Express the value in Cartesian form so that it is in terms of P + jQ.

(d) Calculate the power factor.

- 4. Use component values that were measured and recorded in table 14.2 to answer the following questions about lab Circuit 4. Use the output voltage data that you collected in table 14.6.
  - (a) Use Ohm's law to calculate the complex value of the current flowing through the load (which now contains the compensating capacitor) using RMS values.

(b) Calculate the impedance of the load (which now contains the compensating capacitor).

(c) Use equation 14.2 to calculate the complex power consumed by the load (which now contains the compensating capacitor). Express the value in Cartesian form so that it is in terms of P + jQ.

(d) Calculate the power factor. (As this circuit contained a compensating capacitor, this value should be close to one. If it is not, the most likely issue is a math error in part a or b of this question.)

5. Assuming that there is an inductive load in an AC circuit, how does the frequency affect the value of the capacitor that would need to be used to cancel out the phase difference between voltage and current?

- 6. The following questions pertain to lab Circuit 5, mystery box A, **before** the power factor correction. Use the data that you collected in table 14.7.
  - (a) Use equation 14.1 to calculate the complex power consumed by the load. Express the value in Cartesian form so that it is in terms of P + jQ.

(b) Calculate the power factor.

- 7. The following questions pertain to lab Circuit 5, mystery box A, **after** the power factor correction. Use the data that you collected in table 14.8.
  - (a) Use equation 14.1 to calculate the complex power consumed by the load (which now contains the compensating circuit element). Express the value in Cartesian form so that it is in terms of P + jQ.

(b) Calculate the power factor. (As this circuit was compensated, this value should be close to one. If it is not, the most likely issue is a math error in part a or b of this question.)

- 8. The following questions pertain to lab Circuit 6, mystery box B, **before** the power factor correction. Use the data that you collected in table 14.9.
  - (a) Use equation 14.1 to calculate the complex power consumed by the load. Express the value in Cartesian form so that it is in terms of P + jQ.

(b) Calculate the power factor.

- 9. The following questions pertain to lab Circuit 6, mystery box B, **after** the power factor correction. Use the data that you collected in table 14.10.
  - (a) Use equation 14.1 to calculate the complex power consumed by the load (which now contains the compensating circuit element). Express the value in Cartesian form so that it is in terms of P + jQ.

(b) Calculate the power factor. (As this circuit was compensated, this value should be close to one. If it is not, the most likely issue is a math error in part a or b of this question.)

- 10. Use component values that were measured and recorded in table 14.11 to answer the following questions about lab Circuit 7. Use the output voltage data that you collected in table 14.12.
  - (a) Use Ohm's law to calculate the magnitude of the current flowing through the load using RMS values. Then use the phase measurement to express the RMS current in polar form.

(b) Use equation 14.1 to calculate the complex power consumed by the load. Express the value in Cartesian form so that it is in terms of P + jQ.

(c) Based on the theoretical maximum average power calculated in the pre-lab, what percentage of maximum load power did your circuit obtain? (You may need to recalculate the pre-lab value if the parasitic resistance is much different from the assumption made in the pre-lab.)

(d) If your percentage is less than 90%, first check that your calculations in parts a and b of this question are correct. If they are indeed correct, explain the most likely reasons why the average power consumed by the load was not as high as expected. (Human error is not a reason.)

(e) If your percentage is less than 90%, what could be done to the load circuit to increase the percentage?

# Pre-Lab 15

Carefully read the entirety of Lab 15, then answer the following questions. Attach a separate sheet of paper, if necessary, to show all work and calculations.

- 1. The following questions refer to lab Circuit 2.
  - (a) What is the frequency of the desired output waveform?

(b) What is the amplitude of the desired output waveform?

(c) What is the DC offset of the desired output waveform?

(d) Use superposition to calculate values for resistors  $R_1$  and  $R_2$ .

- 2. The following questions refer to lab Circuit 3.
  - (a) You will be asked to build an op-amp with a voltage gain of -3. Draw a circuit diagram that will accomplish this. Consult the inventory of parts to determine what resistor values are available. Note: op-amps can only source and sink approximately 25 mA of current. Your choice of resistor for this circuit cannot cause an op-amp output current of > 25 mA or < -25 mA. Feel free to confirm these calculations with CircuitJS.

(b) The input voltage is defined by equation 15.1. Sketch what the output wave should look like. Be sure to label the axes with appropriate values for scale.



- 3. The following questions refer to lab Circuit 4.
  - (a) You will be asked to build an op-amp with a voltage gain of 1.5. Draw a circuit diagram that will accomplish this. Consult the inventory of parts to determine what resistor values are available. Note: op-amps can only source and sink approximately 25 mA of current. Your choice of resistor for this circuit cannot cause an op-amp output current of > 25 mA or < -25 mA. Feel free to confirm these calculations with CircuitJS.

(b) The input voltage is defined by equation 15.2. Sketch what the output wave should look like. Be sure to label the axes with appropriate values for scale.



4. Read lab Circuit 8. You will be asked to build an RC circuit with a charging transient response defined by equation 15.3. Calculate the values of  $R_1$ ,  $R_2$ , and C that are required to achieve this response. Consult the inventory of parts to determine what resistor and capacitor values are available. Use only one single available resistor value for  $R_1$ , one single available resistor value for  $R_2$ , and one single available capacitor value for C.

# Lab 15: Course Review

In this lab, we will review many of the concepts that were explored in class this semester.





**Circuit 1:** Obtain one of the four mystery resistor boxes and distinctly record the box number somewhere on this page. The four following schematics shown in figure 15.1 indicate the arrangement of resistors inside of each box. All resistors are 10 k $\Omega \pm 5\%$ . Take as many measurements as you need to determine the arrangement of your chosen mystery box. You will need to justify this answer in your lab homework.



Figure 15.1: Mystery resistor arrangements.

**Circuit 2:** In the pre-lab, you should have derived values for both resistors to obtain the desired output waveform. Record these values in table 15.1.

Parameter	Value
R1	
R2	

 Table 15.1:
 Circuit 2 data table.

Build the circuit, as shown in figure 15.2 using a DC power supply and function generator as sources. (Note that the triangle wave used as an input will have no DC component; the DC component of the output signal is produced entirely by the DC power supply.) Measure the output voltage on an oscilloscope.



Figure 15.2: Circuit diagram for the superposition circuit.

When the output voltage matches that given in the graph in figure 15.3, demonstrate it to your instructor to receive a stamp.



Figure 15.3: Desired output waveform for the superposition circuit.

Circuit 3: Design an op-amp circuit that has a voltage gain of -3. Use a dual supply of at least  $\pm 12$  V to source the op-amp. The input voltage should be a sinusoidal wave defined by equation 15.1.

$$v(t) = 1.5 \text{ V} \cos(2\pi 100t) + 1.5 \text{ V}$$
 (15.1)

Draw the circuit diagram below.

Build the circuit and demonstrate the output to your instructor to receive a stamp.

#### Instructor Stamp: \_\_\_\_\_

**Circuit 4:** Design an op-amp circuit that has a voltage gain of 1.5. Use a dual supply of at least  $\pm 12$  V to source the op-amp. The input voltage should be a sinusoidal wave defined by equation 15.2.

$$v(t) = 3 \text{ V} \cos(2\pi 100t) + 3 \text{ V}$$
 (15.2)

Draw the circuit diagram below.

Build the circuit and demonstrate the output to your instructor to receive a stamp.

**Circuit 5:** Using as few components as possible, design and build a circuit with an equivalent resistance of 620 k $\Omega$ . Draw the circuit diagram below and demonstrate the results to your instructor to receive a stamp.

Instructor Stamp: \_\_\_\_\_

**Circuit 6:** Using as few components as possible, design and build a circuit with an equivalent inductance of 75 mH. Draw the circuit diagram below and demonstrate the results to your instructor to receive a stamp.

Instructor Stamp: \_\_\_\_\_

**Circuit 7:** Using as few components as possible, design and build a circuit with an equivalent capacitance of 330 nF. Use only ceramic capacitors. Draw the circuit diagram below and demonstrate the results to your instructor to receive a stamp.

**Circuit 8:** In the pre-lab, you determined the resistance and capacitance values necessary to build an RC circuit with a charging transient response defined by equation 15.3.

$$v(t) = 6 V (1 - e^{-400t}) u(t)$$
 (15.3)

Record the values in table 15.2.

Parameter	Value
R1	
R2	
С	

Table 15.2: Circuit 8 data table.

Build the circuit, as shown in figure 15.4 and display both the input and output waveforms on the oscilloscope. Use an input wave frequency of approximately 20 Hz. (Any frequency that allows the circuit to fully charge to its steady-state value is acceptable.) Trigger the oscilloscope on a rising edge, and put cursors at v(0) and  $v(\tau)$  to demonstrate the time constant of the output signal. When the output waveform matches the given equation, demonstrate it to your instructor to receive a stamp.



Figure 15.4: Circuit diagram for the RC circuit.

Circuit 9: Gather the components to build the circuit shown in figure 15.5.



Figure 15.5: Circuit diagram for a second order circuit.

Measure the values of the resistance, inductance, parasitic resistance, and capacitance. Record the values in table 15.3.

Parameter	Value
R18	
L	
RL	
С	

 Table 15.3:
 Circuit 9 data table.

Build the circuit **without the potentiometer** (the only resistance is from the 18  $\Omega$  resistor). Display both the input and output waveforms on the oscilloscope. Use an input wave frequency of approximately 10 Hz. (Any frequency that allows the transient response to fully discharge is acceptable.) Trigger the oscilloscope on a falling edge. Measure and record the frequency of any oscillations in the output signal caused by the RLC transient response.

Insert the potentiometer in the circuit as shown in figure 15.5. Dial the pot until you obtain a critically damped response. Measure and record the resistance value of the potentiometer in this situation.

Demonstrate your results to your instructor to receive a stamp.

# Lab 15 Homework

Carefully read each question before answering. Show all work or justify your answers to receive credit. Attach a separate sheet of paper, if necessary, to show all work and calculations.

- 1. Draw the arrangement of resistors that exists in your mystery box in Circuit 1. Justify this answer.
- 2. The following questions pertain to lab Circuit 9.
  - (a) Calculate the resonant frequency,  $\omega_0$ .
  - (b) Calculate the oscillation frequency,  $\beta$ , of the circuit **without** the potentiometer.
  - (c) Convert this value to a frequency with units of Hz, and **meaningfully** compare it to the frequency of any oscillations that you measured.
  - (d) Calculate the value of  $\alpha$  from the circuit **without** the potentiometer. Is this circuit overdamped, critically damped, or underdamped? How do you know?
  - (e) Calculate the value of  $\alpha$  from when you had the potentiometer dialed to a critically damped response.
  - (f) Meaningfully compare this to the value of  $\omega_0$  that you calculated in part a. If the value is off by more than 10%, how would you change the value of the potentiometer to achieve a more critically damped response?